Electric dipole response of light nuclei within the Configuration-Interaction Shell Model approach

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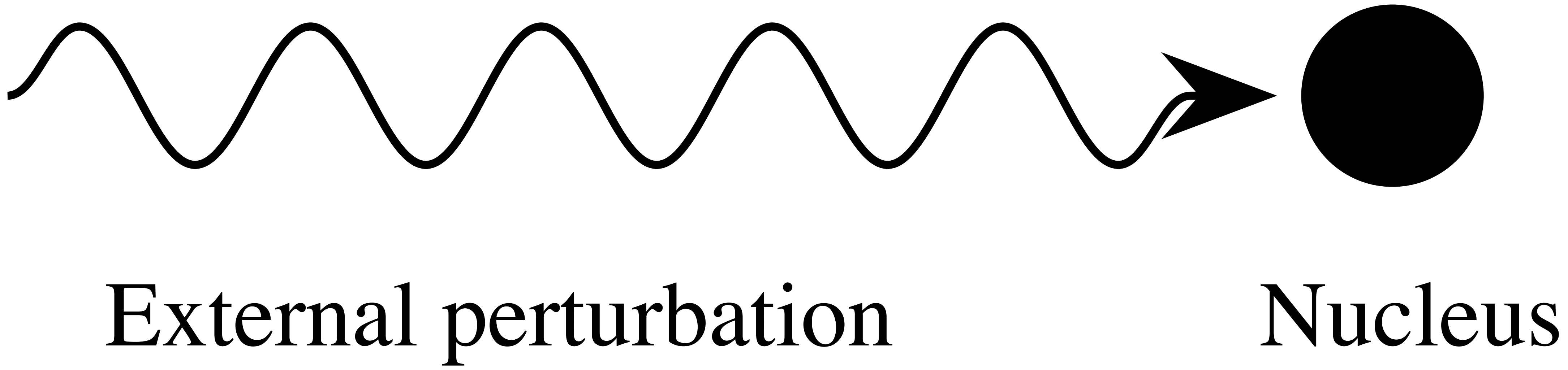
QMat QUANTUM SCIENCE AND NANOMATERIALS

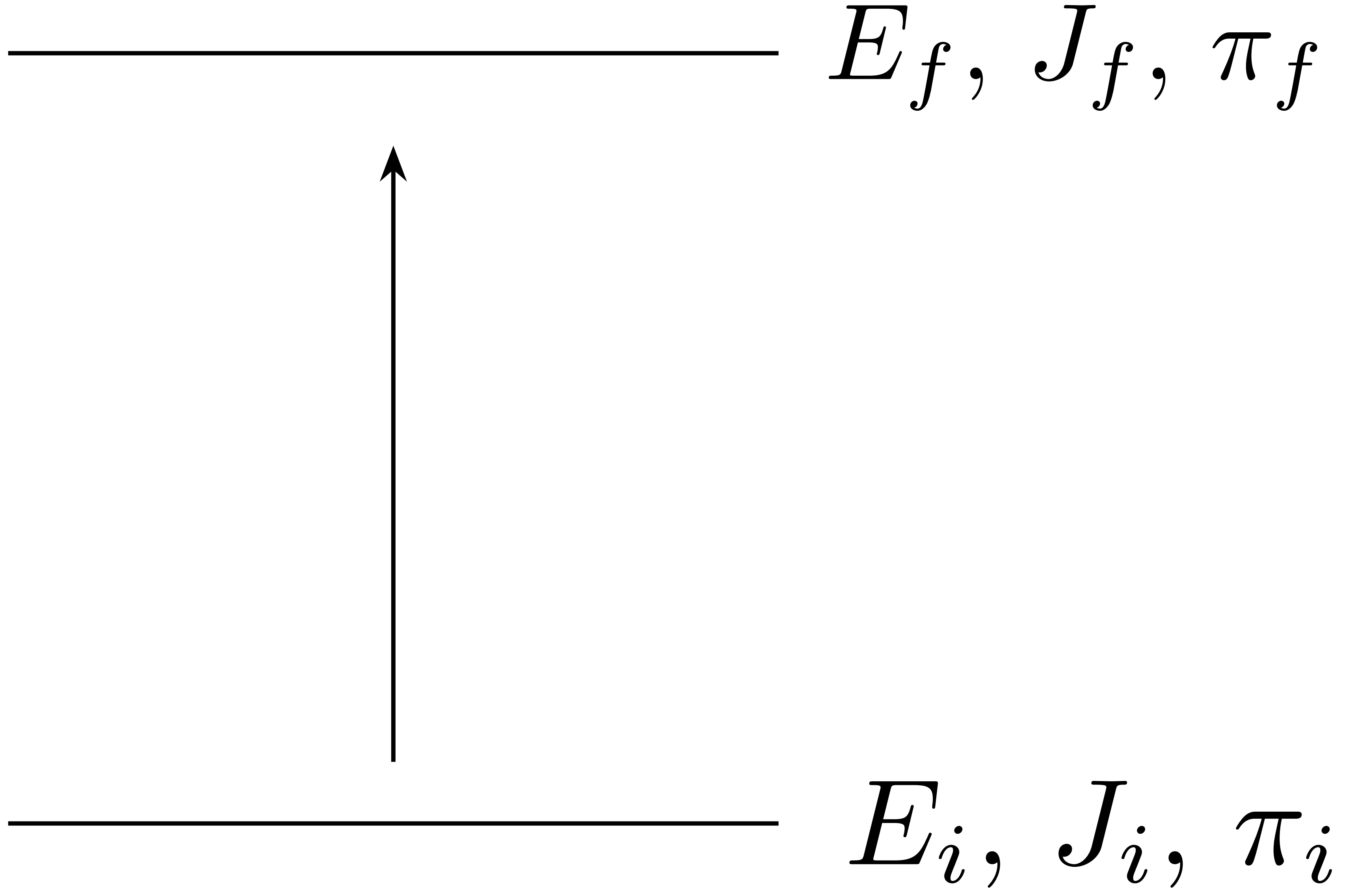
cnrs IN2P3 Les deux infinis

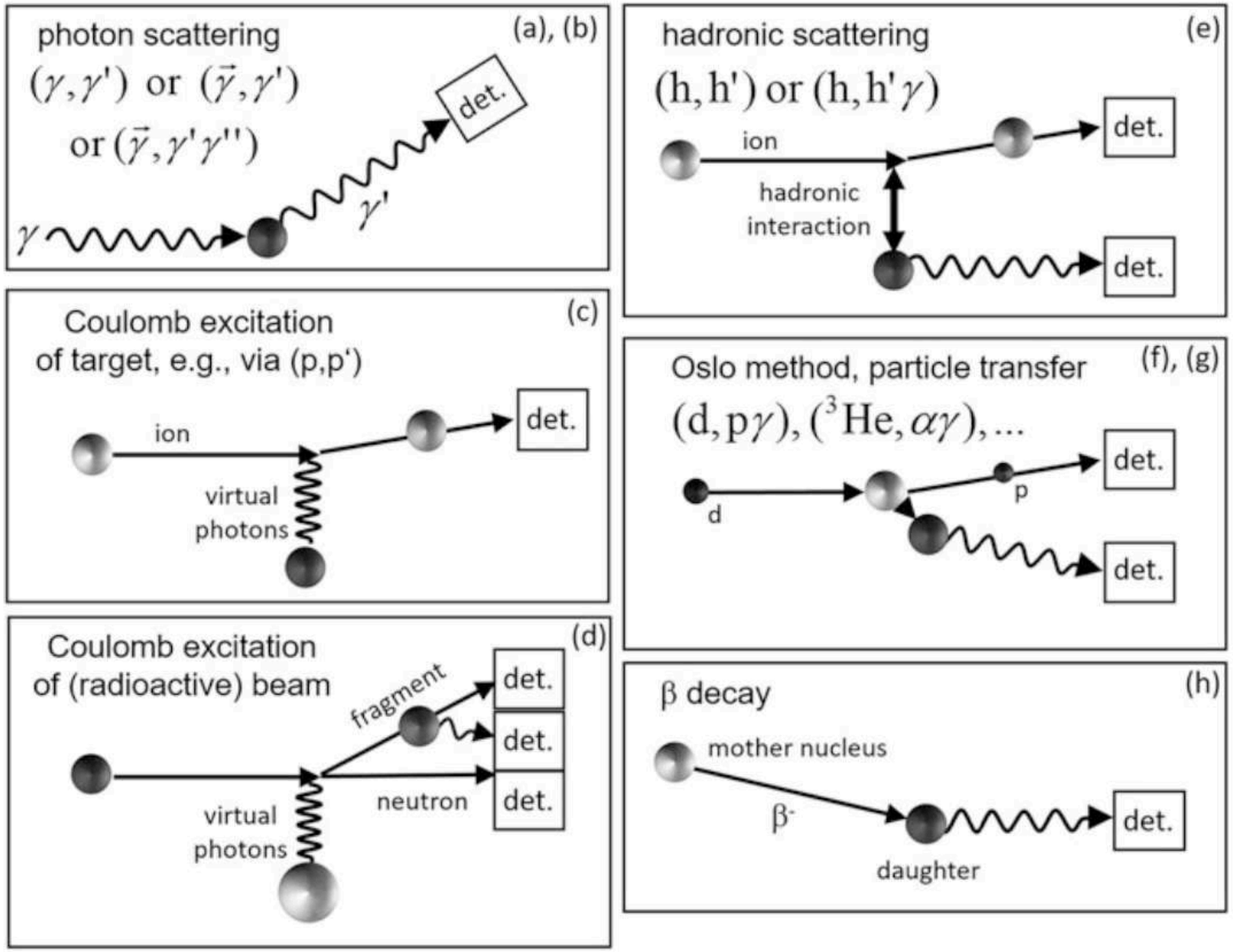
Summary

1. Nuclear electric dipole response: introduction and motivation
2. Theoretical framework: Configuration interaction shell model
3. Systematic electric dipole response of light nuclei
4. Application to ultra high energetic cosmic rays
5. A shell model view on the pygmy dipole resonance

