# Statistical hadronization model for heavy-ion collisions in the few-GeV energy regime

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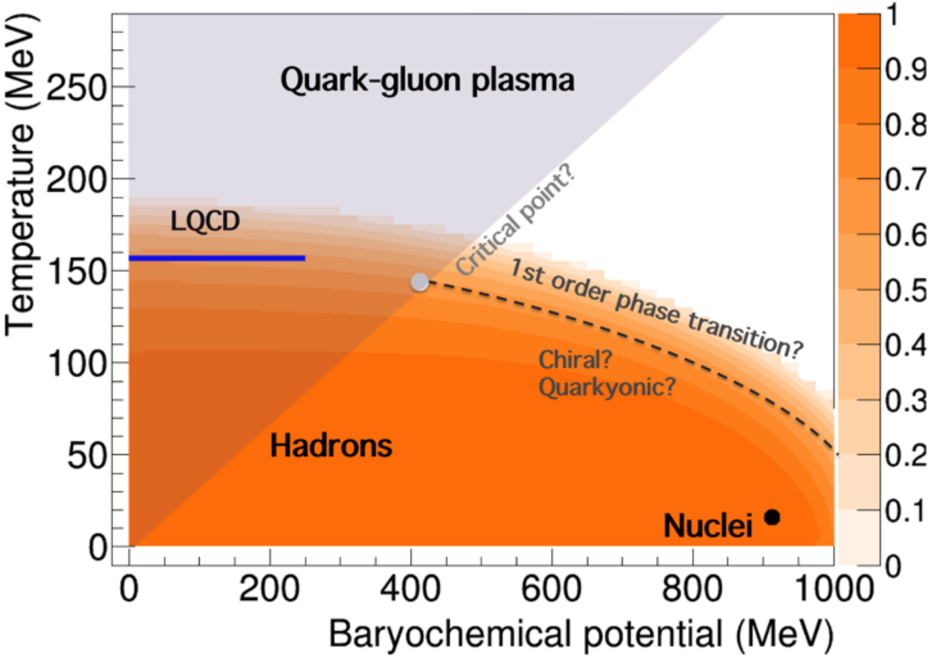
basen on:
PRC 102 (2020) 5, 054903, arXiv: 2003.12992 [nucl-th]

# What is the QCD phase structure?




[P. Steinbrecher (HotQCD), arXiv:1807.05607
Condensate: B.J. Schaefer and J. Wambach]

Vanishing $\mu_B$, high T (lattice QCD)

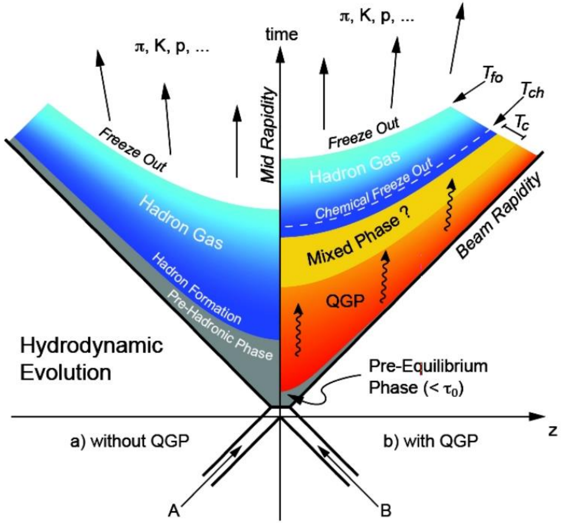
- Crossover, universality
- no CP indicated by lattice QCD at $\mu_B$ < 400 MeV, T >140 MeV

Large $\mu_B$ moderate T (QCD inspired models)

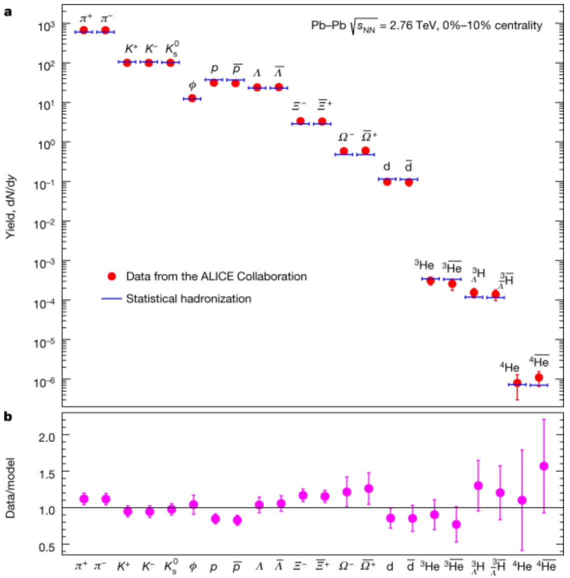
- Thermal equilibrium?
- 1st order transition?
- QCD critical point?
- Melting of the condensate?

$2 < \sqrt{s_{NN}} < 8$ GeV
Large discovery potential!

