Selfconsistent study of genuine ternary fission in (super-)heavy nuclei

Yannen Jaganathen, Janusz Skalski (NCBJ, Warsaw)

Based on: *Phys. Lett. B 868 (2025) 139693*

NARODOWE CENTRUM BADAŃ JĄDROWYCH ŚWIERK

UNIWERSYTET WARSZAWSKI

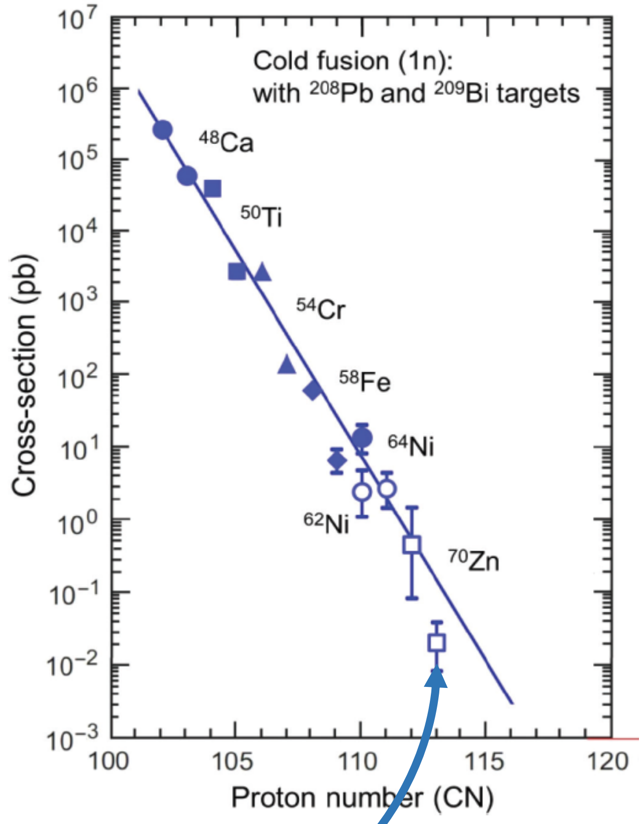
My other project at NCBJ Warsaw: Fusion mechanism

Super-heavy elements with Z > 103 do not occur in nature.
They can only be produced in the laboratory by fusing two lighter nuclei.

Our research project is to gain insight step by step into the fusion reaction mechanisms, in particular on the understanding of the **hindrance mechanism**, **which prevents the formation of super-heavy nuclei**.

→ **A macro-micro model to explain a well-known phenomena.**

Y. Jaganathen, M. Kowal, K. Pomorski, Phys. Lett. B 862 (2025) 139302



Z = 113, 22fb
(Only 3 atoms in 576 days of irradiation)

My other project at NCBJ Warsaw: Fusion mechanism

Thesis topics in Physics

Application deadline: 6th January 2026

- **How the Heaviest Elements Are Born: A New Dynamical Picture of Superheavy Nucleus Formation**
  - **Supervisor**: dr hab. Michał Kowal
  - **Auxiliary Supervisor**: dr Yannen Jaganathen
  - **Description:** The proposed PhD project addresses one of the central unresolved questions of modern nuclear physics: why only a tiny fraction of heavy-ion collisions succeed in forming superheavy nuclei, while most end prematurely in quasifission. The project introduces a novel dynamical perspective in which fusion is understood not as a single barrier crossing, but as a rapidly evolving process governed by the interplay of dissipation, inertia, shape evolution, and mass transfer immediately after the nuclei touch.
    The results of this research will provide new microscopic insight into the mechanisms that control the formation of the heaviest elements, with direct relevance for future synthesis efforts of nuclei around Z=119–120.
    The project will be carried out in scientific collaboration with the world-leading laboratories conducting superheavy-element research:

Contact: michal.kowal@ncbj.gov.pl (Prof. M. Kowal)

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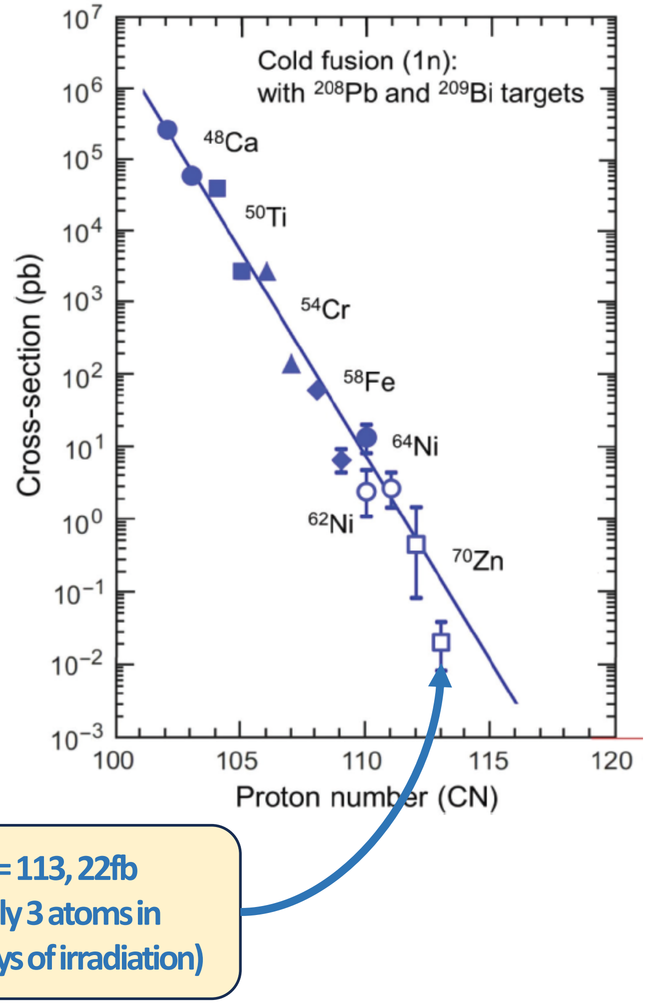
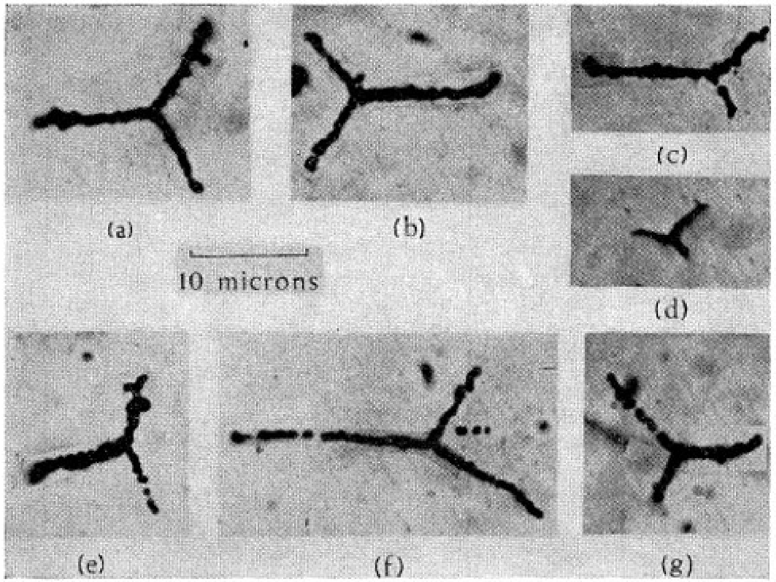


Today's seminar: Self-consistent description of ternary fission

Low-energy ternary fission is a **rare process** that is always **dominated by binary fission**.
The additional fragment is **generally an alpha particle**.

The existence of **genuine ternary fission** with a **heavier fragment** remains an **open question**.

Previous studies rely mostly on **macroscopic-microscopic** or **simplified models**. **We present a self-consistent Hartree-Fock + BCS study with Skyrme-like interactions**



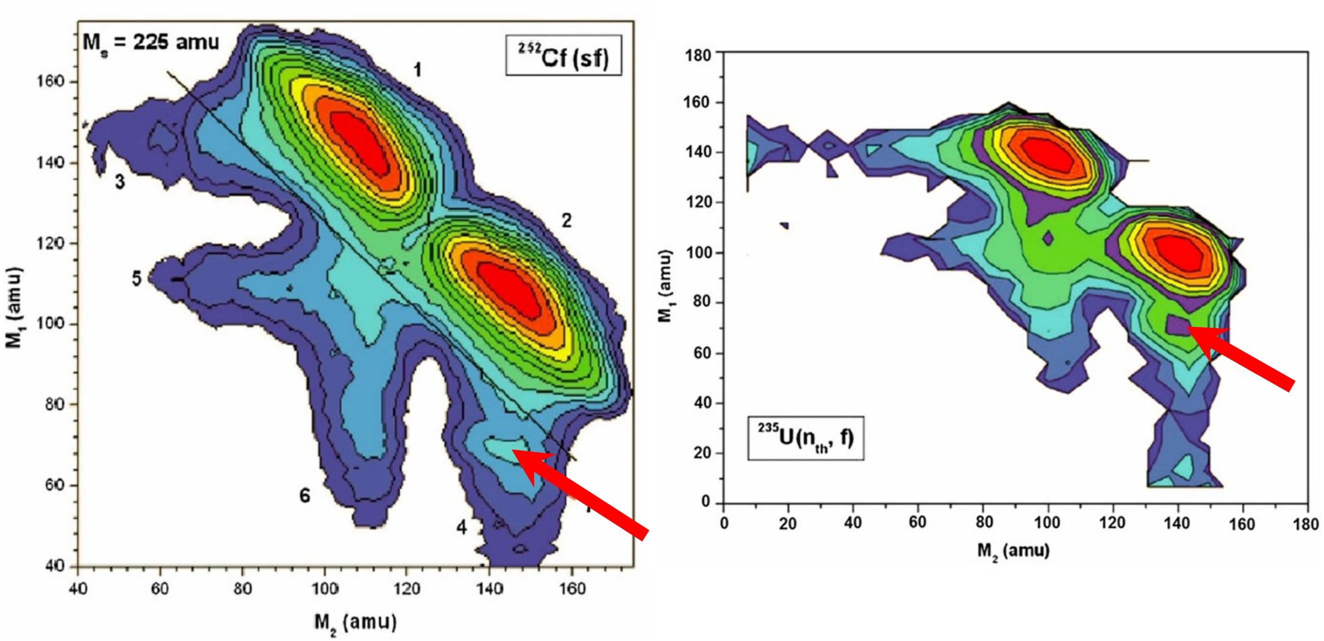
Non-genuine ternary fission
(Emission of an alpha particle)
Muga, Bowman, Thompson, Phys. Rev. 121 (1961) 270

Plan

1. **Genuine ternary fission** and (partial) **experimental evidence**
2. **Selected micro-macro results** up to now
3. **Collinear tripartition** in heavy nuclei (^{252}**Cf** and ^{236}**U**)
   - The **mean-field approach**: Constraints and energy landscapes
   - Estimation of **tripartition probabilities**
4. **Collinear and equatorial tripartition** in superheavy nuclei
5. **Conclusions**

The Dubna experiments

- Dubna experiments[1,2] reported **hints of true ternary mode in ^{252}Cf and ^{236}U (neutron induced), with unexpectedly high relative yields $10^{-3} - 10^{-4}$** w.r.t. binary fission



Mass-mass correlations obtained from the coincidences of the fragments detected by the two arms of the FOBOS detector