Nuclear electric dipole response (E1): definition

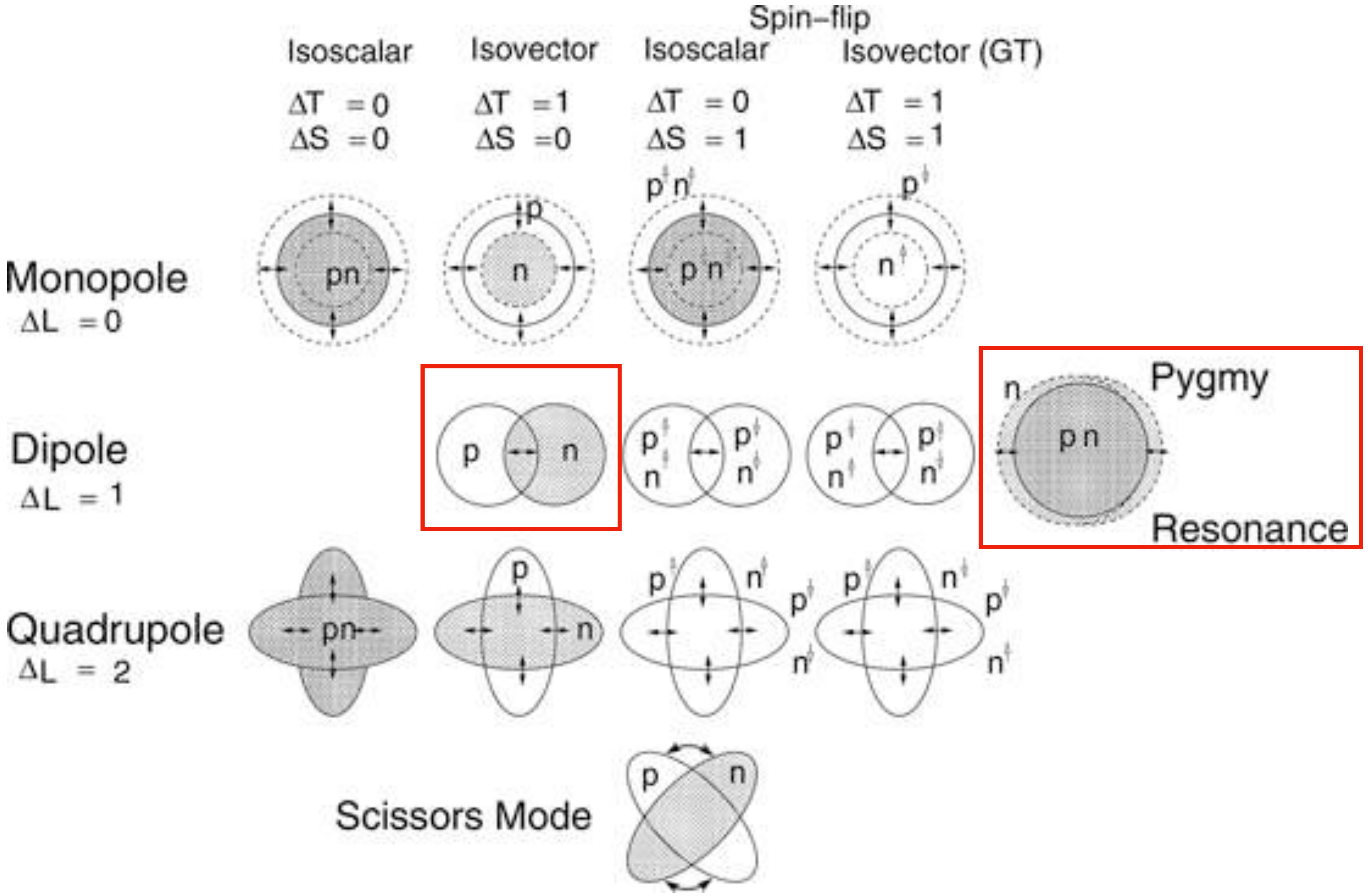




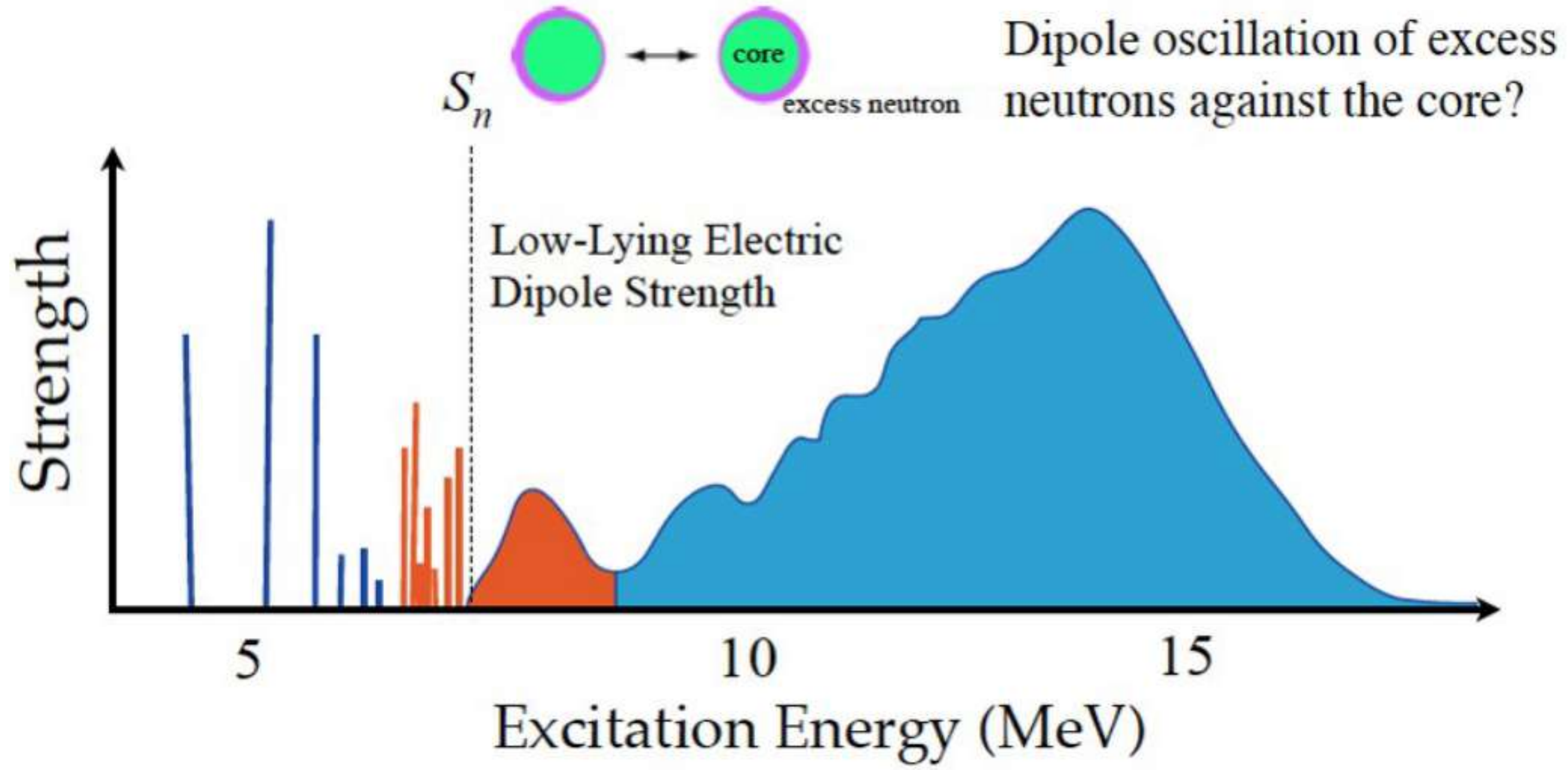


Nuclear dipole response: definition

Nuclear resonance modes

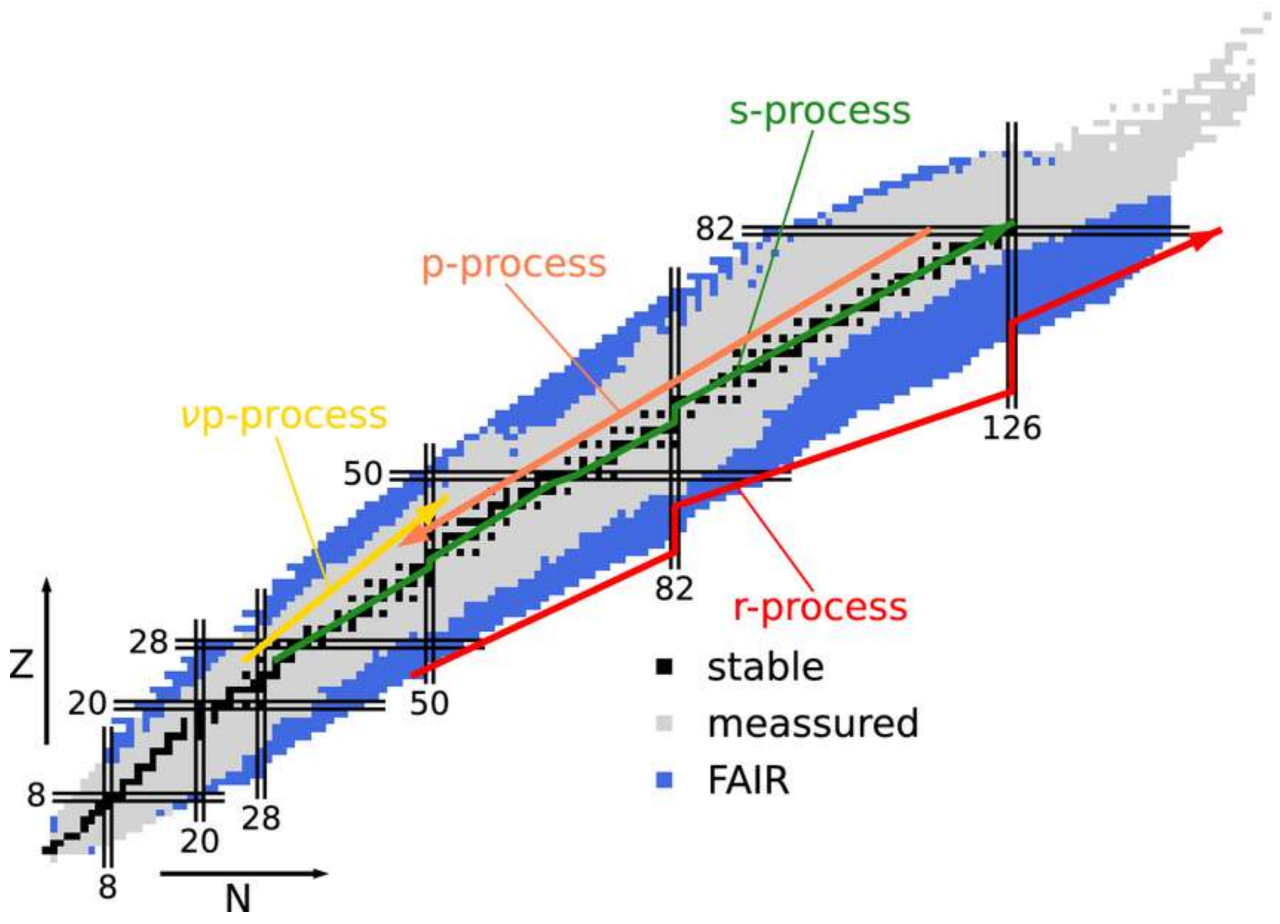


Nuclear electric dipole response :
E1 strength function (PSF)



Nuclear dipole response: motivation

R-process nucleosynthesis



Neutron matter equation of state