# Heavy-ion collisions as a tool to study QCD

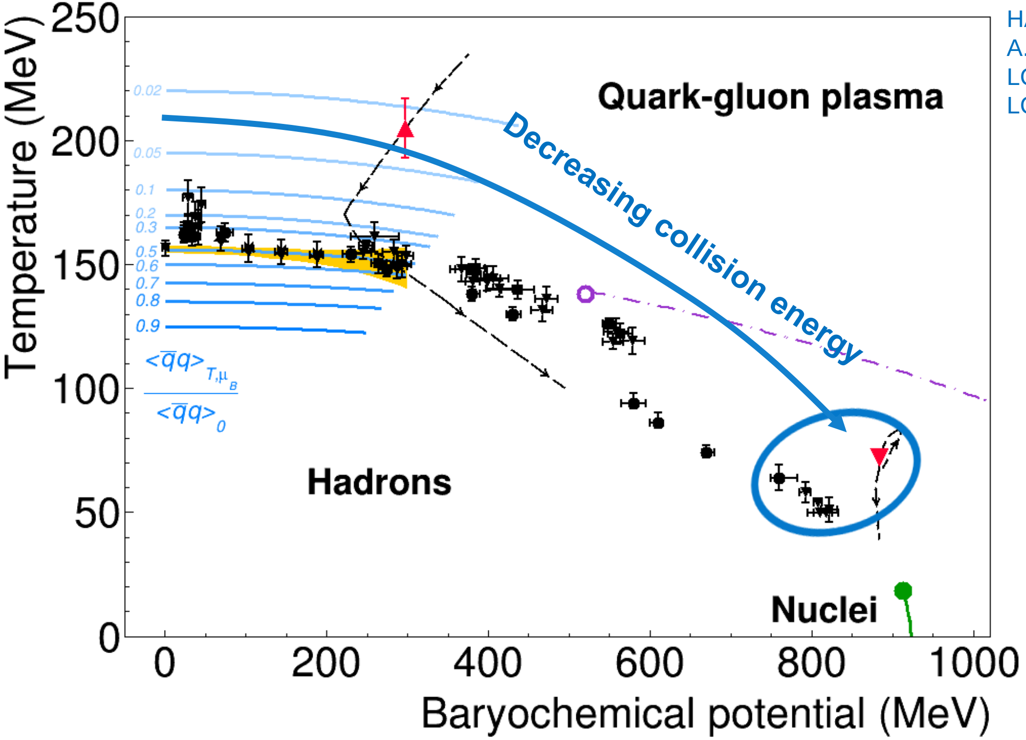



- In chemical equilibrium density of particle *i* can be written as:
  $n_i = \frac{g_s}{2\pi^2} \Upsilon T m^2 K_2\left(\frac{m}{T}\right)$
  Statistical Hadronization Model (SHM)
- One can fit the ratios of measured particle yields and extract free parameters
  – Location in the phase diagram




A. Andronic et al., Nature 561 (2018) 7723

# Mapping the phase diagram with the Statistical Hadronization Model




HADES, Nature Phys. 15 (2019) 10, 1040-1045
A. Andronic *et al.*, Nature 561 (2018) no.7723
LQCD: S. Borsanyi *et al.* [Wuppertal-Budapest Collab.], JHEP 1009 (2010) 073
LQCD: A. Bazavov *et al.*, PLB 795 (2019) 15-21

- Is it valid at all to use equilibrium methods at low energies?
  - Particles with strange quarks produced deep below the NN threshold
  - Low number of newly produced particles in the interaction zone: ~40 in central events (mainly pions)
- On the other hand:
  - Original nucleons stopped in the interaction zone (~300 particles in central events)
  - Longer life-time of the system – enough to thermalize

# Dynamic description of heavy-ion collisions

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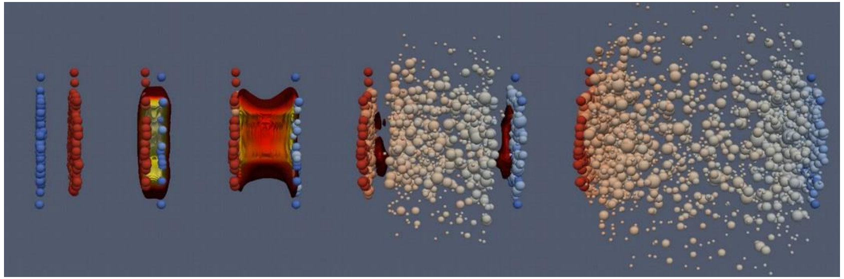
**Standard prescription at high beam energies (RHIC/LHC):**

- Non-equilibrium initial conditions
- Viscous hydrodynamic evolution
  - Equilibrium
  - People often assume: fluid = QGP
- Hadronic final-state rescattering



**Standard prescriptipn at "low" beam energies (GSI/FAIR/NICA/…):**

- Hadronic transport
- Importance of:
  - Resonance dynamics
  - Nuclear potentials

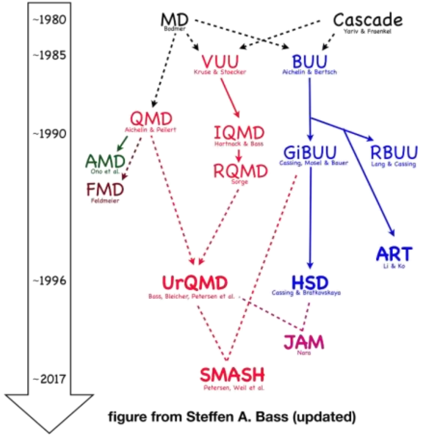
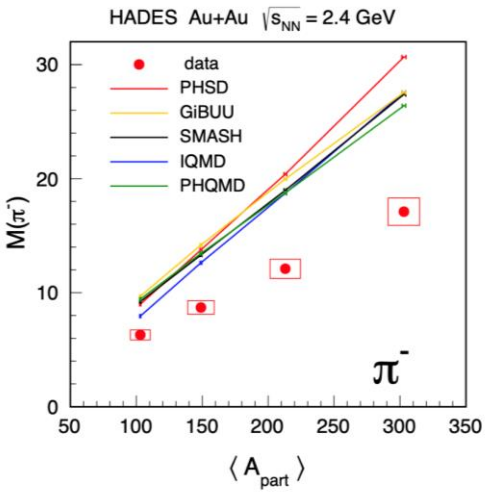