[1] Pyatkov et al., Phys. Rev. C 96 (2017) 064606
[2] W. von Oertzen, A. K. Nasirov, Eur. Phys. J. A 56 (2020) 299

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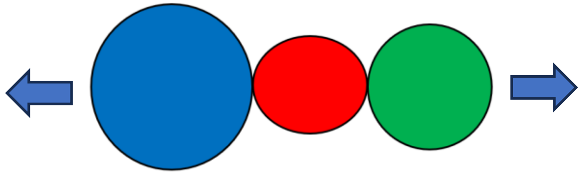

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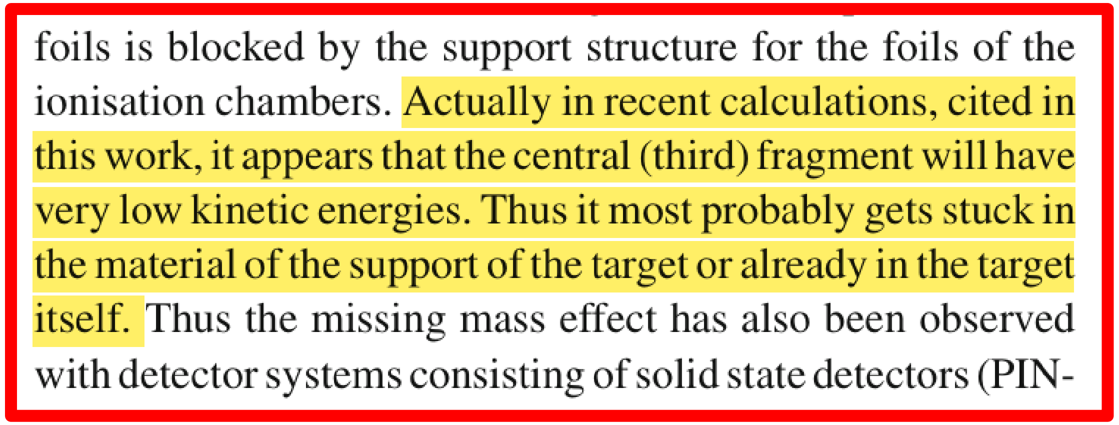
- The **lightest fragment** (Ca, S, Si) **was never detected** due to its supposedly **low velocity** which most probably arises from the **linear configuration**.


foils is blocked by the support structure for the foils of the ionisation chambers. Actually in recent calculations, cited in this work, it appears that the central (third) fragment will have very low kinetic energies. Thus it most probably gets stuck in the material of the support of the target or already in the target itself. Thus the missing mass effect has also been observed with detector systems consisting of solid state detectors (PIN-

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Selected macro-micro results

- **Karpov's macro-micro description within a three-center shell model**
  - "Presumably" sequential, with a **well-deformed second neck after the first scission** (tripartition)
  - Predicts a barrier of only a few MeV for ^{252}Cf [1]
  - No barrier in the theoretical superheavy nucleus ^{296}Lv [2]

- **Shell effects/magic numbers** "may enhance the probability of true ternary fission":
  - $^{252}\mathrm{Cf} \rightarrow {}^{132}_{50}\mathrm{Sn}_{82} + {}^{48}_{20}\mathrm{Ca}_{28} + {}^{72}_{28}\mathrm{Ni}_{44}$
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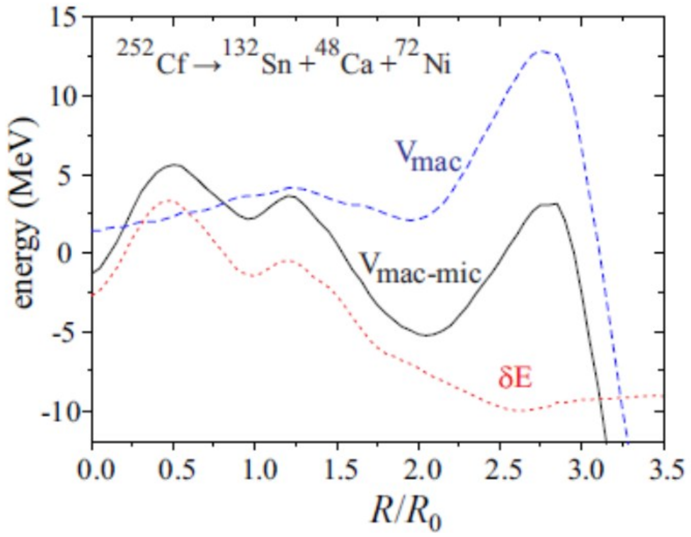

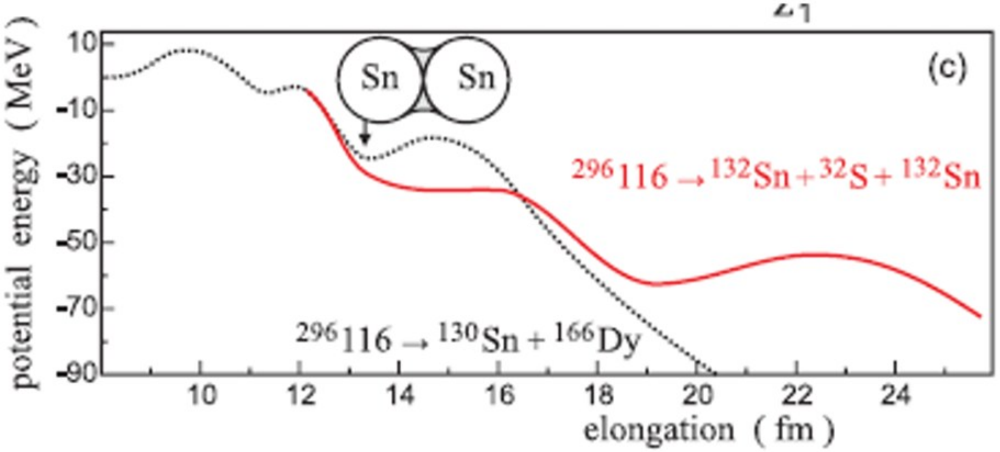

[1] A. V. Karpov, Phys. Rev. C, 94 (2016) 064615
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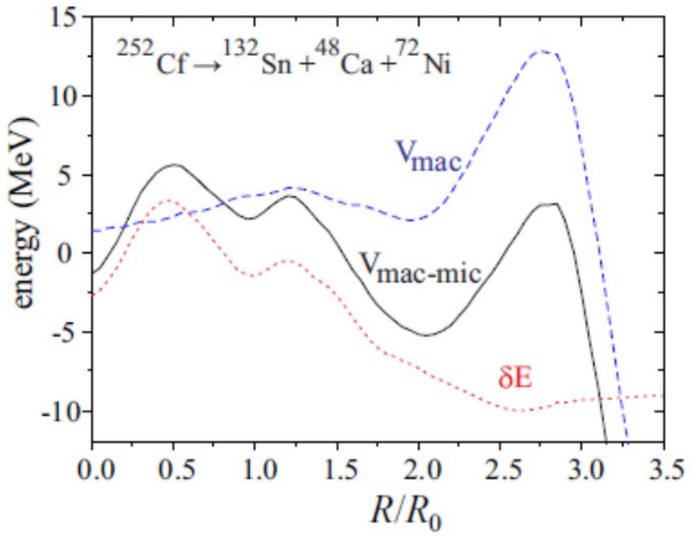
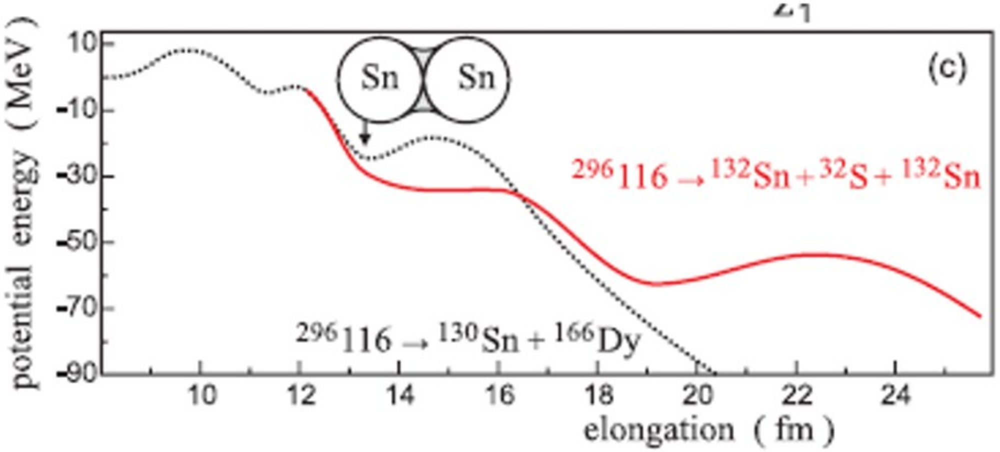
- **How much can a self-consistent model reduce the barrier?**





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The Framework

- **HF + BCS procedure with the Skyrme SLy6** effective interaction and a delta pairing interaction (state-dependent gaps)
- **Known to perform well in the studied regions**

- Minimization procedure:

$$\delta\left(\left\langle \phi \;\middle|\; \hat{H} - \sum_i \lambda_i \hat{Q}_i + \cdots \;\middle|\; \phi \right\rangle\right) = 0$$

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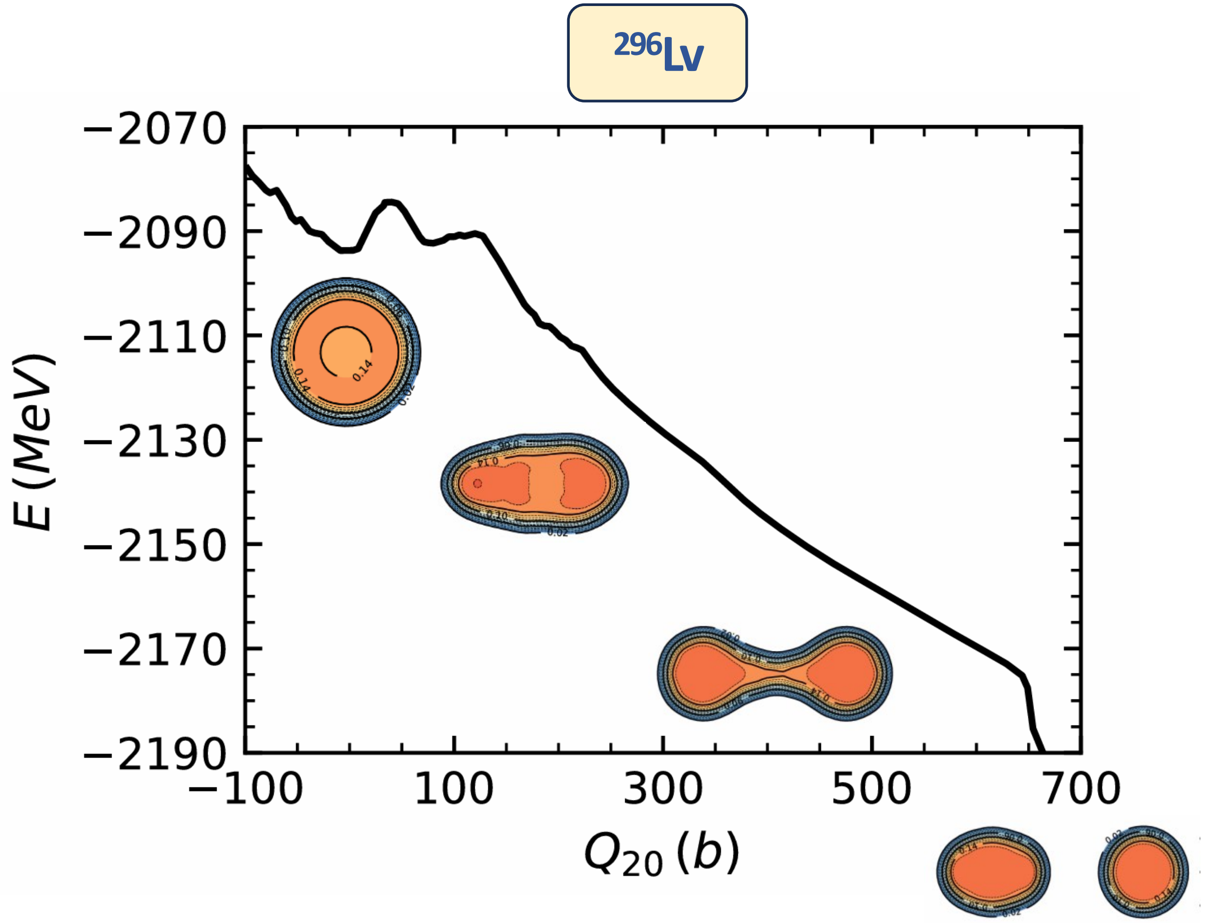
  - **Q_{20} ↔ Elongation**