$$\mathcal{E}(\rho,\alpha) = \mathcal{E}_{\mathrm{SNM}}(\rho) + \alpha^2 \mathcal{S}(\rho) + \mathcal{O}(\alpha^4)$$

$\rho = (\rho_n + \rho_p)$ $\alpha = (\rho_n - \rho_p)/\rho$

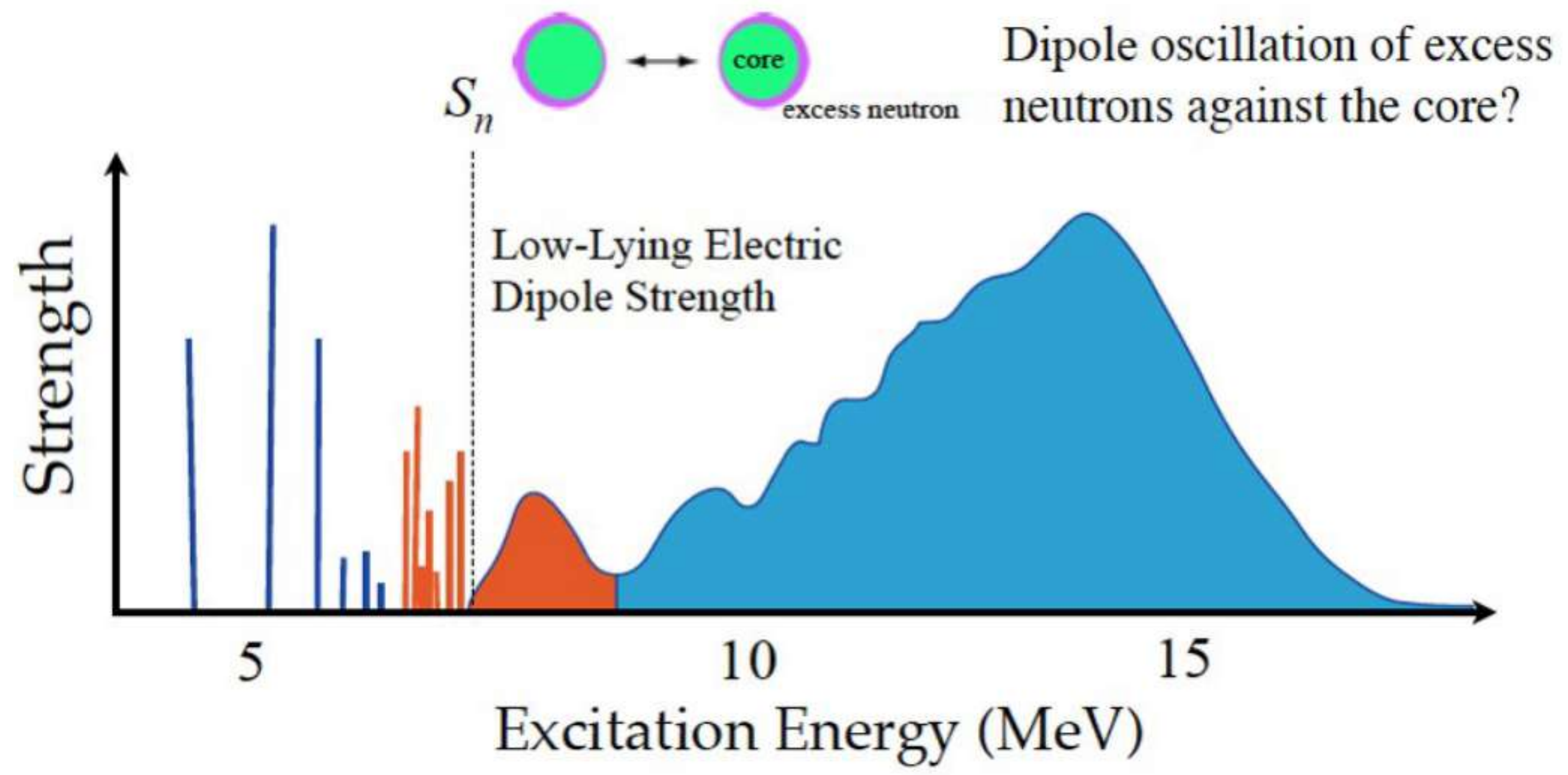
where the density-dependent symmetry energy is:

$$\mathcal{S}(\rho) = J + L\frac{(\rho-\rho_0)}{3\rho_0} + \ldots.$$

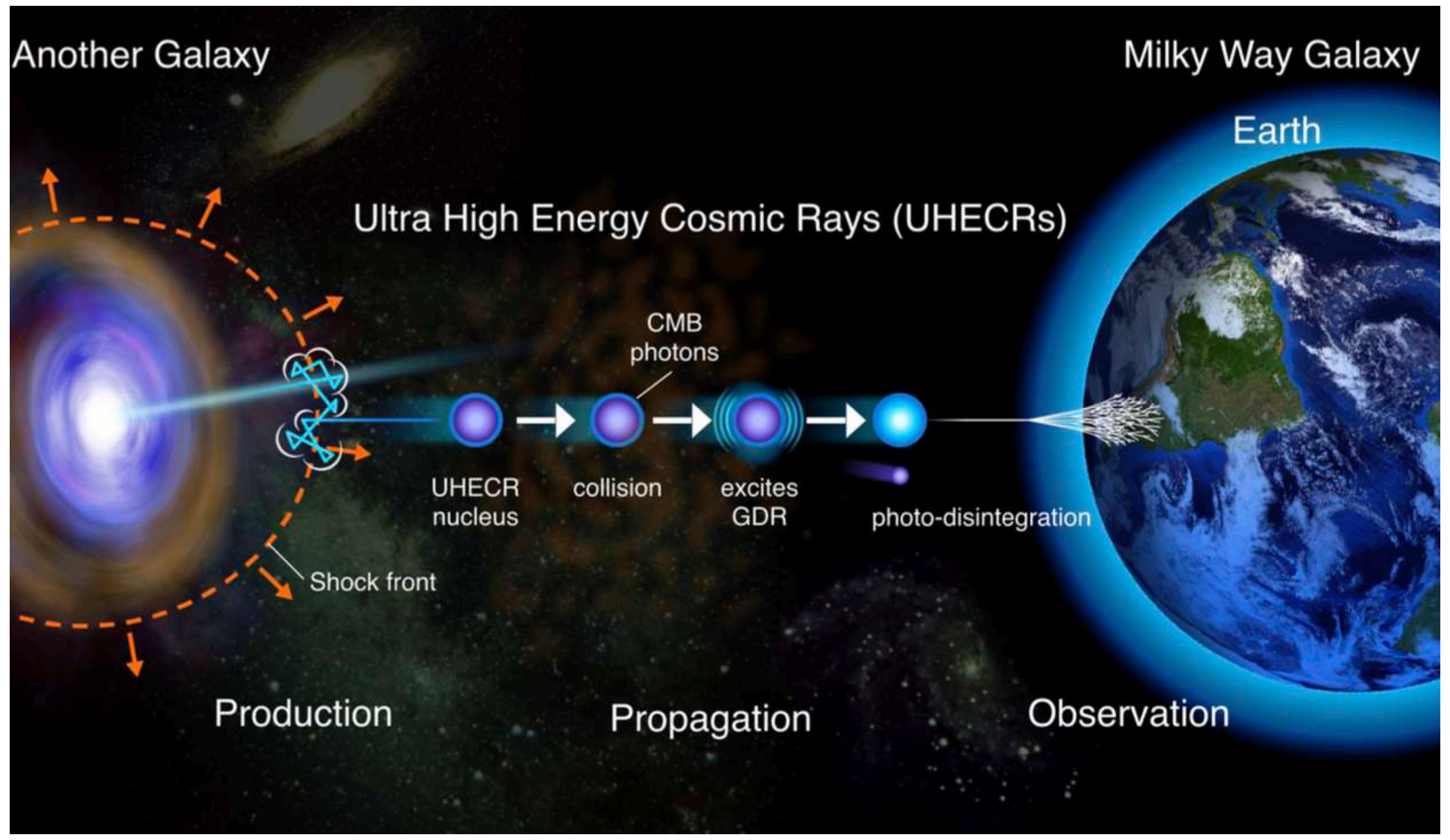
symmetry energy at saturation density

slope parameter, related to pressure of pure neutron matter at saturation density