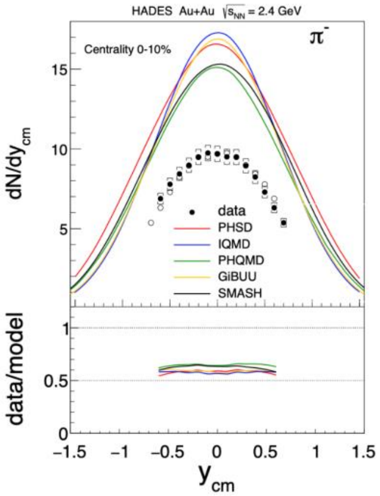

figure from Steffen A. Bass (updated)

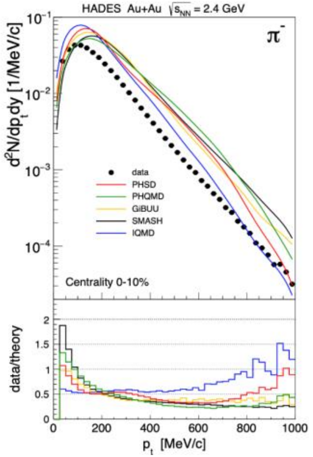
# Not everything is known yet about few-GeV HIC

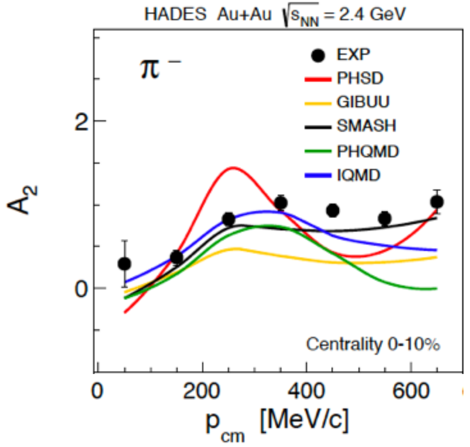
Example for $\pi^-$, same holds for $\pi^+$

HADES Collaboration, EPJA 56 (2020) 10, 259
https://www.hepdata.net/record/ins1796710




$$\frac{dN}{d(cos\theta_{cm})} = C(1 + A_2 cos^2\theta_{cm})$$

- Only width of the rapidity distributiion is correctly described by the models
- Is there something fundamentally missing?

Pion and Proton “Temperatures” in HIC
R. Brockmann *et al.*, PRL 1984

# HADES data vs. other experiments

HADES Collaboration, EPJA 56 (2020) 10, 259
https://www.hepdata.net/record/ins1796710

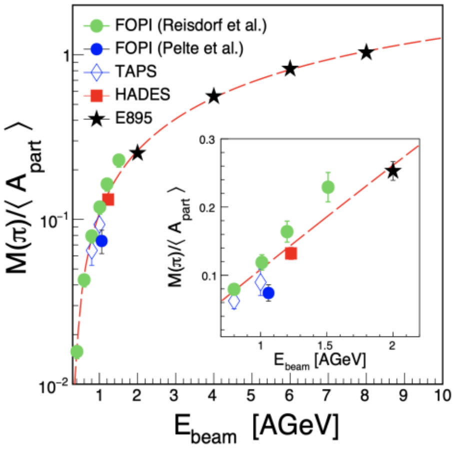



**Fig. 8** Pion multiplicity $M(\pi)$ per mean number of participating nucleon $\langle A_{part} \rangle$ as a function of the kinetic beam energy $E_{beam}$. The dashed curve is a fit to the data points except for the one labeled "FOPI (Pelte et al.)", as suggested in [4]. The inset magnifies the energy region around the HADES point.

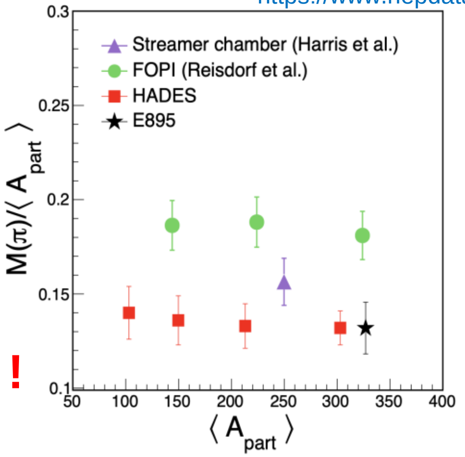



**Fig. 9** Comparison of the centrality dependence of $M(\pi)/\langle A_{part} \rangle$ in Au+Au collisions to earlier measurements at similar energies. The results from FOPI, E895, and from the BEVALAC Streamer Chamber group (the latter for $La + La$ collisions) have been scaled to 1.23 A GeV; note the suppressed zero on the ordinate.

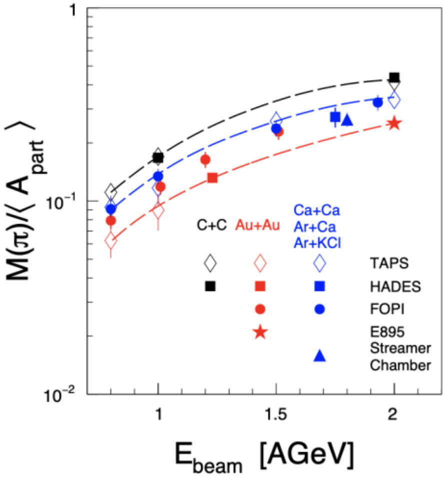



**Fig. 10** Pion multiplicity per participating nucleon as a function of beam energy for three different systems: C+C (black) [7, 22, 39], Ar+KCl (blue) [4, 7–9, 40] and Au+Au (red) [4, 6, 7, 11]. The curves are polynomial fits to these data used to interpolate the multiplicities as a function of bombarding energy for corresponding systems.