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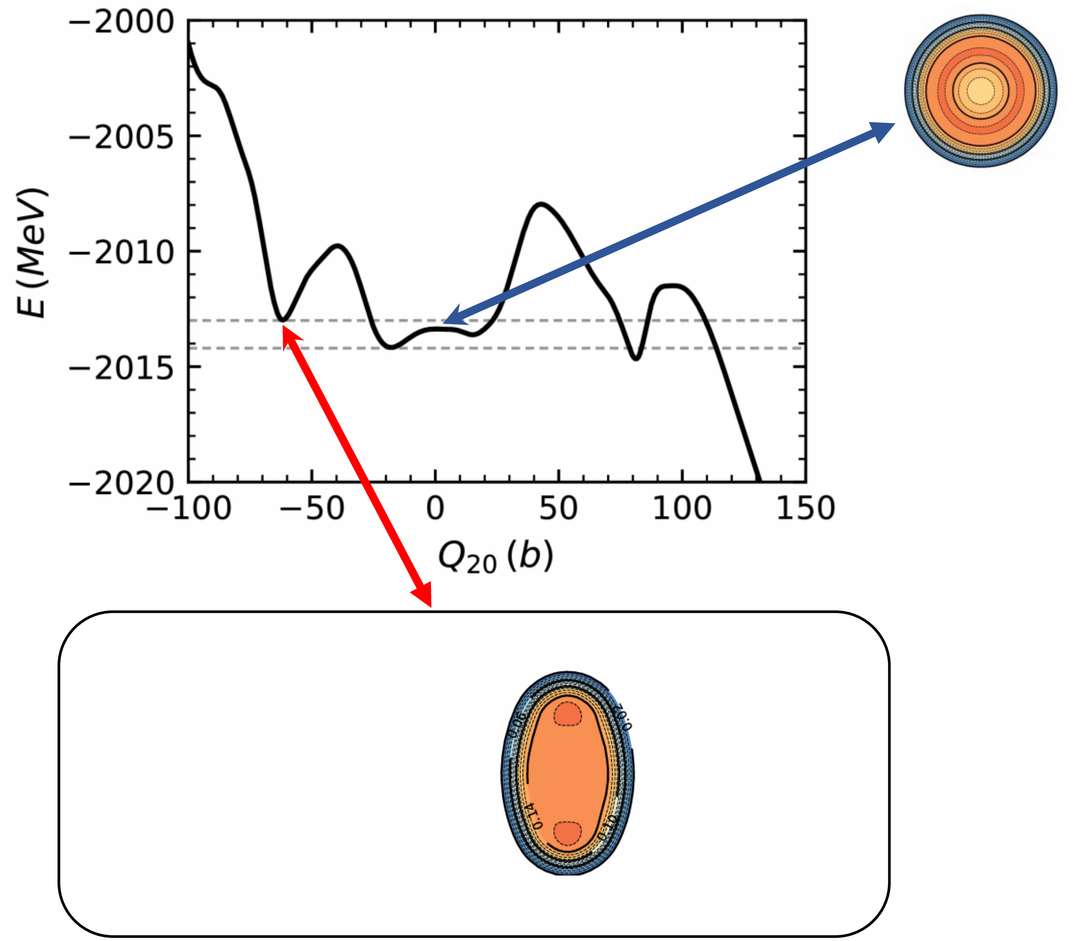
^{288}Ubn



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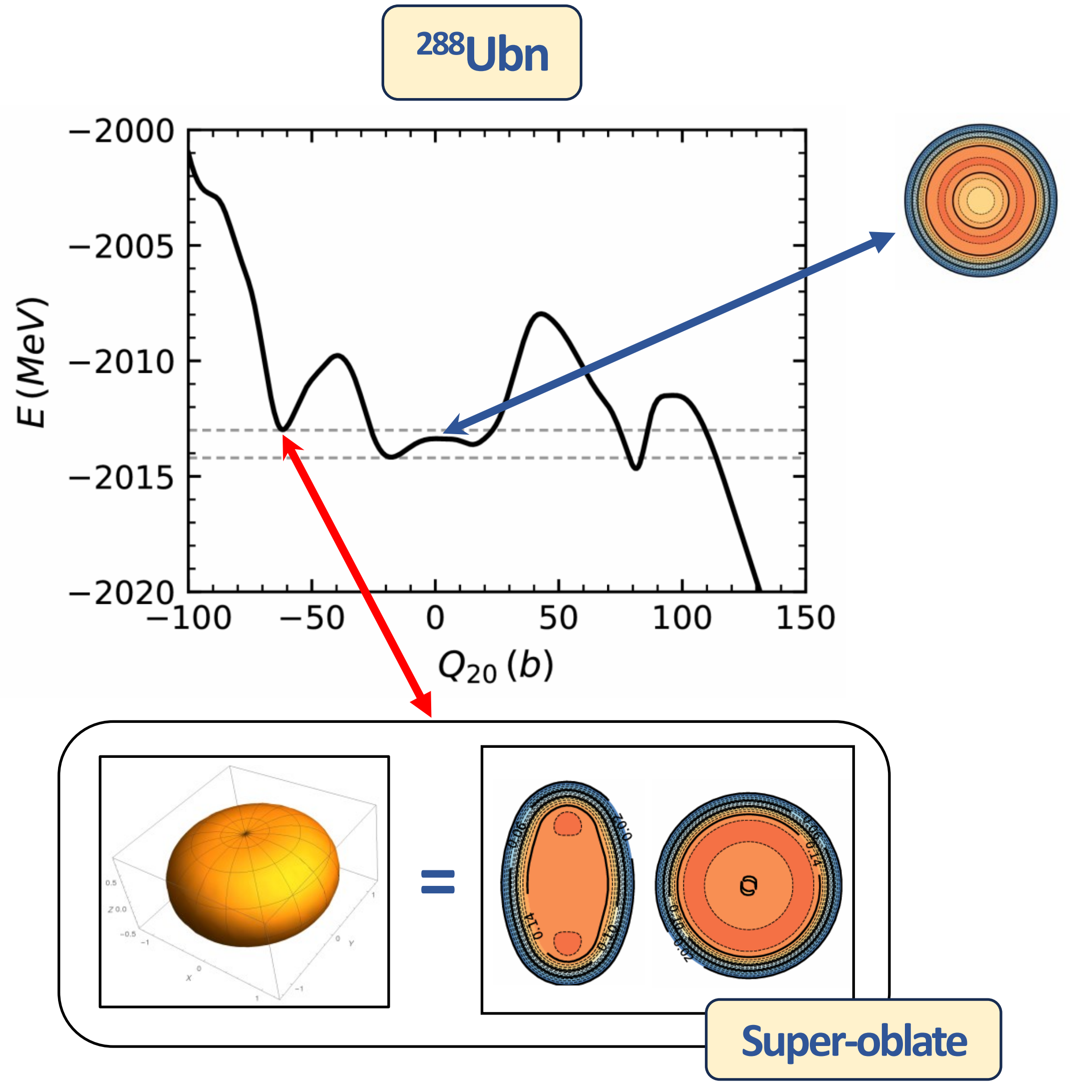
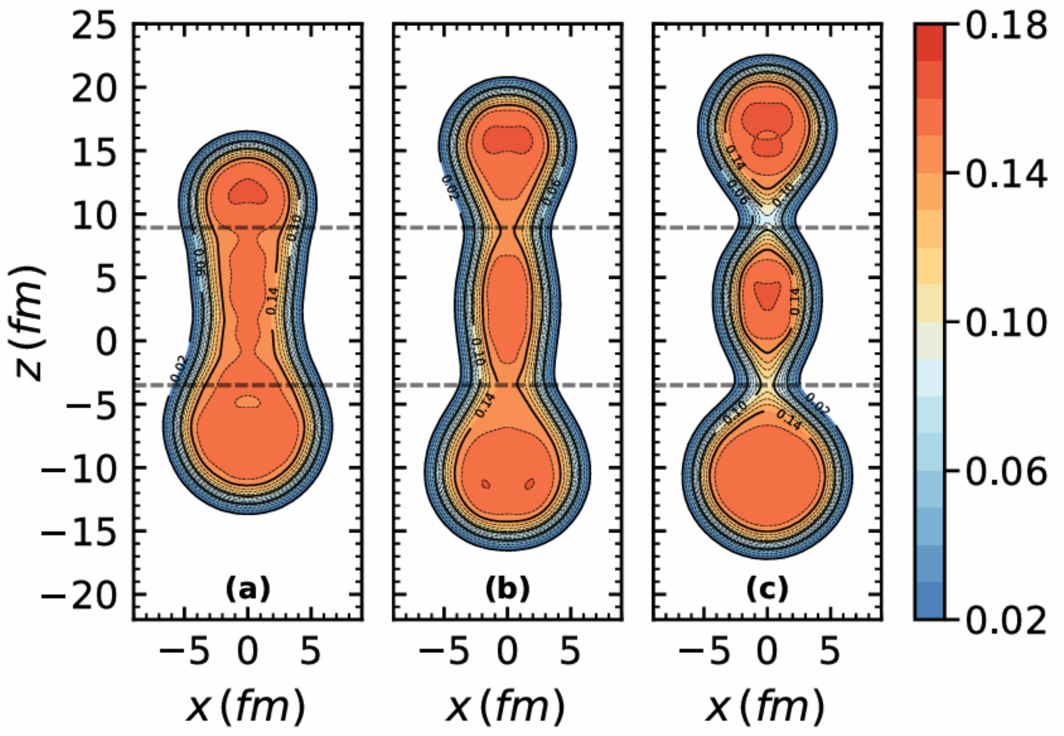


Constraints and Forbidden Shapes

- **Neck constraint:**
  $\hat{Q}_{n,1,2} = \exp\left(-\left(\frac{z-z_{1,2}}{a_0}\right)^2\right)$, with the width $a_0 = 1$ fm.
- **Formation of the necks when $Q_n \to 0$**