Nuclear structure

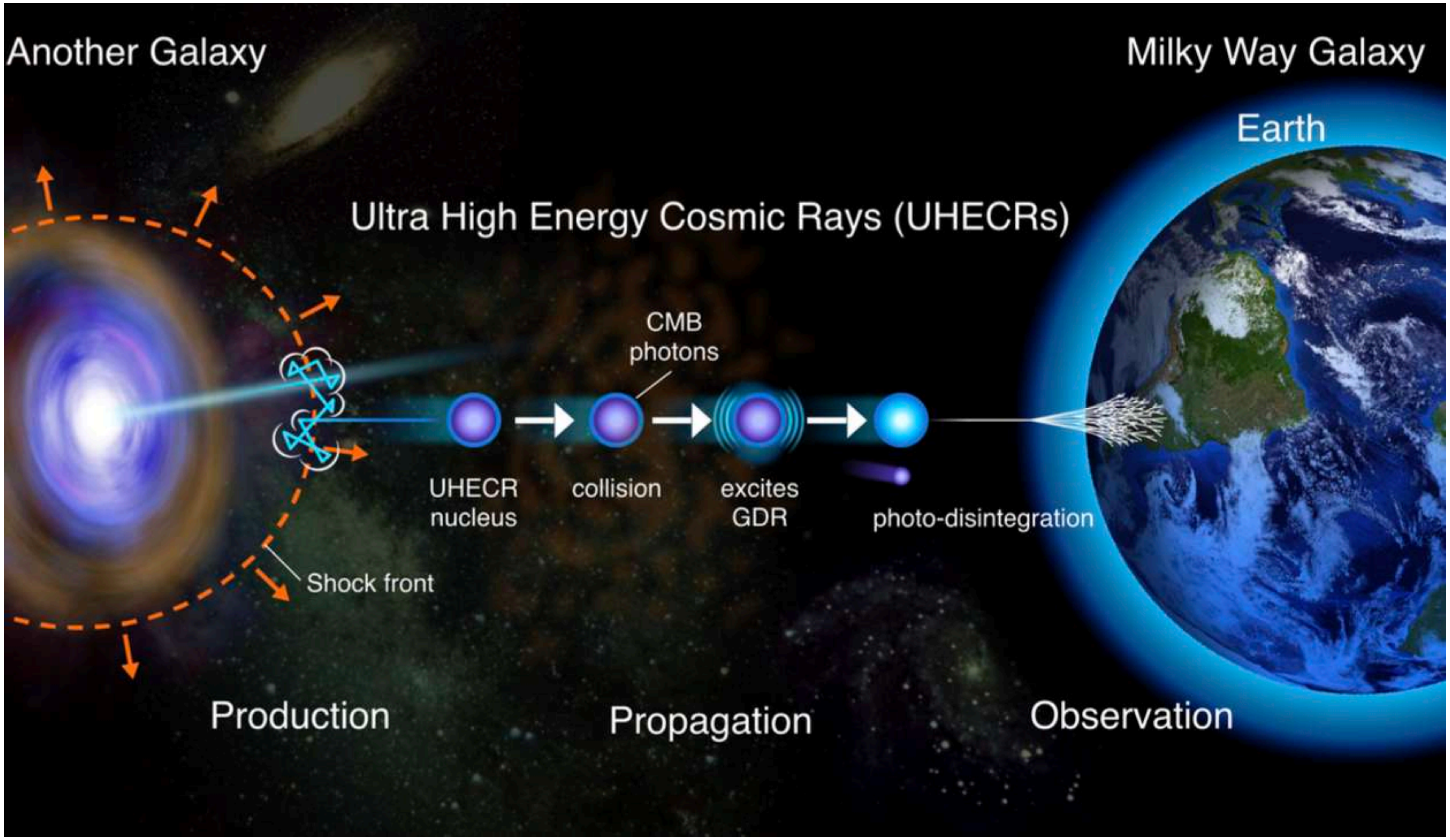


Ultra High Energetic Cosmic Rays



Nuclear dipole response: motivation

Ultra High Energetic Cosmic Rays



PANDORA Project

$\longrightarrow$ Need all PSFs for isotopes below $A \sim 60$

$\longrightarrow$ Very few exp in this mass region and often mutually incompatible

$\longrightarrow$ Need reliable theoretical predictions especially for unstable nuclei

Theoretical landscape

Models commonly used to predict PSFs

Purely phenomenological approaches

- Simple Modified Lorentzian (SMLO)
- Artificial Neural Network (ANN)

Microscopic approaches

Linear response:

- Quasi-particle Random Phase approximation (QRPA)
- Quasi-particle Finite Amplitude Method (QFAM)

Beyond linear response:

- Second QRPA (SQRPA)
- QRPA + phonon coupling
- Projected Generator Coordinates Method (PGCM)
- Configuration Interaction Shell Model (CI-SM)
- …