HADES results are consistent with the trends established by previous experiments at similar beam energies

# Hydro-inspired models

## of particle production at the freeze-out

- First idea:

  P. J. Siemens and J. O. Rasmussen, PRL 42 (1979) 880

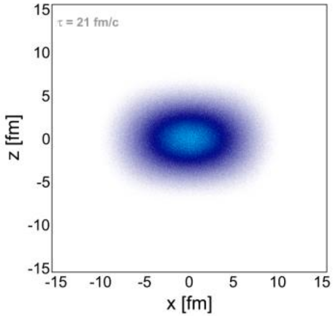
  – Used for Ne+NaF at $E_{kin}/A$ = 800 MeV!
  – Thermal source of spherical geometry and spherically symmetric expansion
  – Constant radial velocity (non-physical for *r* = 0?)

- Modification:

  E. Schnedermann, J. Sollfrank, U. W. Heinz, PRC 48 (1993) 2462:

  – Appropriate for higher-energy collisions (originally S+S at $E_{kin}/A$ = 200 GeV)
  – Cylindrically-symmetric geometry and expansion
  – Boost invariance in Z direction – "Bjorken scaling"
  – Velocity profile: $\beta(r) = \beta_{\max}(r/\mathrm{r}_{\max})^n$

Guidance from dynamic models




- Density evolution in Au+Au at $E_{kin}/A$ = 1230 MeV
- Coarse-grained hadronic transport

  T. Galatyuk et al., EPJA 52 (2016) 5, 131

- Spherical symmetry clearly more realistic than boost invariance

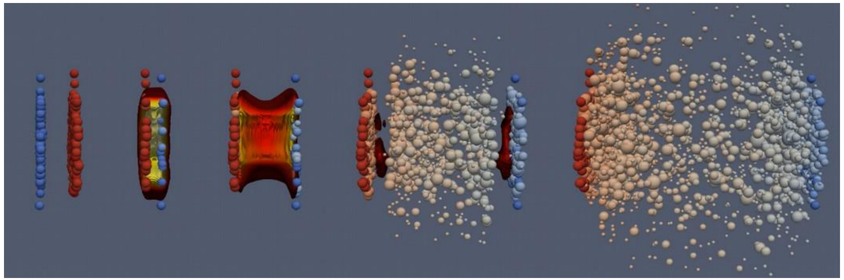
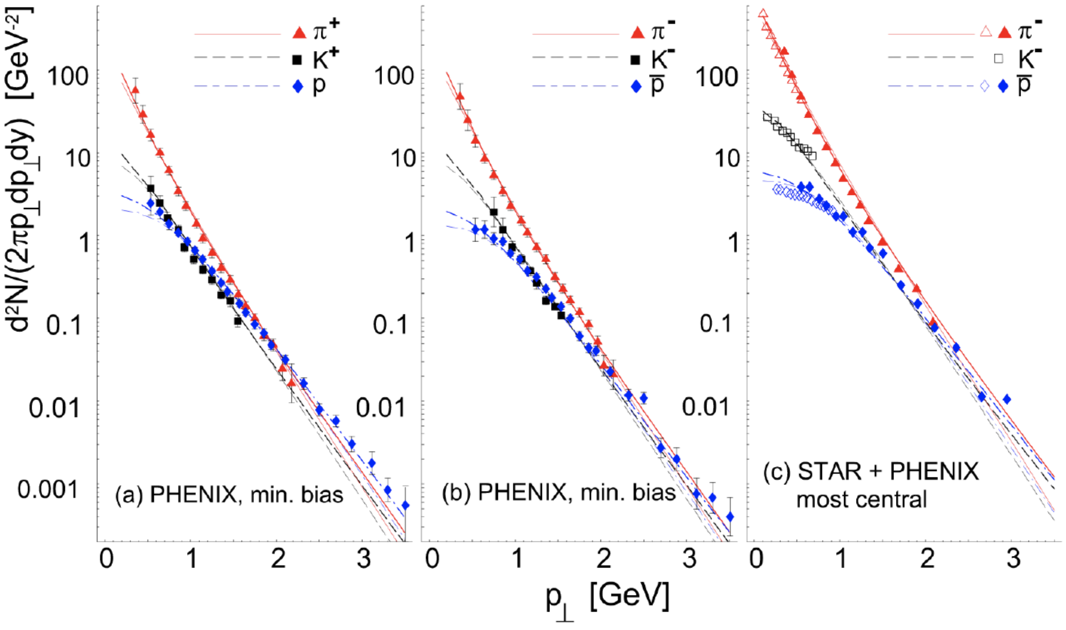


Figure: MADAI collaboration, Hannah Petersen and Jonah Bernhard

# Single freeze-out scenario

´W. Broniowski and W. Florkowski, PRL 87 (2001) 272302

- Chemical freeze-out coincides with kinetic freeze-out
- Hadron yields are given by the integrals of hadron spectra
- Feed-down from resonance decays included
- Successful at RHIC, how it works at SIS18 synchrotron?
- Idea is implemented in the Thermal Event Generator (Therminator 2)