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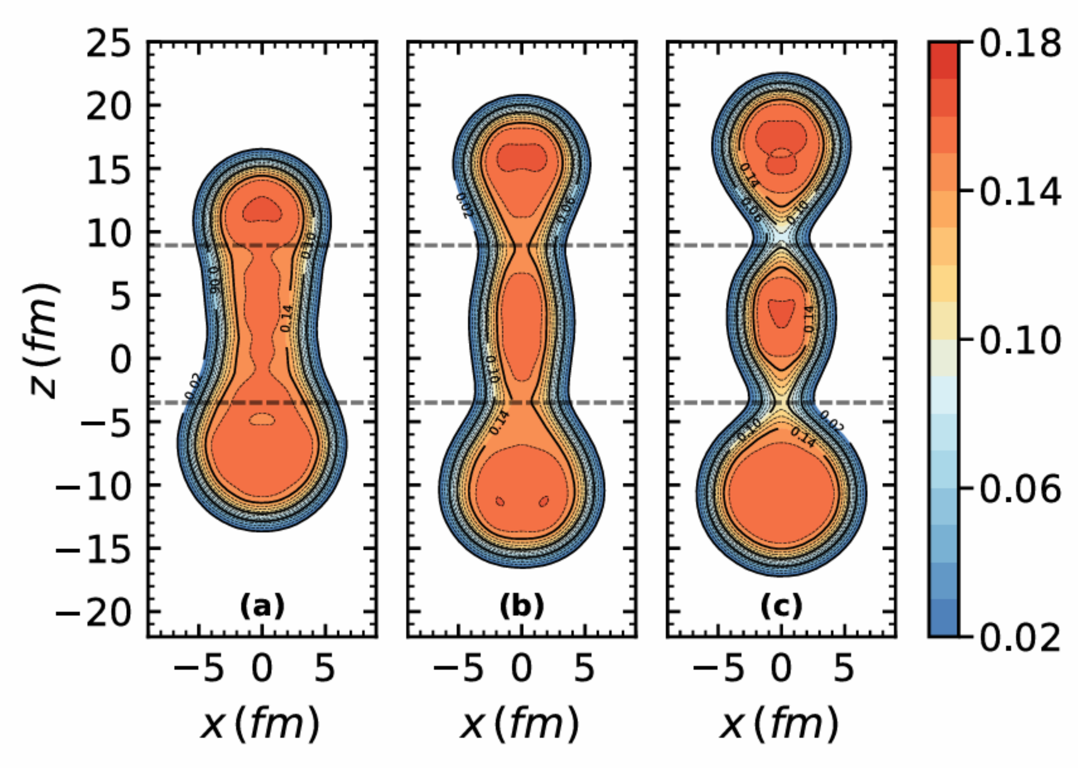
  - **Center-of-mass + quadrupole moment** Q_{20}
  - **Two correlated neck constraints**
  - Constraint **on A_{mid} to prevent bipartition (leakage)**
  - Constraint on the **distance between the two necks:**

    $\mathbf{z_2 - z_1 = d_{sph} + \delta, \delta = 0, 2, 4, 6\ fm}$



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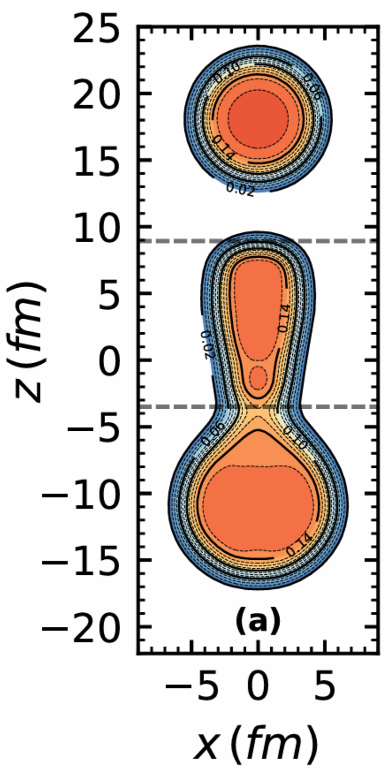
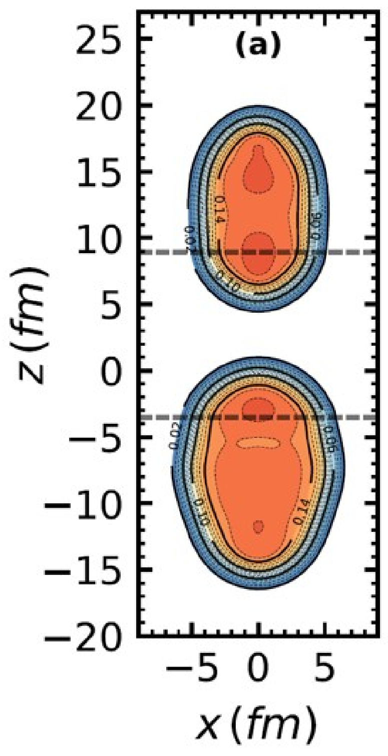


Type I



Type II

Constraints and Forbidden Shapes

[1] Y. Jaganathen, J. Skalski, Phys. Lett. B 868 (2025) 139693

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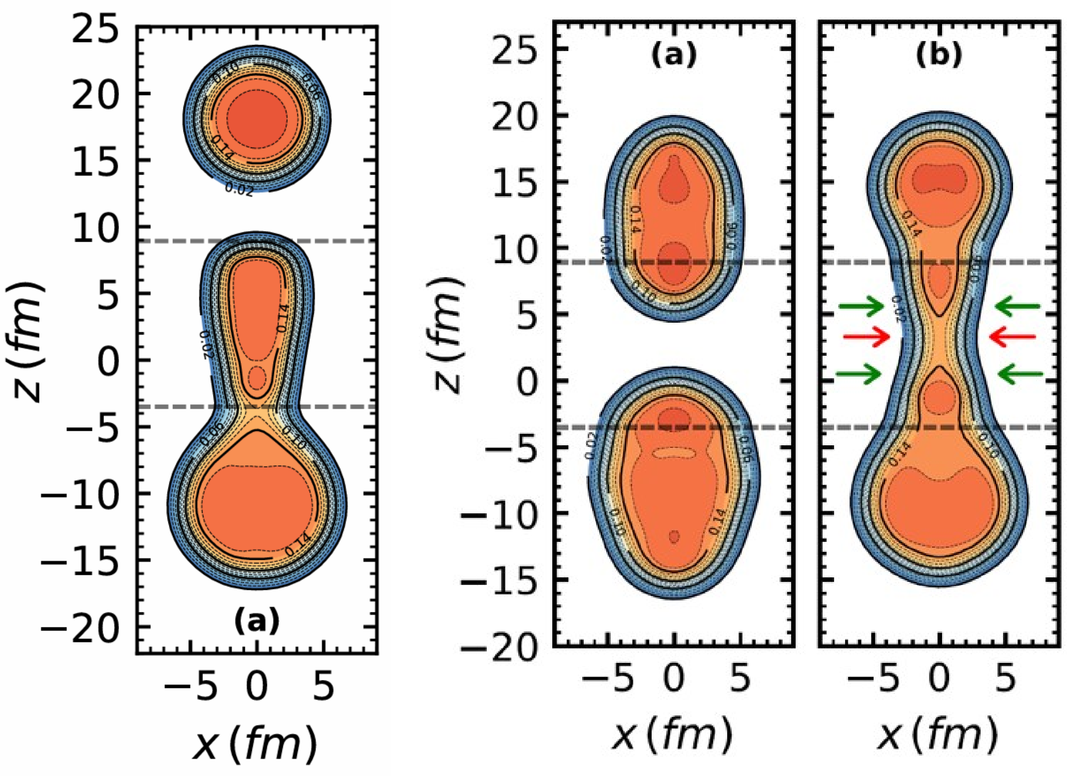


Type I

Type II

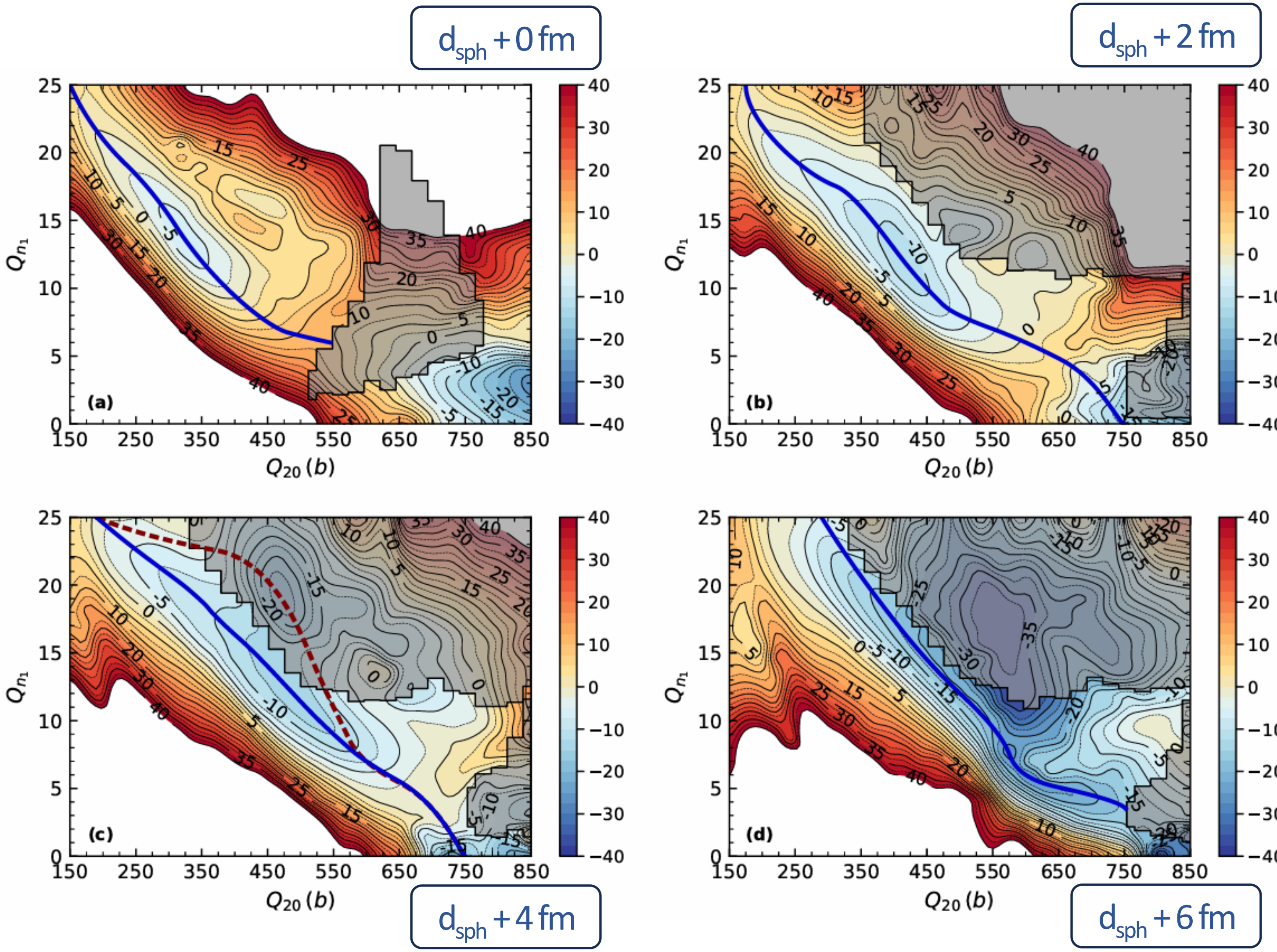
(Double)-neck vs Elongation maps – The ^{252}Cf test case