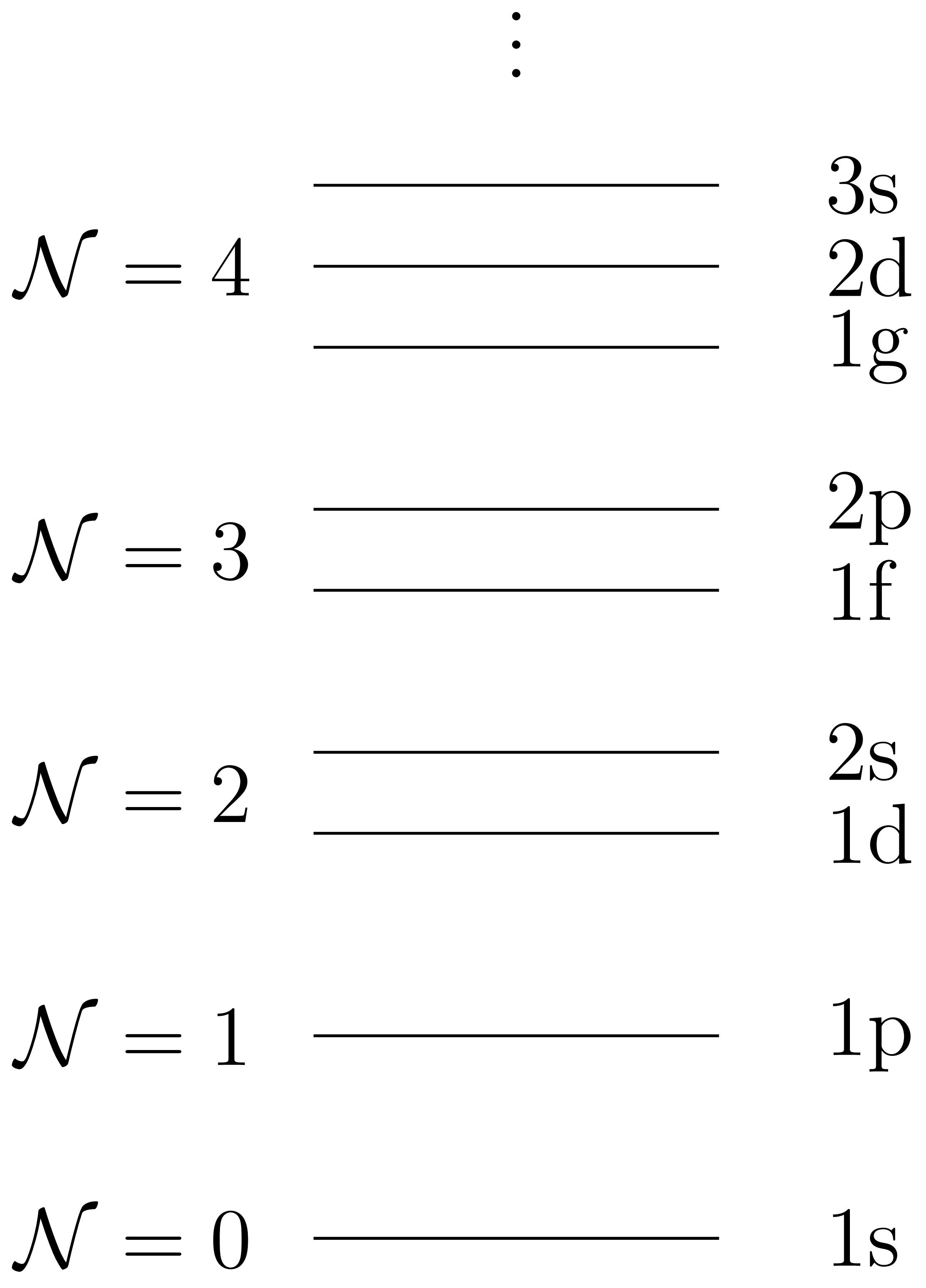
Theoretical framework: CI-SM

Quantity of interest: transition probability amplitudes

$$\langle \psi_f || O_\lambda || \mathrm{GS} \rangle$$

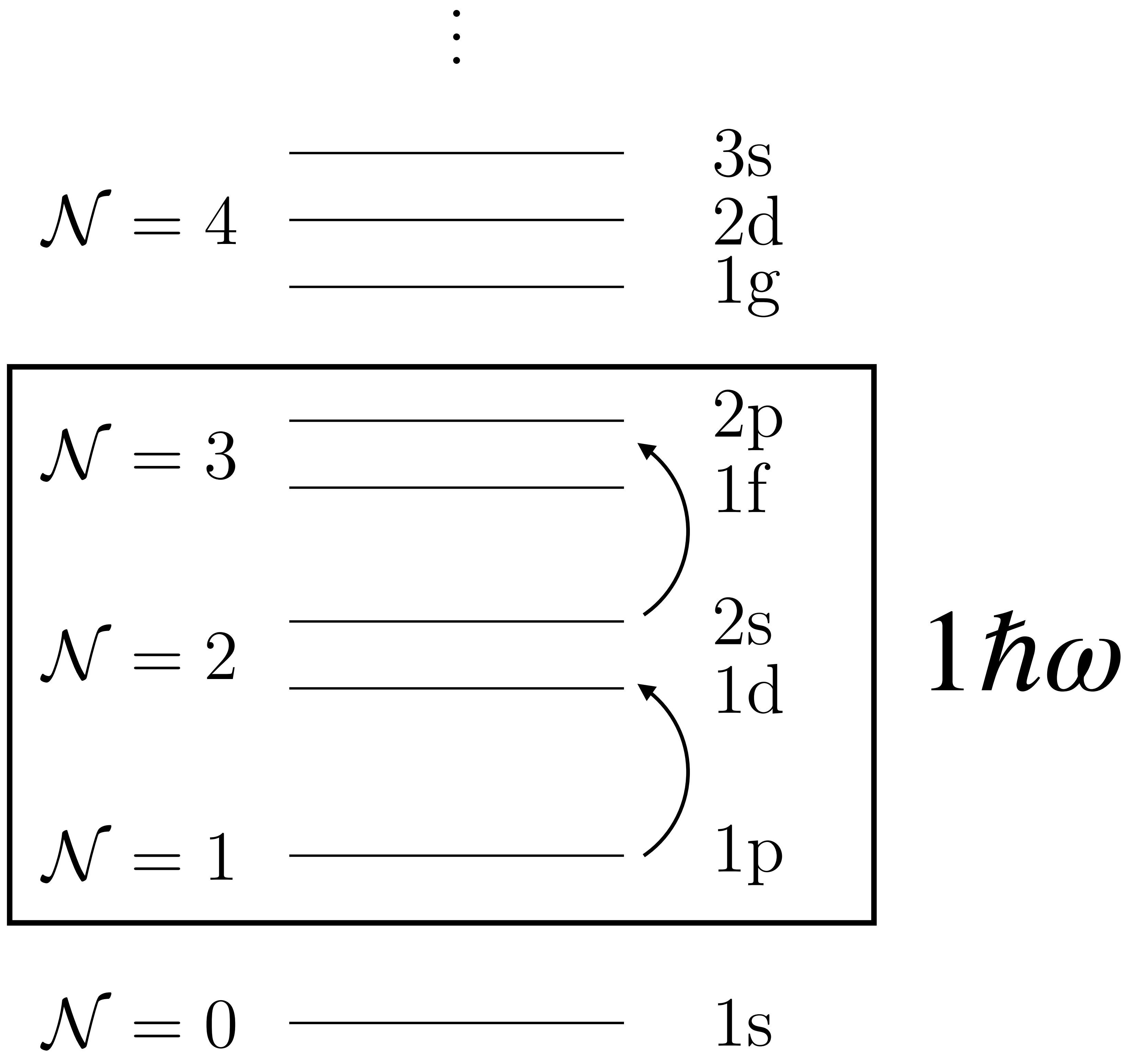
Discrete strength function

$$S(E) = \sum_f \frac{|\langle \psi_f || O_\lambda || \mathrm{GS} \rangle|^2}{2J_{\mathrm{GS}} + 1} \delta(E_f - E)$$



Valence space mapping:
E1 sd-shell nuclei

$$\mathcal{H} \longrightarrow \mathcal{H}_{\mathrm{valence}}$$
$$H \longmapsto H_{\mathrm{eff}}$$



Theoretical framework: CI-SM

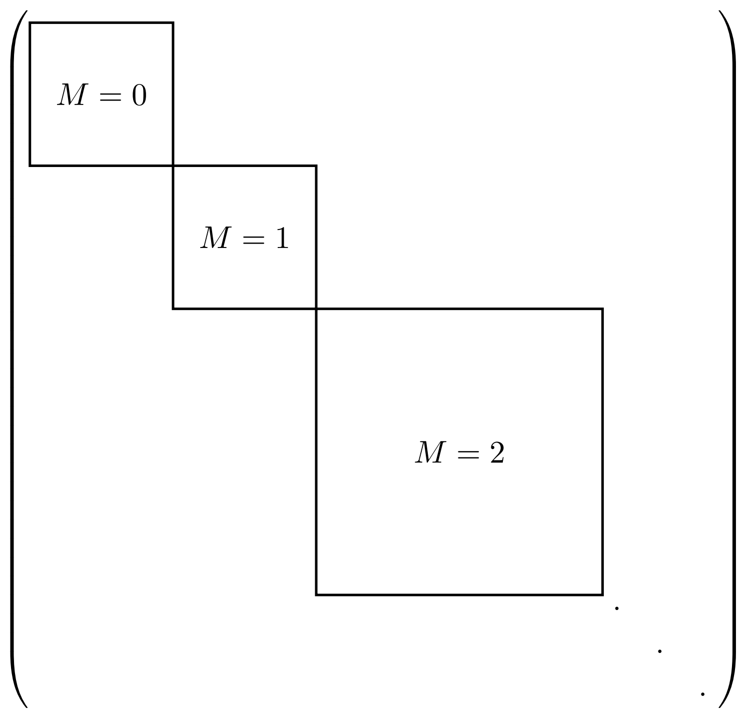
M-scheme Shell Model

Spherical harmonic single particle basis: $|\alpha\rangle = |n_\alpha, l_\alpha, j_\alpha, m_\alpha\rangle \longrightarrow$ Many-body basis: Fock states $|\Phi\rangle = \sum_{i=1}^{A} a^\dagger_{\alpha_i} |0\rangle = |\cdots 0010110 \cdots\rangle$

Bit-string representation of Slater determinants

Good total M value: $J^z |\Phi\rangle = \sum_{i=1}^{A} j^z_i |\Phi\rangle = \sum_{i=1}^{A} m_i |\Phi\rangle = M |\Phi\rangle$

$$[H_{\text{eff}}, J^z] = 0 \implies H_{\text{eff}} \text{ bloc diagonal in } M$$



M-scheme dimension for ^{18}O (GS) $= 14$

M-scheme dimension for ^{60}Zn (GS) $\sim 10^9$ $\longrightarrow$ Iterative diagonalization procedure

Theoretical framework: CI-SM

Lanczos iterative diagonalization

$\longrightarrow$ Extremal eigenvalues converge first (GS + first excited states)

$\longrightarrow$ Very efficient for sparse matrix (H_{eff} connects only basis states differing by 1 or 2 single particle occupation)

Initialization: initial pivot state e.g. random pivot: $$|\mathcal{L}_1\rangle \equiv \frac{1}{||r||}\sum_{i=1}^{d(M)} r_i\,|\Phi_i\rangle \longrightarrow H_{11} = \langle\mathcal{L}_1|\,H\,|\mathcal{L}_1\rangle$$

Construction of the Lanczos orthonormal basis in which H_{eff} is tridiagonal:

$$H\,|\mathcal{L}_1\rangle = H_{11}\,|\mathcal{L}_1\rangle + |\tilde{\mathcal{L}_2}\rangle$$

$$\longrightarrow |\mathcal{L}_2\rangle = \frac{|\tilde{\mathcal{L}_2}\rangle}{||\tilde{\mathcal{L}_2}||} \longrightarrow H_{12} \quad H_{22}$$

$$|\tilde{\mathcal{L}_2}\rangle = (H - H_{11})\,|\mathcal{L}_1\rangle$$

$$H\,|\mathcal{L}_2\rangle = H_{12}\,|\mathcal{L}_1\rangle + H_{22}\,|\mathcal{L}_2\rangle + H_{23}\,|\mathcal{L}_3\rangle$$

$$H_{23}\,|\mathcal{L}_3\rangle = (H - H_{22})\,|\mathcal{L}_2\rangle - H_{12}\,|\mathcal{L}_1\rangle$$

$$\vdots$$

$$H_{nn+1}\,|\mathcal{L}_{n+1}\rangle = (H - H_{nn})\,|\mathcal{L}_n\rangle - H_{n-1n}\,|\mathcal{L}_{n-1}\rangle$$

$$H^{(n)} = \begin{pmatrix} H_{11} & H_{12} & 0 & \cdots & 0 \\ H_{21} & H_{22} & H_{23} & \ddots & \vdots \\ 0 & H_{32} & H_{33} & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & H_{n,n+1} \\ 0 & \cdots & 0 & H_{n+1,n} & H_{n+1,n+1} \end{pmatrix}$$

Usual diagonalization of $H^{(n)}$ (e.g. QR algorithm) $\longrightarrow \{E_j, |\psi_j\rangle\}$

Theoretical framework: CI-SM

Lanczos Strength function

Step 1: compute the GS $[H_{\mathrm{eff}}, J^z] = 0 \implies H_{\mathrm{eff}}$ bloc diagonal in M