# Cooper-Frye formula

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F. Cooper and G. Frye, PRD 10 (1974) 186

*"Single-particle distribution in the hydrodynamic and statistical thermodynamic models of multiparticle production"*

$$E_p \frac{dN}{d^3p} = \int d^3\Sigma_\mu(x)\, p^\mu\, f(x,p)$$

- Spherically symmetric system:

$$x^\mu = (t(r), r\mathbf{e_r})$$

- Spherical expansion of the "fluid":

$$u^\mu = \frac{1}{\sqrt{1-v^2(r)}}(1, v(r)\mathbf{e_r})$$

- Sudden freeze-out in the "lab" frame ($t = \text{const}(r)$):

$$d^3\Sigma_\mu \equiv \varepsilon_{\mu\alpha\beta\gamma} \frac{\partial x^\alpha}{\partial\zeta}\frac{\partial x^\beta}{\partial\phi}\frac{\partial x^\gamma}{\partial\theta} d\zeta\, d\phi\, d\theta$$
$$= (r^2 \sin\theta\, d\theta\, d\phi\, dr, 0, 0, 0)$$

Parameter of $\zeta \to (t(\zeta), r(\zeta))$

**Local thermodynamic equilibrium**

$$f(x,p) = \frac{g_s}{2\pi}\left[\Upsilon^{-1}\exp\left(\frac{p_\mu u^\mu}{T}\right) \pm 1\right]^{-1}$$

Fugacity factor:

$$\Upsilon \equiv \gamma_q^{N_q+N_{\bar{q}}}\gamma_s^{N_s+N_{\bar{s}}}\exp\left(\frac{\mu_B B + \mu_S S + \mu_{I_e} I_3}{T}\right)$$

(in this work we assume $\gamma_q = 1$)

- Integrating over the freeze-out hypersurface and phase-space gives back particle multiplicity
- Right sets of assumptions recover the original Siemens-Rasmussen and Schnedermann-Sollfrank-Heinz formulas
- But we assume Hubble-like expansion:

$$v(r) = \tanh(Hr)$$

# Resonance treatment

R. Dashen, S. K. Ma and H. J. Bernstein, Phys. Rev. 187 (1969) 345 (1969)
R.Venugopalan, and M. Prakash, Nucl. Phys. A 546 (1992) 718
W.Weinhold,, and B. Friman, Phys. Lett. B 433 (1998) 236
Pok Man Lo, Eur. Phys.J. C77 (2017) no.8, 533

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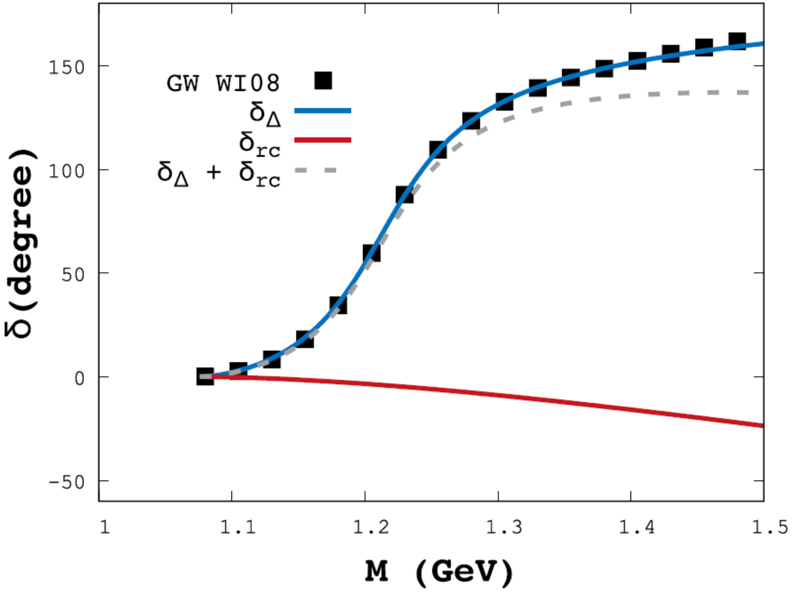
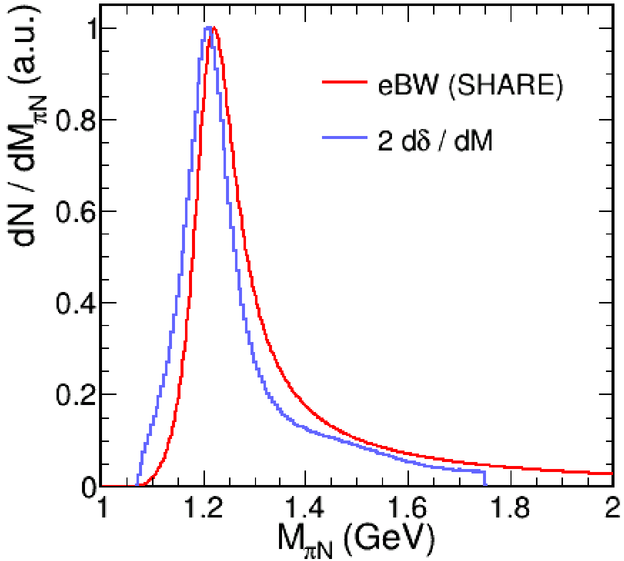
$\pi$N phase shift in the $P_{33}$ channel

[Pok Man Lo *et al.*, PRC 96, 015207,
GW WI08: R.L. Workman et al. PRC 86, 035202]