- $^{252}\mathrm{Cf} \rightarrow {}^{132}_{50}\mathrm{Sn}_{82} + {}^{48}_{20}\mathrm{Ca}_{28} + {}^{72}_{28}\mathrm{Ni}_{44}$

- **Energies w.r.t the ground state**
- **Shaded areas = forbidden shapes**

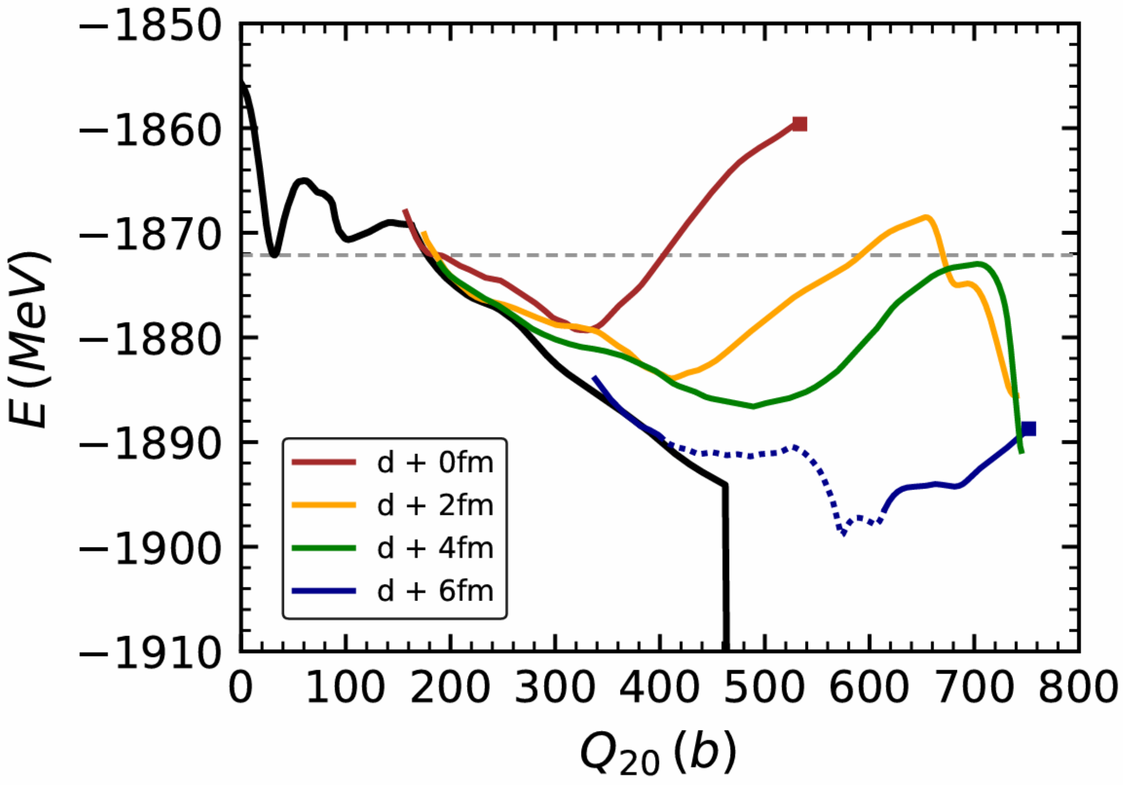
- **The trajectories are close/connected to the forbidden area**

- No energy barrier at $d_{sph} + 4$ fm?



Spontaneous tripartition?

- Connection to the **super-elongated path to binary fission**
- For a neck distance of d + 4fm, there is **no barrier to tripartition**
  → **Spontaneous tripartition**?



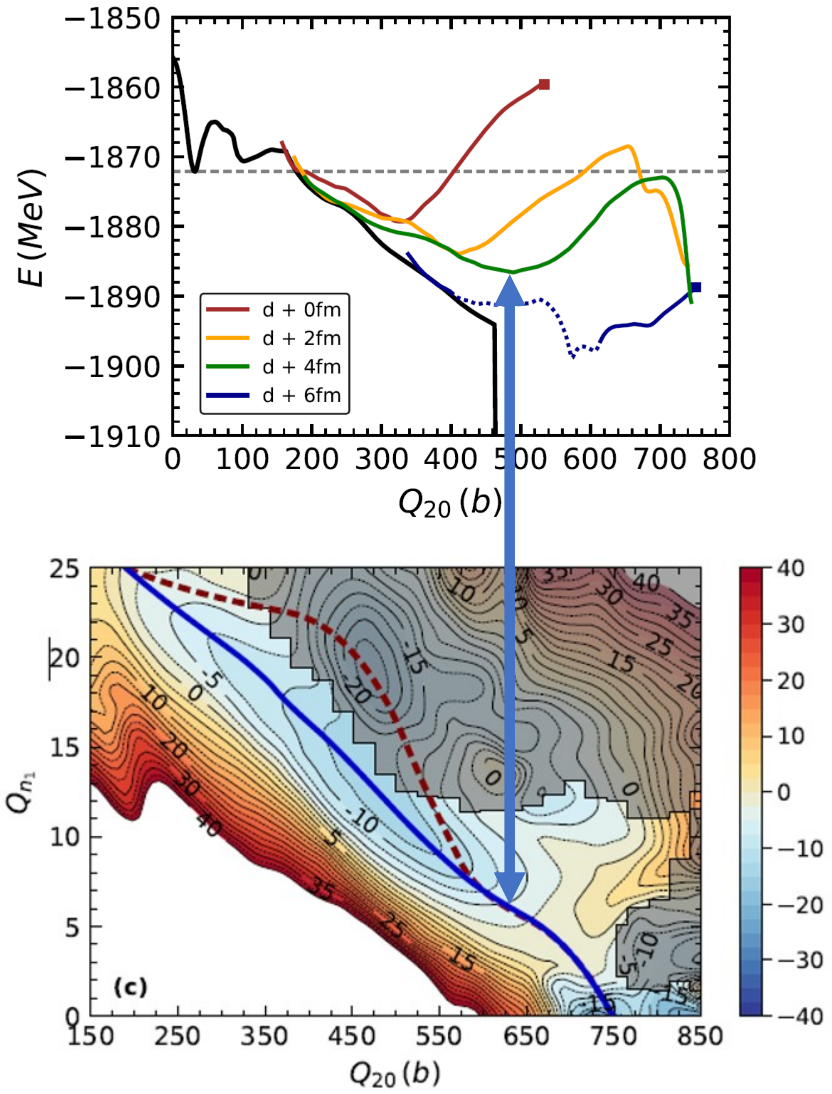
Dynamical suppression of tripartition

- There is a **strong drift toward the bipartition region**
- The formation of the central fragment is **dynamically suppressed along the entire trajectory**.
- We **estimated the tripartition probability** with a simplified Langevin-based approach (constant friction and temperature):

$$P(\boldsymbol{X}_i \rightarrow \boldsymbol{X}_f) \propto \exp\left[-\frac{1}{2T}\left(\Delta V + \int_{\boldsymbol{X}_i}^{\boldsymbol{X}_f} |\nabla_X V|\, \mathrm{d}s\right)\right]$$

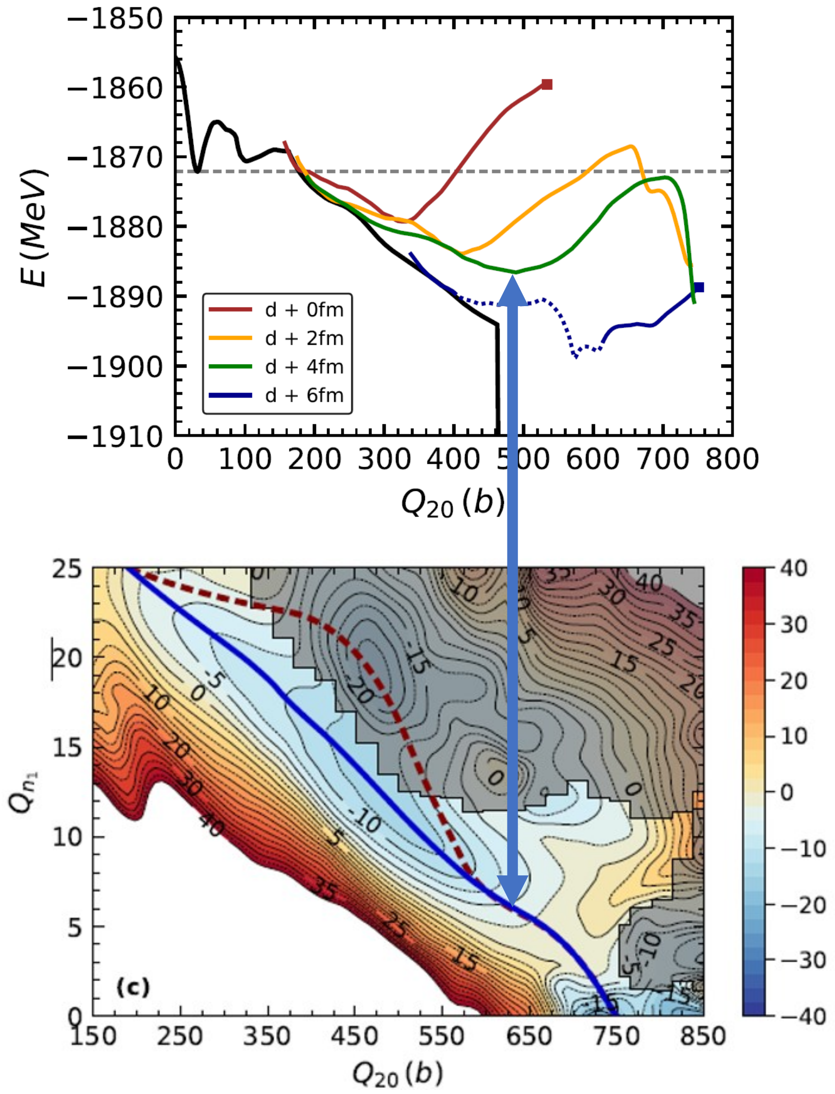
- **Overestimation** of:
  - the tunneling probability (if a barrier is present)
  - the dynamical suppression,
  - and the (quantum-corrected) temperature T = 2 MeV

[1] Y. Jaganathen, J. Skalski, Phys. Lett. B 868 (2025) 139693



Results (actinide region)

- <u>$^{252}Cf \longrightarrow {}^{132}Sn + {}^{48}Ca + {}^{72}Ni$:</u>
  - $z_{12} = d_{sph} + 4\,\text{fm}$: $\boldsymbol{P_{tri} < 10^{-7}}$, $\Delta t_{tri} \sim 10^{-21}s$
  - **Without A_{mid} constraint** (middle fragment formation): $P_{tri} < 10^{-4}$
  - Red trajectory: $P_{tri} < 10^{-15}$
    → **validation of the forbidden areas**