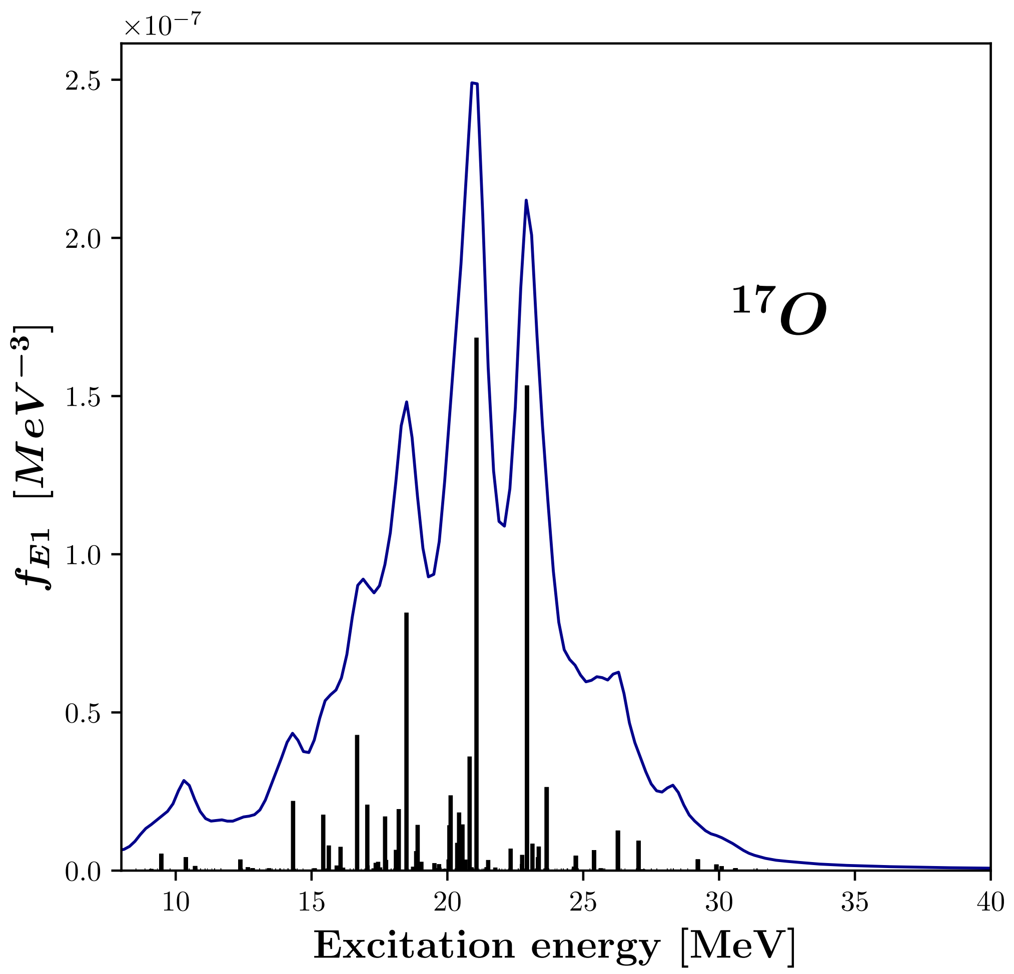
For a 0^+ GS we diagonalize H_{eff} in the $M = 0$ subspace using Lanczos algorithm $\implies |\mathrm{GS}\rangle$

Step 2: compute the sum rule state $|\mathrm{SR}\rangle = O\,|\mathrm{GS}\rangle$

Step 3: Lanczos calculation using $|\mathrm{SR}\rangle$

Unitary matrix $U_{ij} = \langle \mathcal{L}_i | \psi_j \rangle$ diagonalizing H_{eff} such that $U_{1j} = \langle \mathrm{GS} | O \, |\psi_j\rangle$

Step 4: Lorentzian folding



Theorem: for N_{it} Lanczos iterations the first $2N_{\mathrm{it}}$ moments S_k of the distribution are exact

$$S_k = \sum_j E_j^k |\langle \psi_j | O \, |\mathrm{GS}\rangle|^2 = \langle \mathrm{SR} | H^k |\mathrm{SR}\rangle$$

Theoretical framework: CI-SM

Dressing of the E1 operator $$O(\mathrm{E1}) = e\sum_{j=1}^{Z} rY_{10}$$

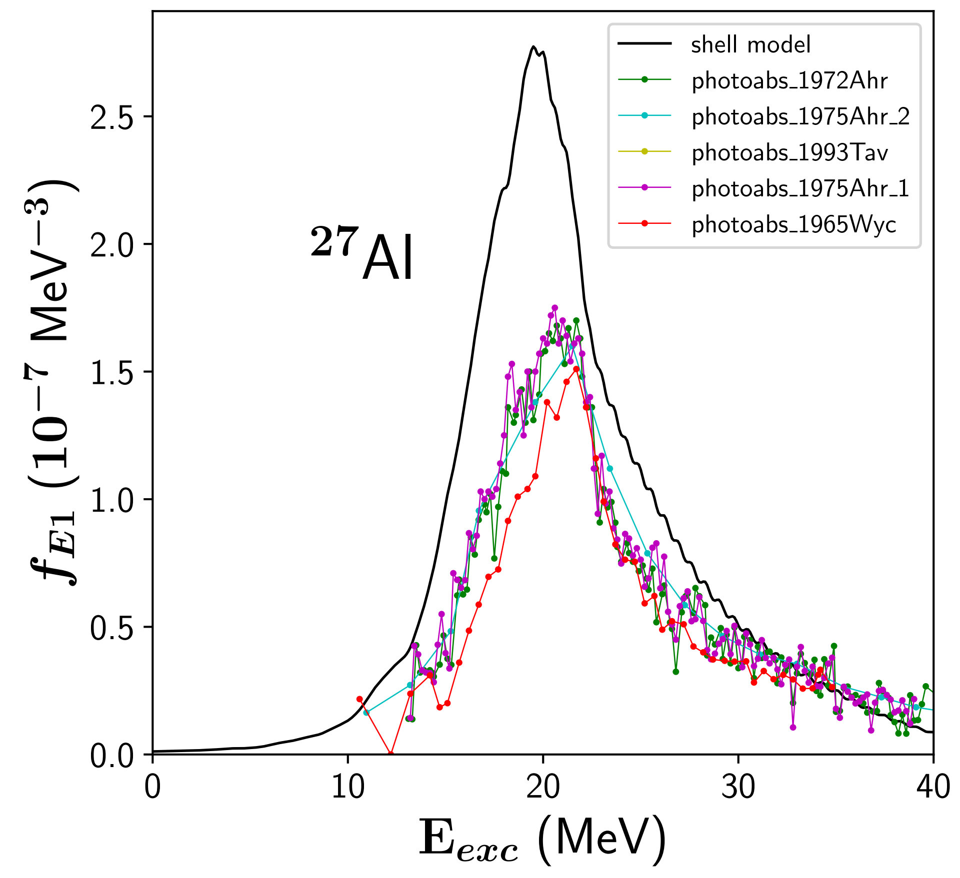
Valence space mapping: $$\begin{array}{ccc} \mathcal{H} & \longrightarrow & \mathcal{H}_{\text{valence}} \\ H & \longmapsto & H_{\text{eff}} \end{array}$$ Likewise for all observables $$\begin{array}{ccc} \mathcal{H} & \longrightarrow & \mathcal{H}_{\text{valence}} \\ O(\mathrm{E1}) & \longmapsto & O_{\text{eff}}(\mathrm{E1}) \end{array}$$

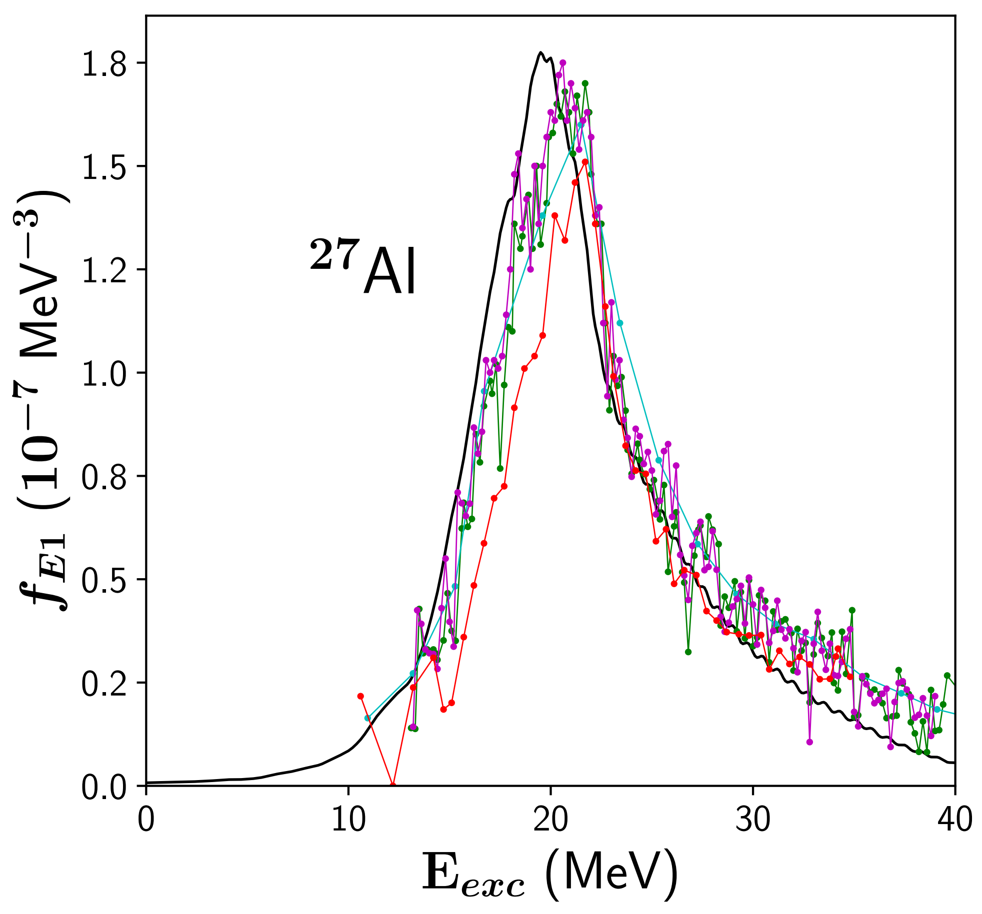
How to address this issue ?