Spectral function: $B_l(M) = 2\frac{d}{dM}\delta_l$



# Thermal Event Generator (Therminator 2)

M. Chojnacki et al., Comput. Phys. Comm. 103 (2012) 746-773
SH, W. Florkowski, T. Galatyuk *et al.*, PRC 102 (2020) 5, 054903

Ingredients of the method:

- Single (chemical and kinetic) freeze-out on a **spherically symmetric hypersurface** (Siemens-Rasmussen blast-wave model)
- Fix thermodynamic paremeters with multiplicities of particles:

  → Solve numerically 6 equations for 6 parameters:

| | |
|---|---|
| protons (incl. those bound in light nuclei): | 124.1 |
| $\pi^+$: | 9.3 |
| $\pi^-$: | 17.1 |
| $K^+$: | 0.0598 |
| $K^-$: | 0.00056 |
| $\Lambda$: | 0.0822 |

⇒

**$T$ = 49.6 MeV**

**$\mu_B$ = 776 MeV**

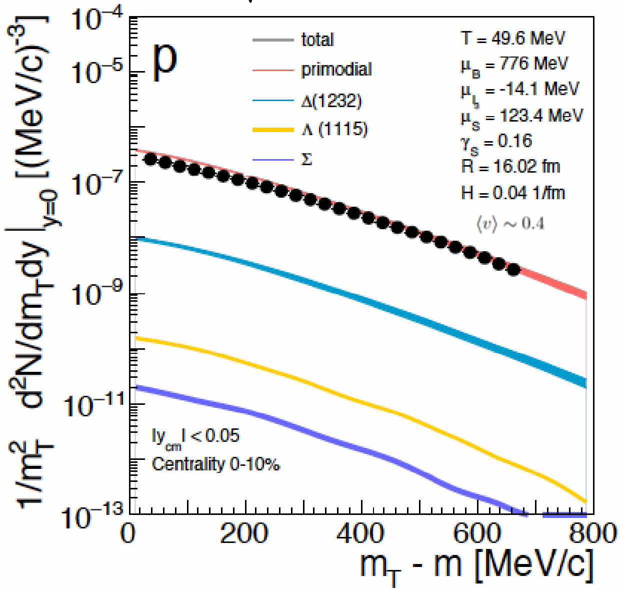
**$\mu_{I3}$ = -14.1 MeV**

**$\mu_S$ = 123.4 MeV**

**$\gamma_S$ = 0.16**

**$R$ = 16.02 fm**

Au+Au $\sqrt{s_{NN}}$ = 2.42 GeV, 0-10%

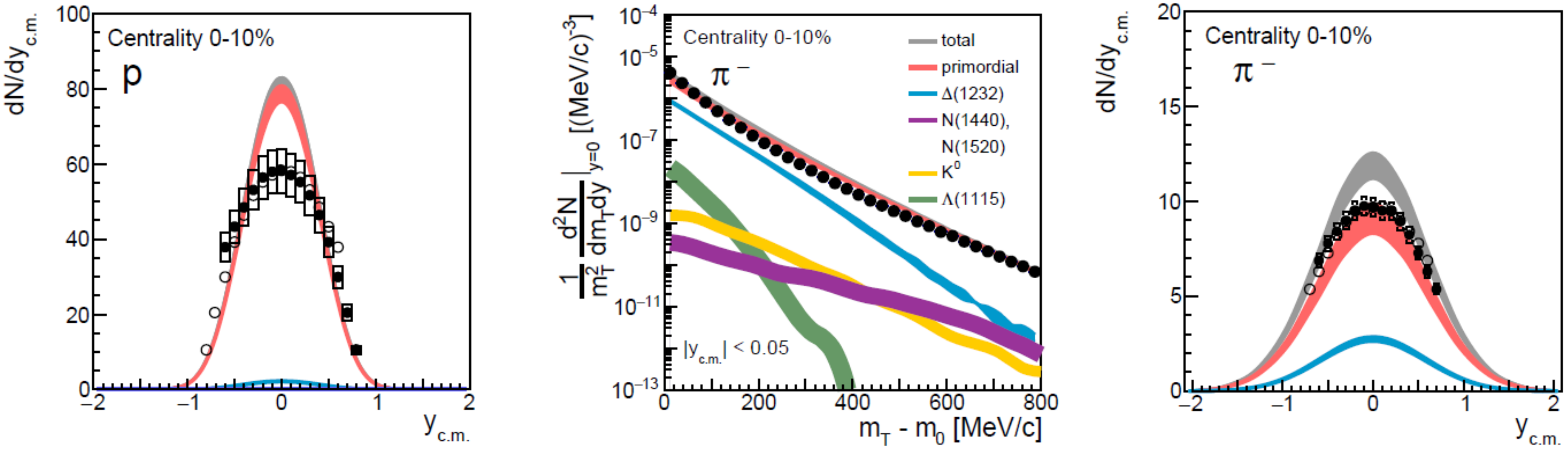



- Proton $m_t$ spectrum at mid-y is fitted to get the expansion velocity profile: ***H* = 0.04 fm$^{-1}$**
- $\Delta$ spectral function from $\pi$N phase shift