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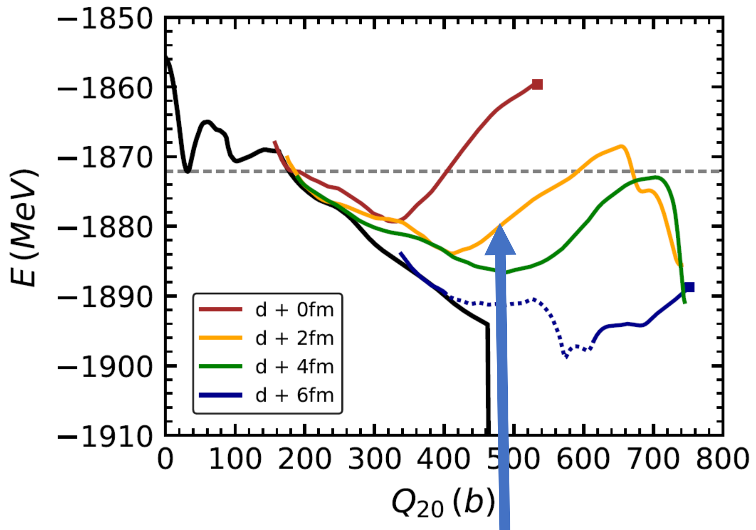
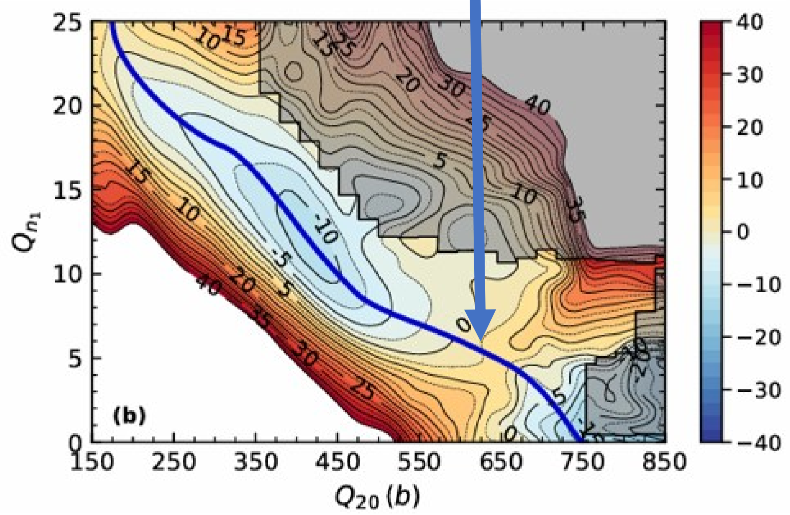
  - $z_{12} = d_{sph} + 2\,\text{fm}$: $\boldsymbol{P_{tri} < 10^{-9}}$ (10^{-3}due to quantum tunneling)





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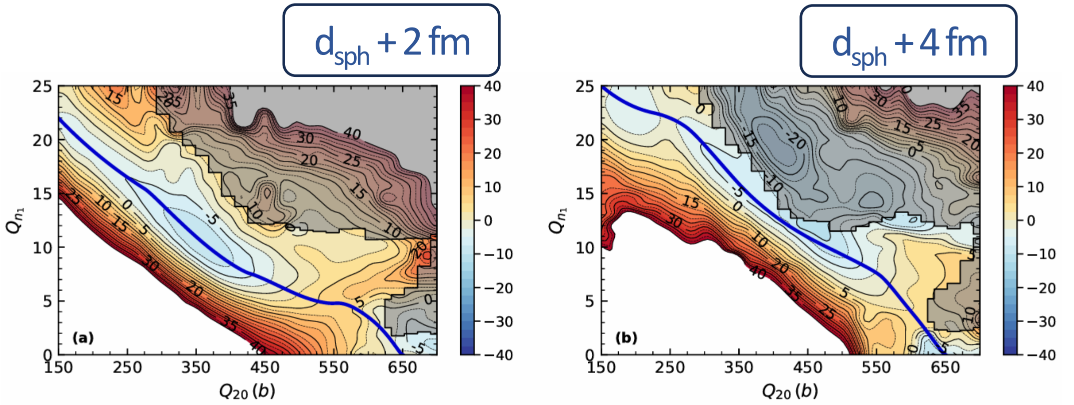
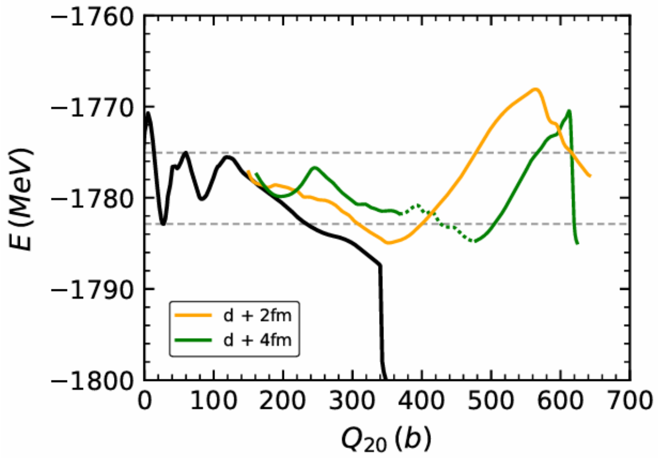
  - $z_{12} = d_{sph} + 2\,\text{fm} : \boldsymbol{P_{tri} < 10^{-9}}$ (10^{-3} due to quantum tunneling)
- **$^{236}U \longrightarrow {}^{132}Sn + {}^{34}Si + {}^{70}Ni$ (neutron induced):**
  - $\boldsymbol{P_{tri} < 10^{-8}}$
- **Conclusion: Strong dynamical suppression**





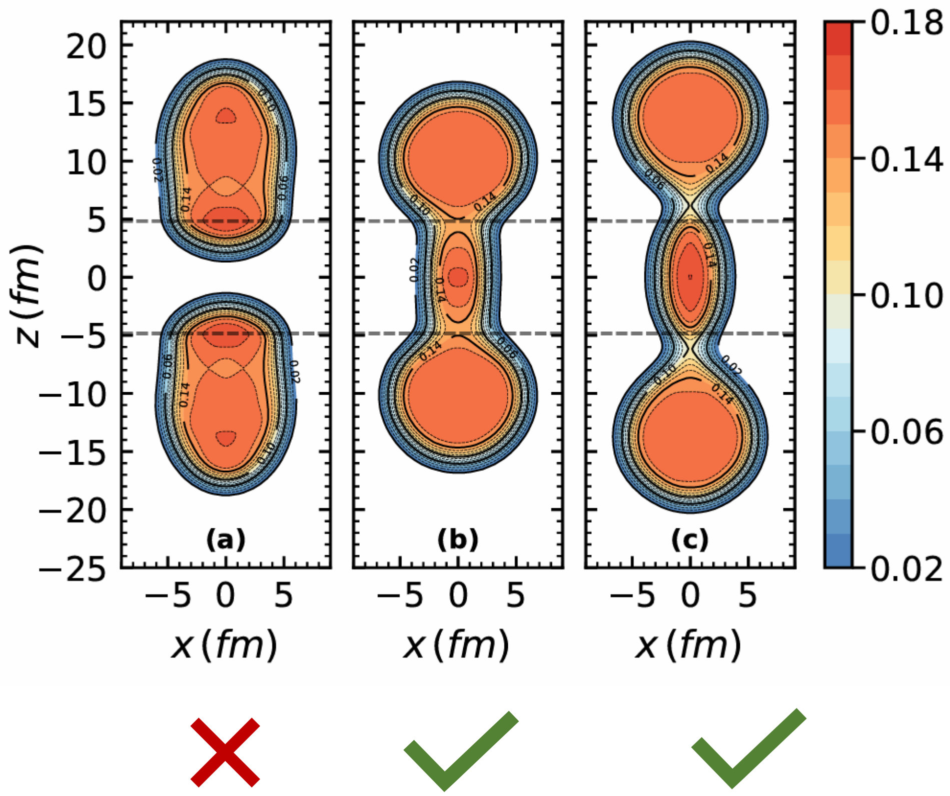
[1] Y. Jaganathen, J. Skalski, Phys. Lett. B 868 (2025) 139693

Collinear tripartition in superheavy nuclei - Densities

- **Large number of protons → Stronger Coulomb repulsion**
- Larger probability of tripartition, but larger suppression by bipartition

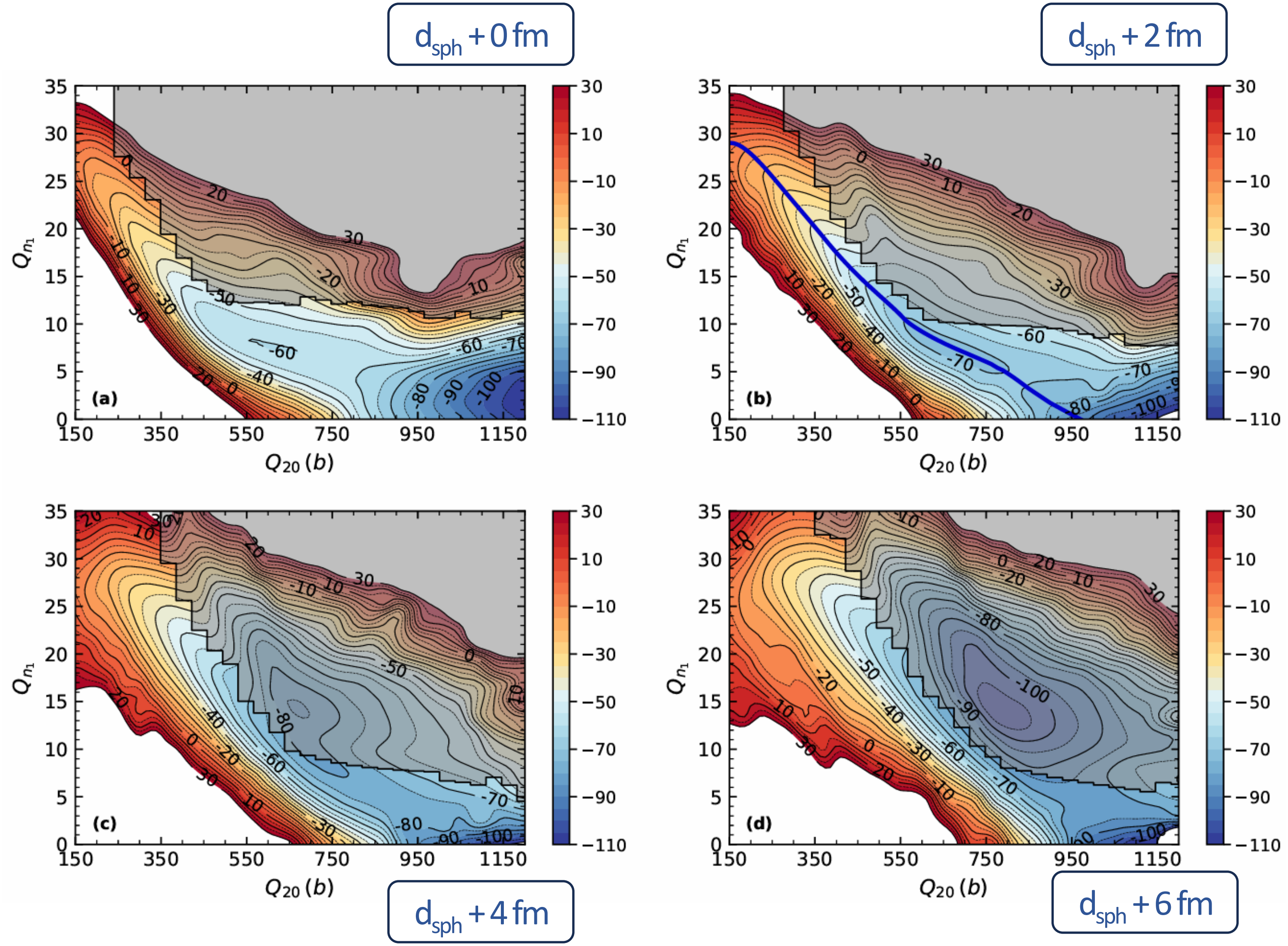
- $^{296}\mathrm{Lv} \rightarrow {}^{132}_{50}\mathrm{Sn}_{82} + {}^{32}_{16}\mathrm{S}_{16} + {}^{132}_{50}\mathrm{Sn}_{82}$ [expected, Karpov]
- $^{296}\mathrm{Lv} \rightarrow {}^{130}_{50}\mathrm{Sn}_{80} + {}^{36}_{16}\mathrm{S}_{20} + {}^{130}_{50}\mathrm{Sn}_{80}$ [obtained, otherwise ${}^{32}_{18}\mathrm{Si}_{14}$]
- **Large number of neutrons in the superheavy, stabilization of the middle fragment with an excess of neutrons.**