Microscopic approaches (many body perturbation): Lee-Suzuki, IMSRG

Phenomenological approaches: scaling to data (effective charges),

scaling to TRK $$S_1^{TRK} = \frac{9\hbar^2 e^2}{8\pi m}\frac{NZ}{A}$$





Details of numerical calculations

300 Lanczos iterations

Lawson's method for COM

Effective interactions:

- E1 sd-shell: PSDPF
- E1 p-shell: WBP

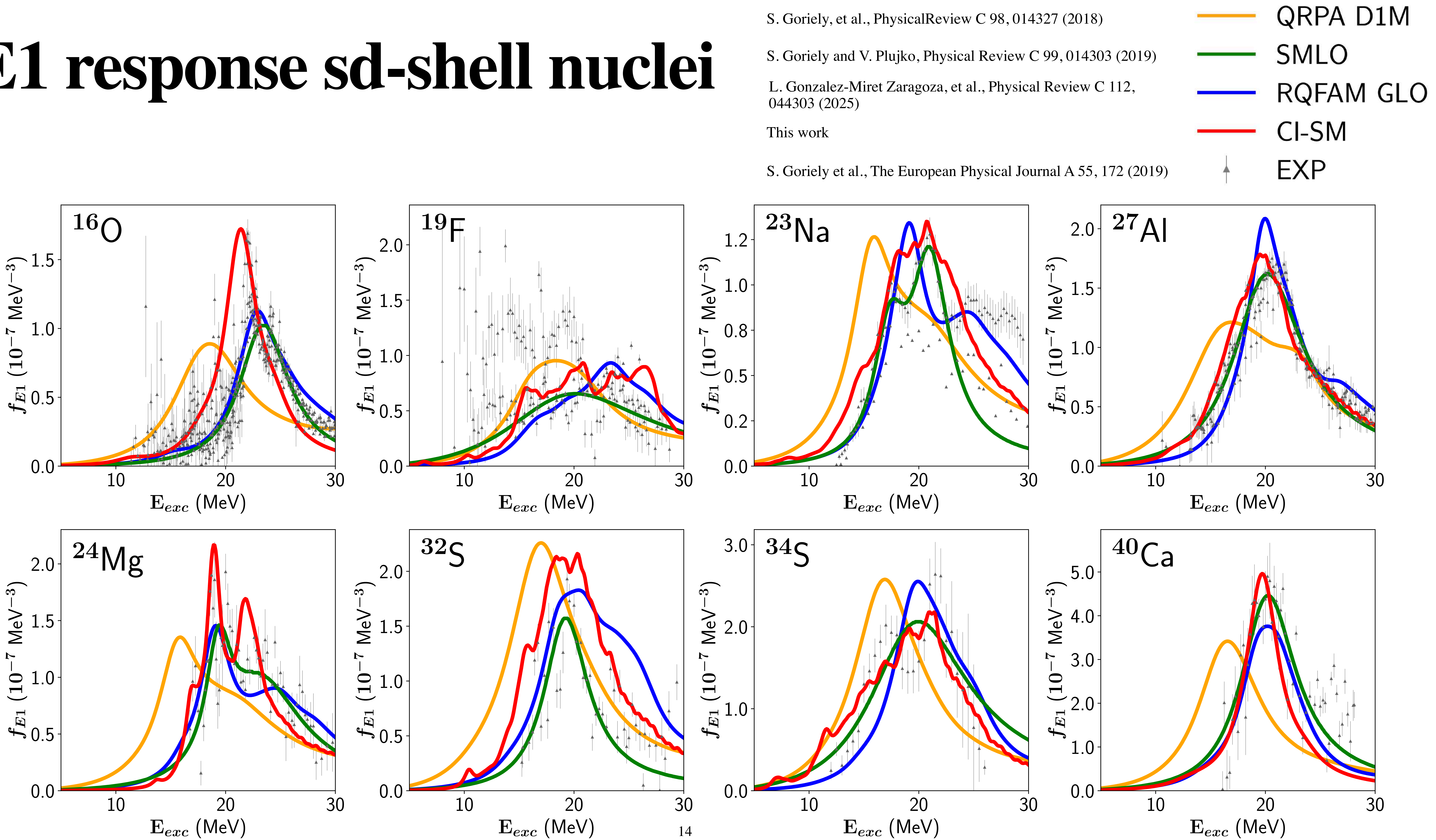
E1 response p-shell nuclei

L. Gonzalez-Miret Zaragoza, et al., Physical Review C 112, 044303 (2025) — RQFAM GLO

This work — CI-SM

S. Goriely et al., The European Physical Journal A 55, 172 (2019) — EXP

E1 response sd-shell nuclei



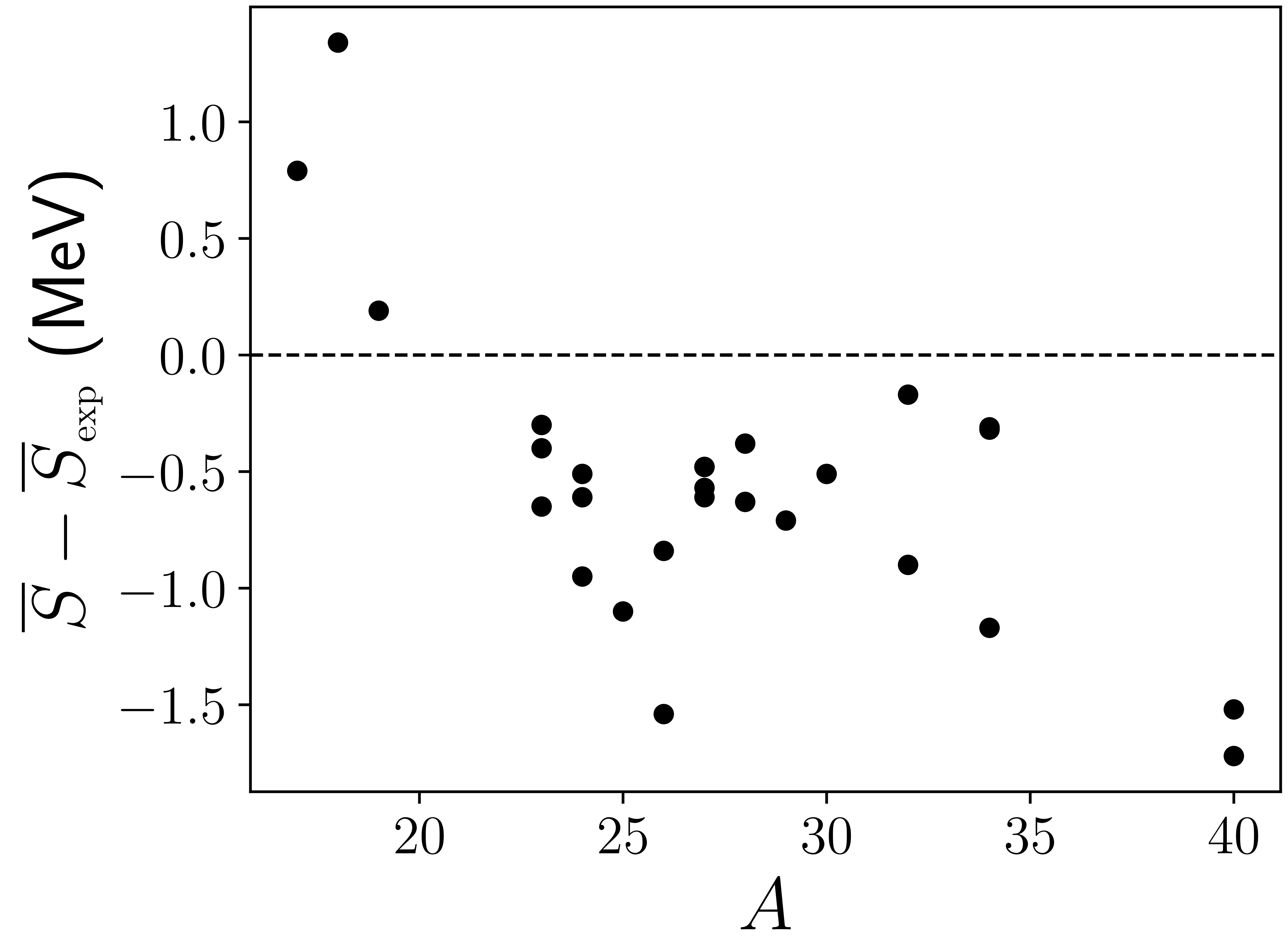
E1 sd-shell nuclei systematics

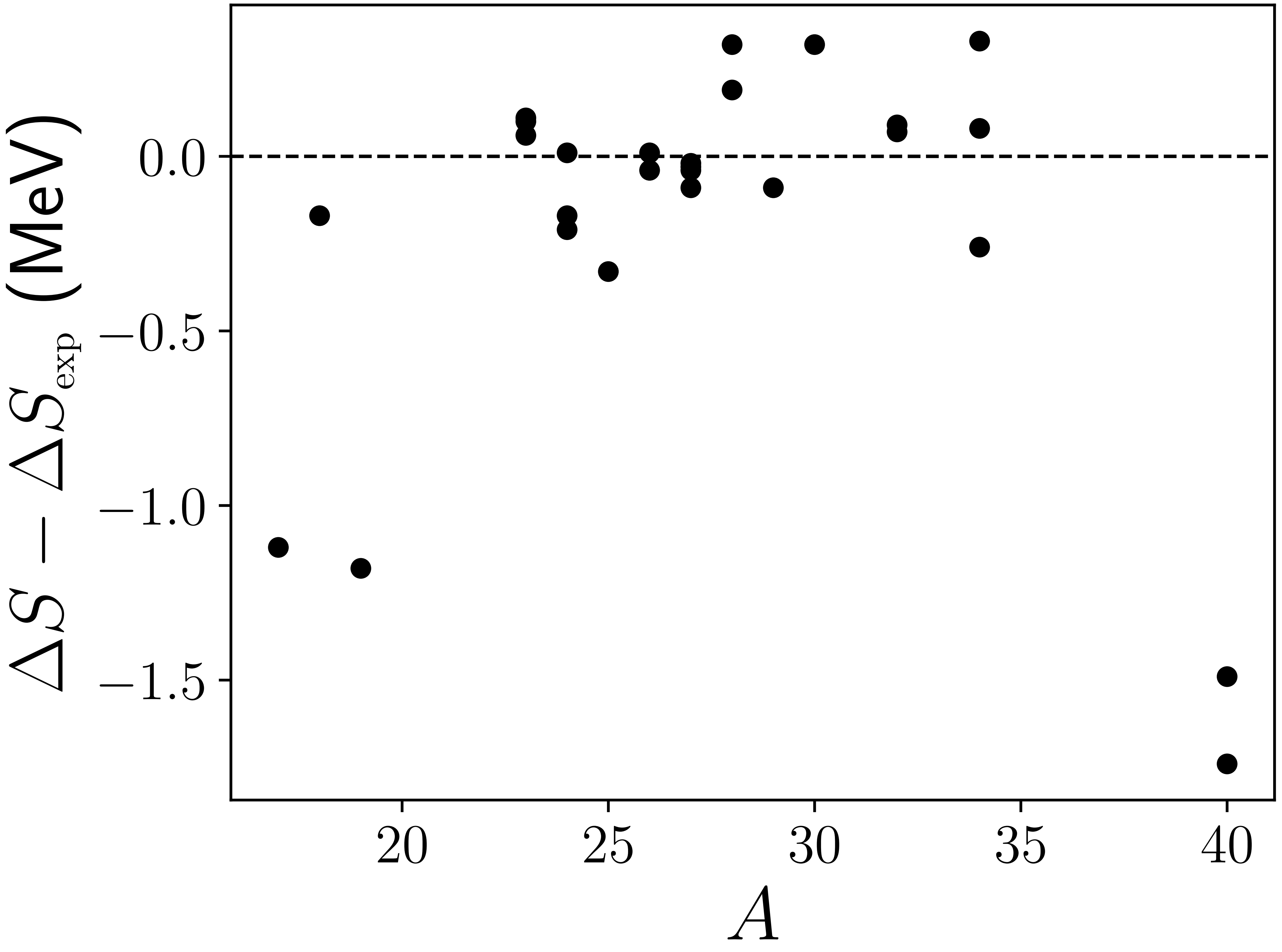
132 nuclei (all isotopic chains within the valence space)