P.M. Lo *et al.*, PRC 96 (2017) 015207

# Spectra of bulk particles

Au+Au $\sqrt{s_{NN}}$ = 2.42 GeV, 0-10%



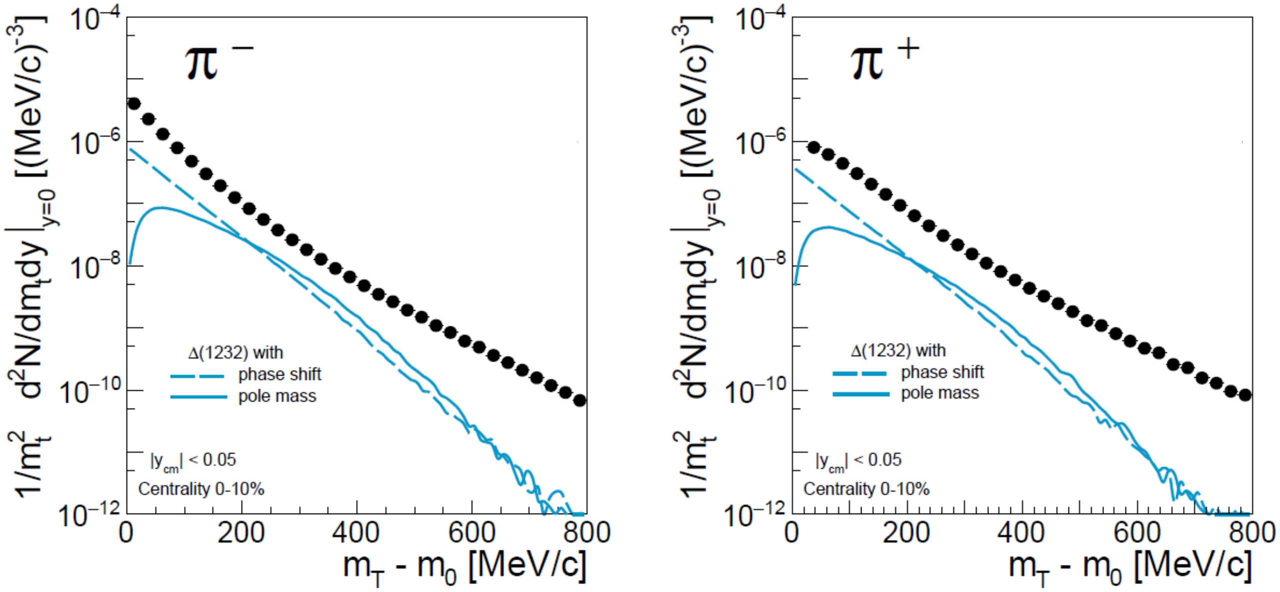

- These spectra are **not fitted**, but **predicted** by the model
- Bands: uncertainty from errors on hadron yields
- For $\pi^+$ similar level of agreement as for $\pi^-$
- Pion slope at high mt described with T ~ 50 MeV and Hubble

- Rapidity too narrow in the model

  → Spherical symmetry is not exactly fulfilled

  → Further improvements are ongoing

# Influence of the Δ shape on pion spectra



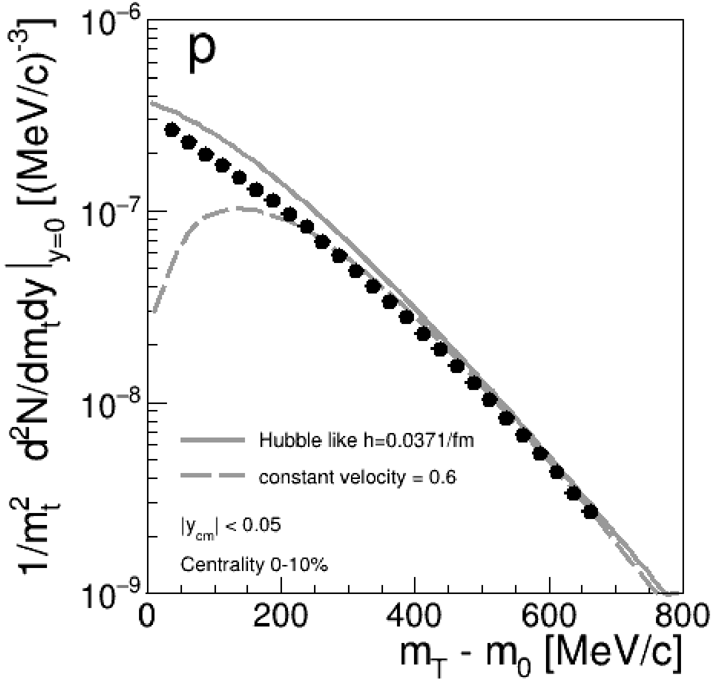

Transverse mass of pions from Δ decay for different spectral functions:

- Δ with fixed mass of 1.232 GeV
- Spectral function from the πN phase shift in the $P_{33}$ channel

Finite Δ width:
→ more pions at low $m_t$

# Influence of the velocity profile

- Hubble-like fireball expansion:
  $$v(r) = \tanh(Hr)$$
- The parameter *H* fitted to the proton $m_t$ spectra:
  $H = 0.037\ \mathrm{fm}^{-1}$
- Mean value:
  $$\langle v \rangle = \frac{2}{3} HR \left(1 - \frac{1}{5} H^2 R^2\right) \approx 0.4$$
- Best fit with constant velocity
  - gives $\langle v \rangle = 0.6$
  - fails to describe the data at low $m_t$