- No rupture of one neck before the other (symmetry)
- Only "type II"-forbidden areas



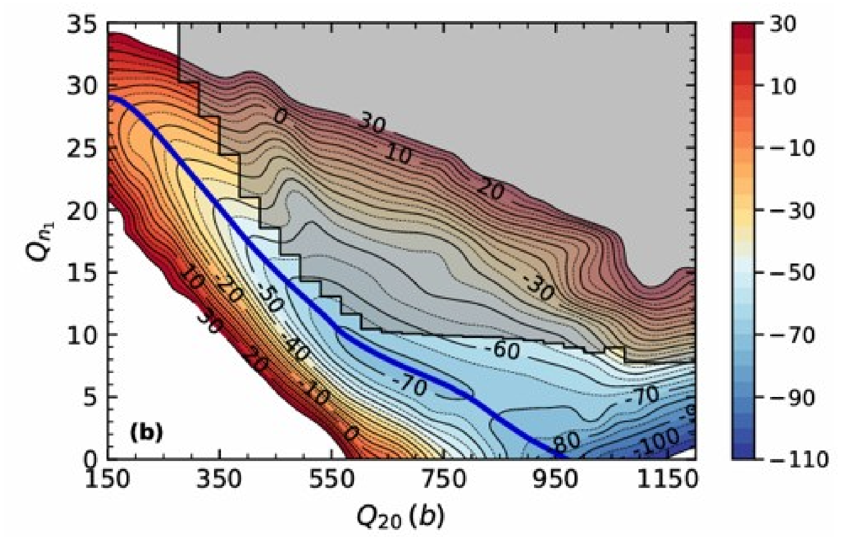
Collinear tripartition in superheavy nuclei - Maps

- **Large drop in energy**
  (expected in superheavy nuclei)
- Study of "d_{sph} + 2 fm" (no barrier)

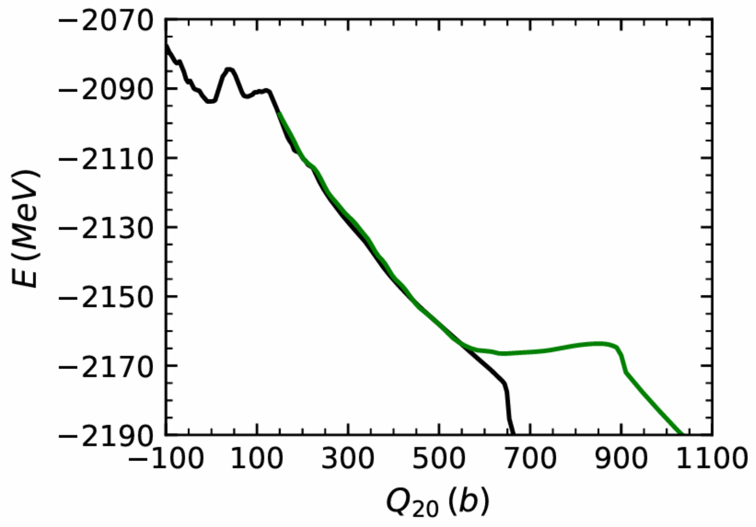


Collinear tripartition in superheavy nuclei - Probabilities

- Same **Langevin-based approximation** with **overestimation**
- Despite the drop of ~ 80 MeV (larger temperature), the **suppression remains high**:
  - $P_{\rm tri} < 10^{-6}$



$d_{sph} + 2$ fm

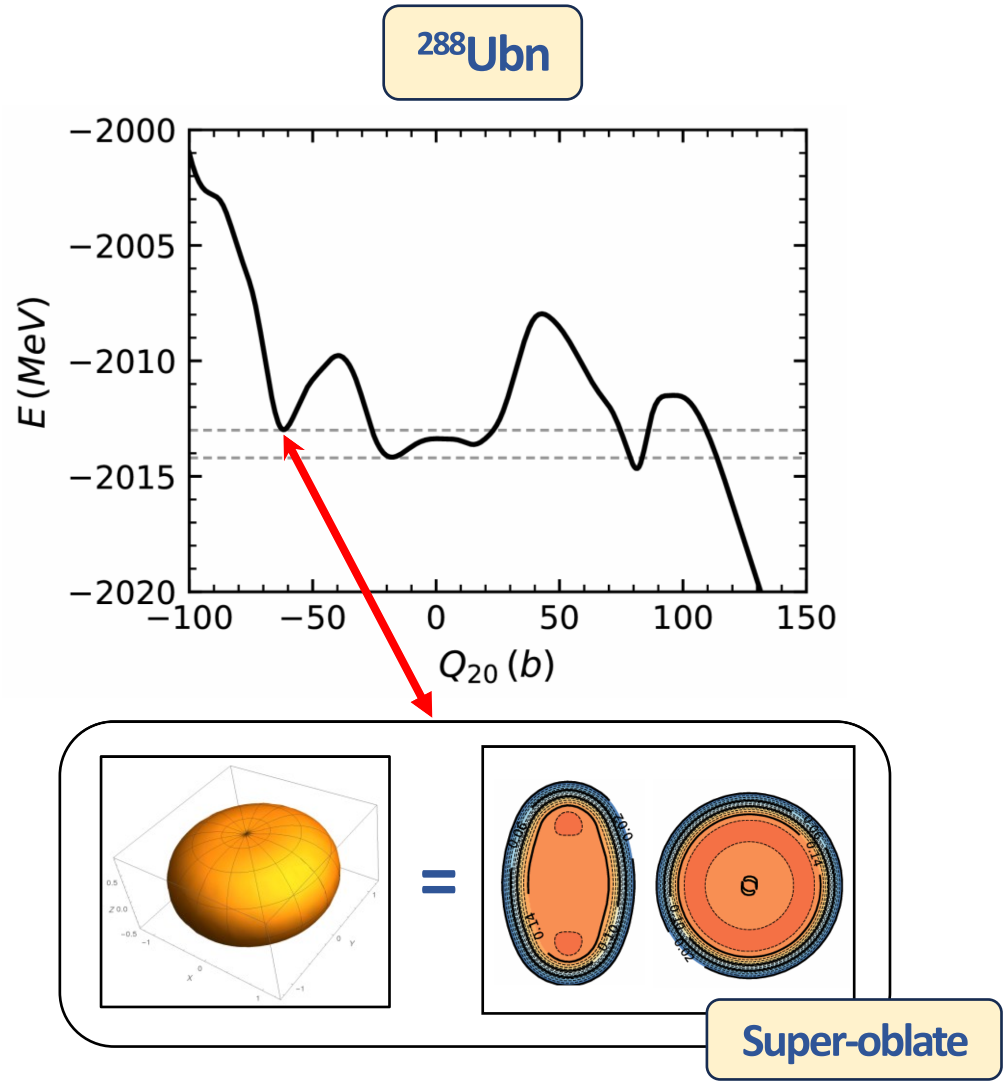


[1] Y. Jaganathen, J. Skalski, to be submitted

Equatorial tripartition in superheavy nuclei

- The **hypothetical** $^{288}_{120}\mathbf{Ubn}$ is expected to have a **super-oblate** state
- The second minimum of ^{288}Ubn corresponds to a $Q_{20} \simeq -60\mathbf{b}$.

- **Question**: Could this pronounced oblate shape make an **equatorial (triangular-like)** ternary fission probable?



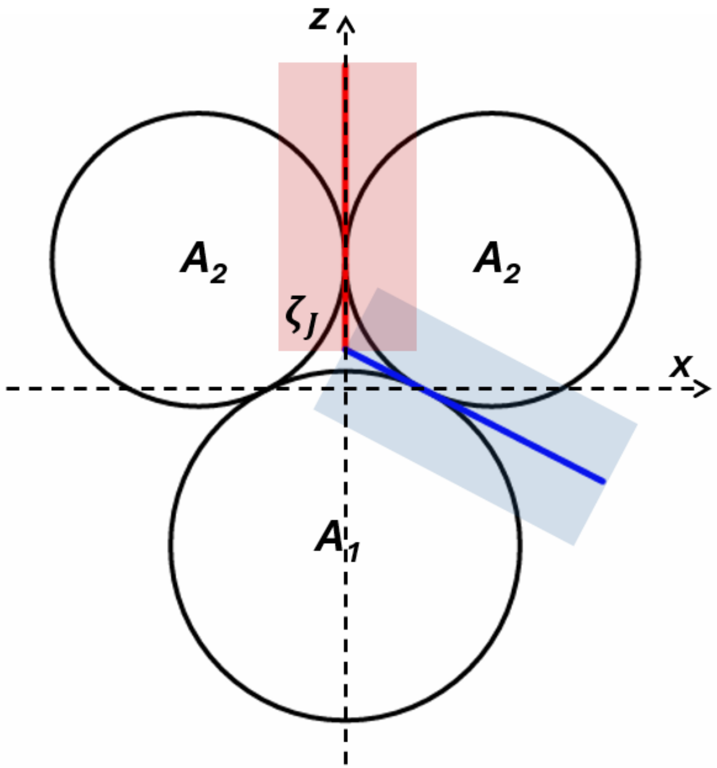
Equatorial tripartition in superheavy nuclei – Constraints

^{288}Ubn

- <u>**Extension of the previous formalism:**</u>
  - New basis to satisfy the symmetry of the system/coordinates
    Studied partitions: $\mathbf{A_2} + A_1 + \mathbf{A_2}$
  - **Generalization of the neck constraints**
  - Elongation $Q_{20} \rightarrow$ **Expansion** Q_Δ :

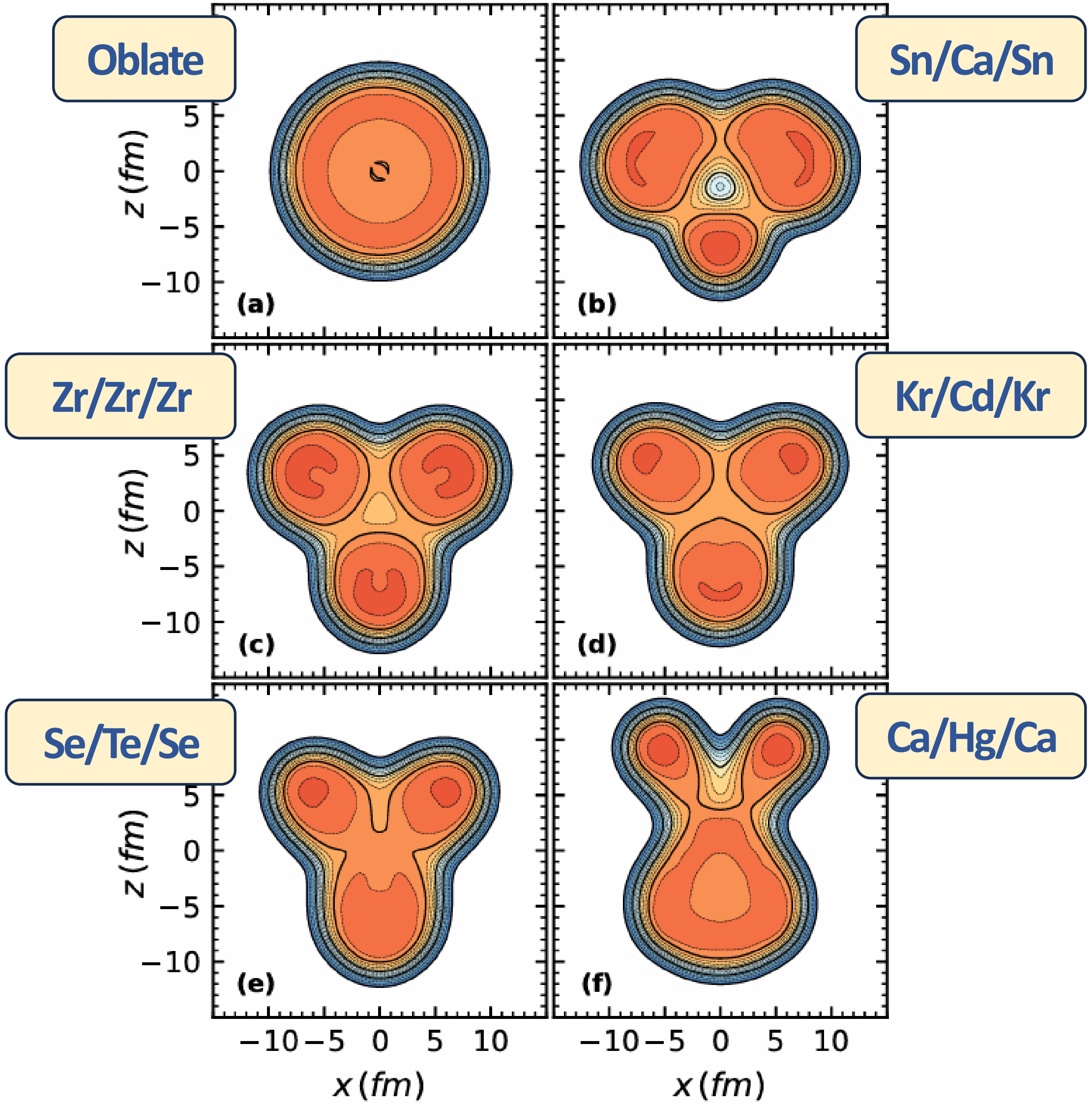
$$\hat{Q}_\Delta = (z - \zeta_J)^2 + x^2$$

where ζ_J is the **junction point of the two necks**.



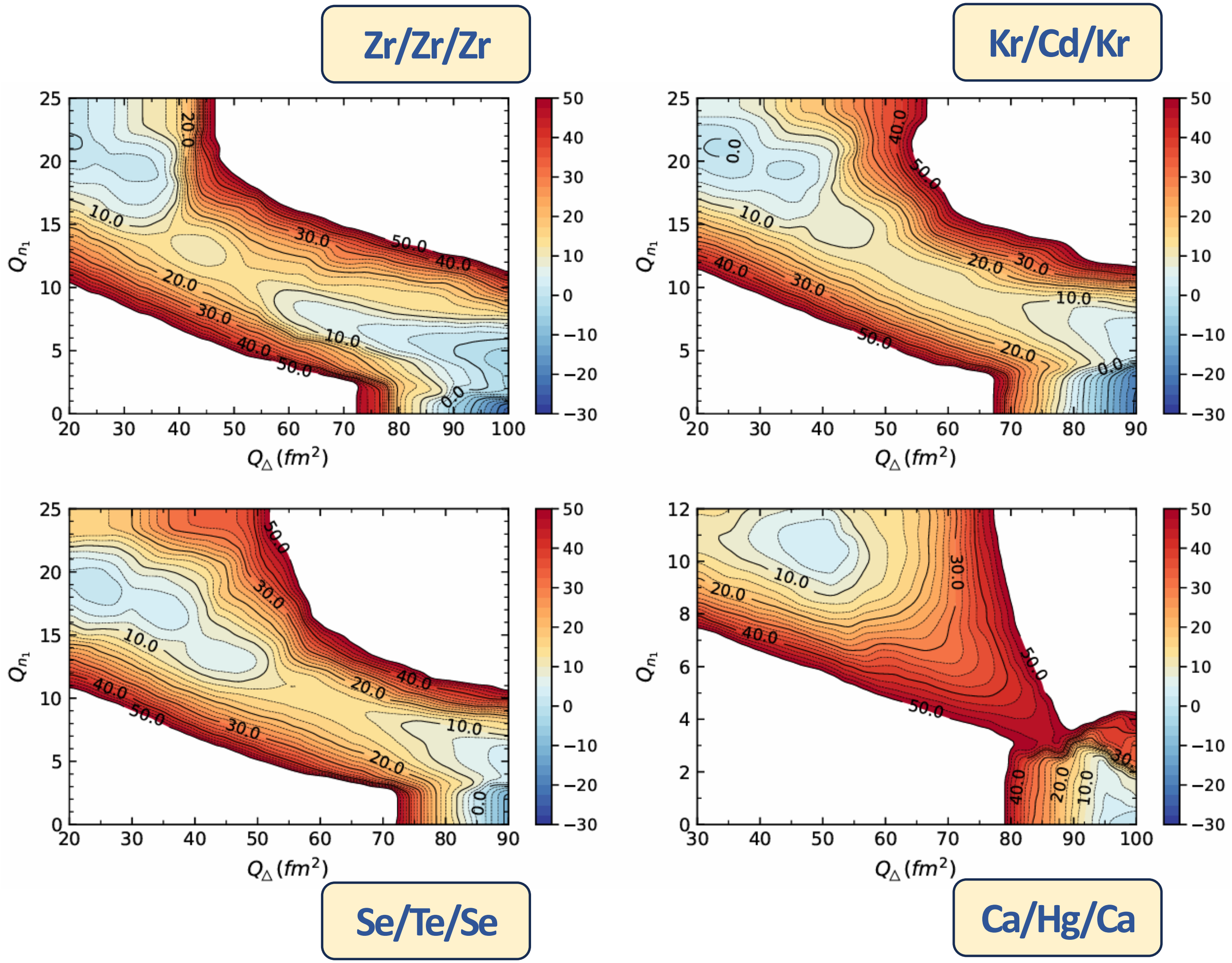
Equatorial tripartition in superheavy nuclei – Densities

- **Lowest expected energies from equilateral triangles**:
  - **Not possible for Sn/Ca/Sn**: tetrapartition
  - **Not possible for Ca/Hg/Ca**
  - **Only possible for the very symmetric systems**



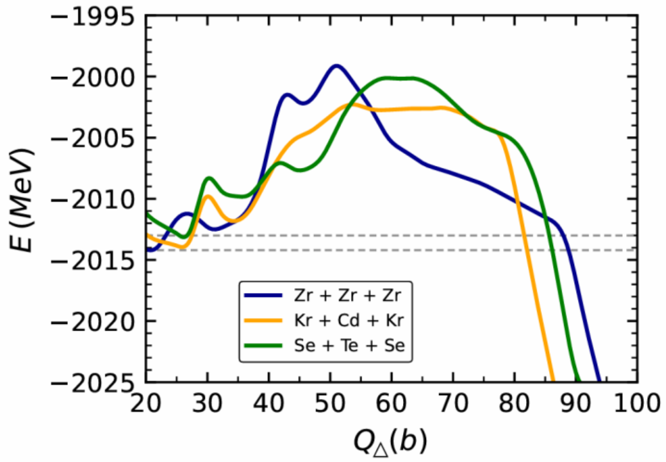
Equatorial tripartition in superheavy nuclei – Maps

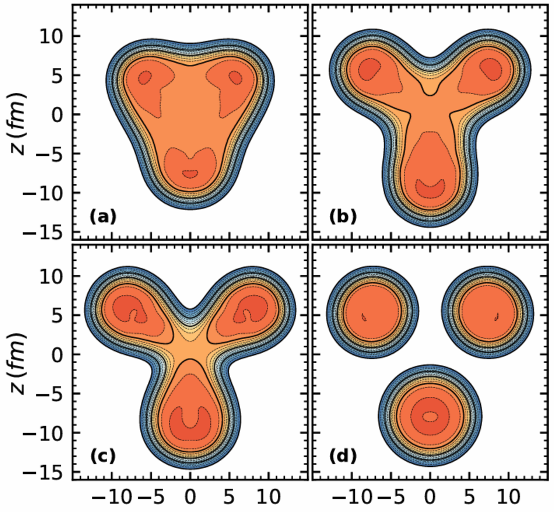
- Remnance of the spherical and oblate states on the maps
- **Clear paths to equatorial tripartition**
  (**No dynamical suppression** from bipartition)
- BUT, **relatively large barriers**
  (Huge barrier in the case of the **impossible Ca/Hg/Ca**)



Equatorial tripartition in superheavy nuclei – Energies

- **Zr / Zr / Zr : E ~ 15 MeV**
- **Kr / Cd / Kr : E ~ 11 MeV**, slightly lower probably due to the **elongation of the fragments**





[1] Y. Jaganathen, J. Skalski, to be submitted

Conclusions

- We have developped **constraints adapted to describe the collinear and equatorial tripartitions** within the **HF-BCS framework.**
- Collinear ternary pathway emerge from the super-elongated fission valleys.
- **A purely static description is insufficient to describe tripartition.**
- **Along the trajectory, energy gradients drive the system towards bipartition. Only fluctuations counteract this trend.**
- As a result, **genuine ternary fission is dynamically suppressed**, **even in the absence of a static energy barrier.**
- The barrier decreases with increasing elongation of the middle fragment, but the dynamical suppression becomes stronger.
- Our (intentionally overestimated) ternary-to-binary probability from the **simplified Langevin model** is **10^{-8} – 10^{-9} for ^{252}Cf and ^{236}U**, and **slightly higher (10^{-6} – 10^{-7}) for ^{296}Lv.**
- For ^{288}Ubn, **equatorial tripartition barriers** are lower than in simplified models (**$E \sim 10$ MeV**), yet still **large enough to prevent genuine ternary breakup**.

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Thank you for your attention!