# Outlook:

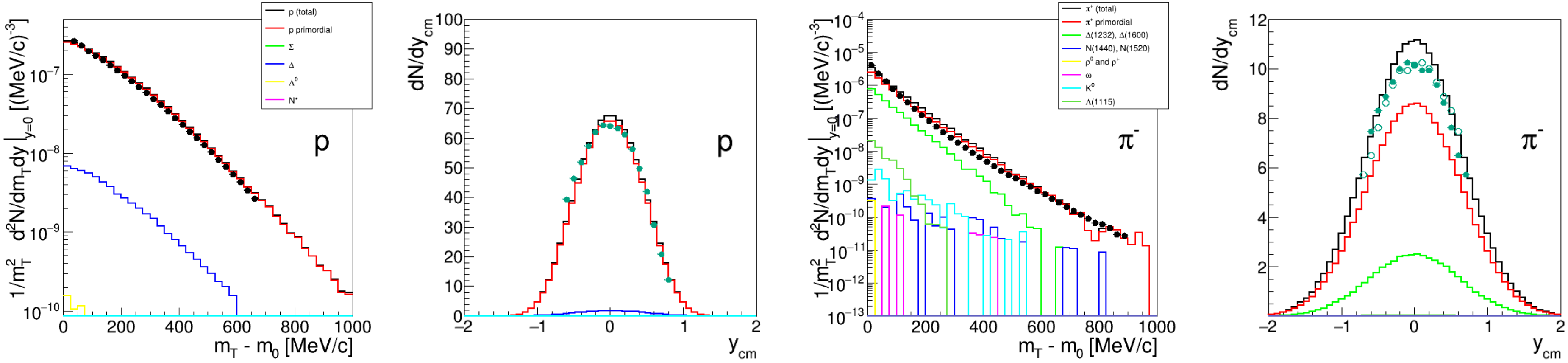
## moving from spherical to spheroid symmetry

- Transverse momentum spectra are well described, and
- Rapidity spectra are too narrow compared to experiment

  → Expansion in longitudinal direction should be stronger than in transverse direction
- Guidance from dynamic models

  → Freeze-out hypersurface should be narrower in the longitudinal direction

**Ongoing work on systematic fitting the shape parameters**



# Outlook:

## Afterburner for final-state EM interaction

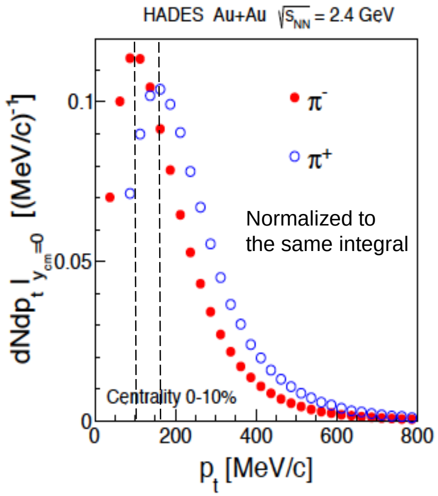
After the freeze-out, particles a propagated according to standard formulas:

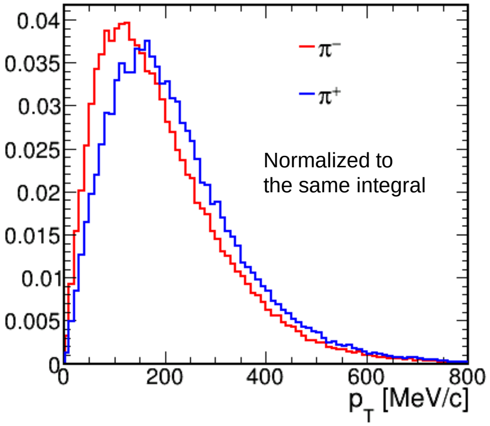
$$\mathbf{E}(\mathbf{r},t) = \frac{q}{4\pi\epsilon_0}\frac{R}{(\mathbf{R}\cdot\mathbf{u})^3}[(c^2 - v^2)\mathbf{u} + \mathbf{R}\times(\mathbf{u}\times\mathbf{a})]$$

$$\mathbf{B}(\mathbf{r},t) = \frac{1}{c}\hat{\mathbf{R}}\times\mathbf{E}(\mathbf{r},t)$$

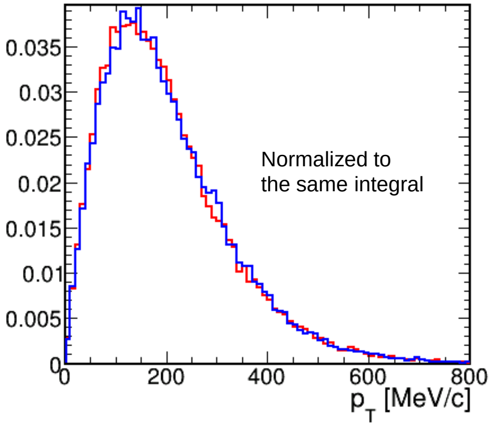
$$\mathbf{R} \equiv \mathbf{r} - \mathbf{w}(t_r), \qquad \mathbf{v} \equiv \dot{\mathbf{w}}(t_r)$$

$$\mathbf{u} \equiv c\hat{\mathbf{R}} - \mathbf{v}, \qquad |\mathbf{r} - \mathbf{w}(t_r)| = c(t - t_r)$$




Without afterburner



# Conclusions

- Statistical hadronization model can describe not only multiplicities, but also spectra of bulk particles produced in heavy-ion collisions in $\sqrt{s_{NN}}$ of few GeV
- Ingredients:
  - Spherical, Siemens-Rasmussen-type fireball expansion
  - Hubble-like velocity profile
  - Sudden freeze-out
  - Careful treatment of baryonic resonances

**Outlook:**

- Spheroidal instead of spherical symmetry
- Final-state EM interactions
- HBT radii, nucleon coalescence, data from STAR fixed-target, FAIR, NICA…

# THANK YOU FOR YOUR ATTENTION