M1 and E1 PSF available in the Talys database

Centroids $\bar{S} = \frac{S_1}{S_0}$

Width $\Delta S = \sqrt{\frac{S_2}{S_0} - \bar{S}^2}$





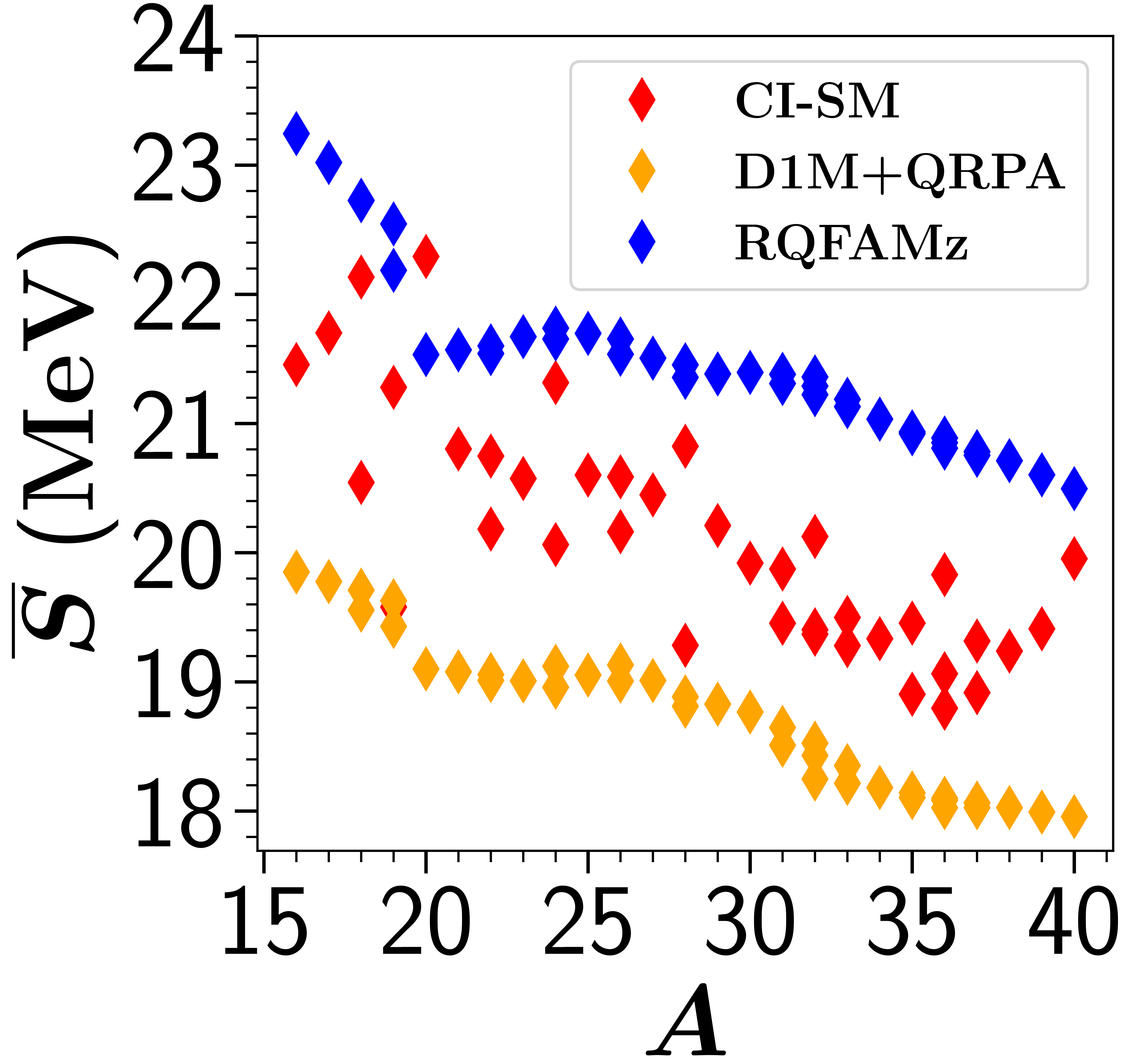
RMS $0.84 \quad \mathrm{MeV}$

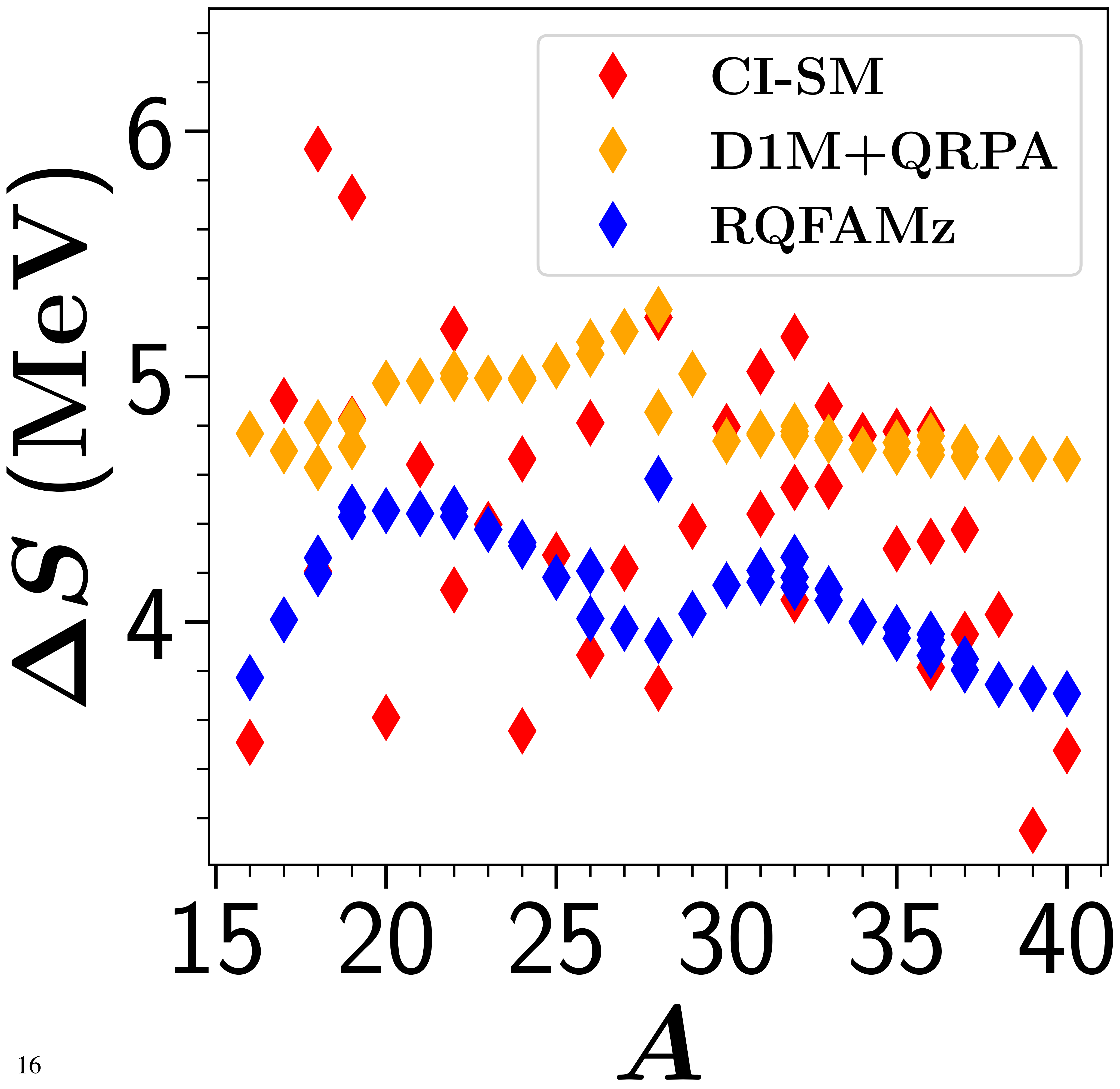
RMS $0.56 \quad \mathrm{MeV}$

E1 sd-shell nuclei systematics

Centroids $\bar{S} = \frac{S_1}{S_0}$

Width $\Delta S = \sqrt{\frac{S_2}{S_0} - \bar{S}^2}$

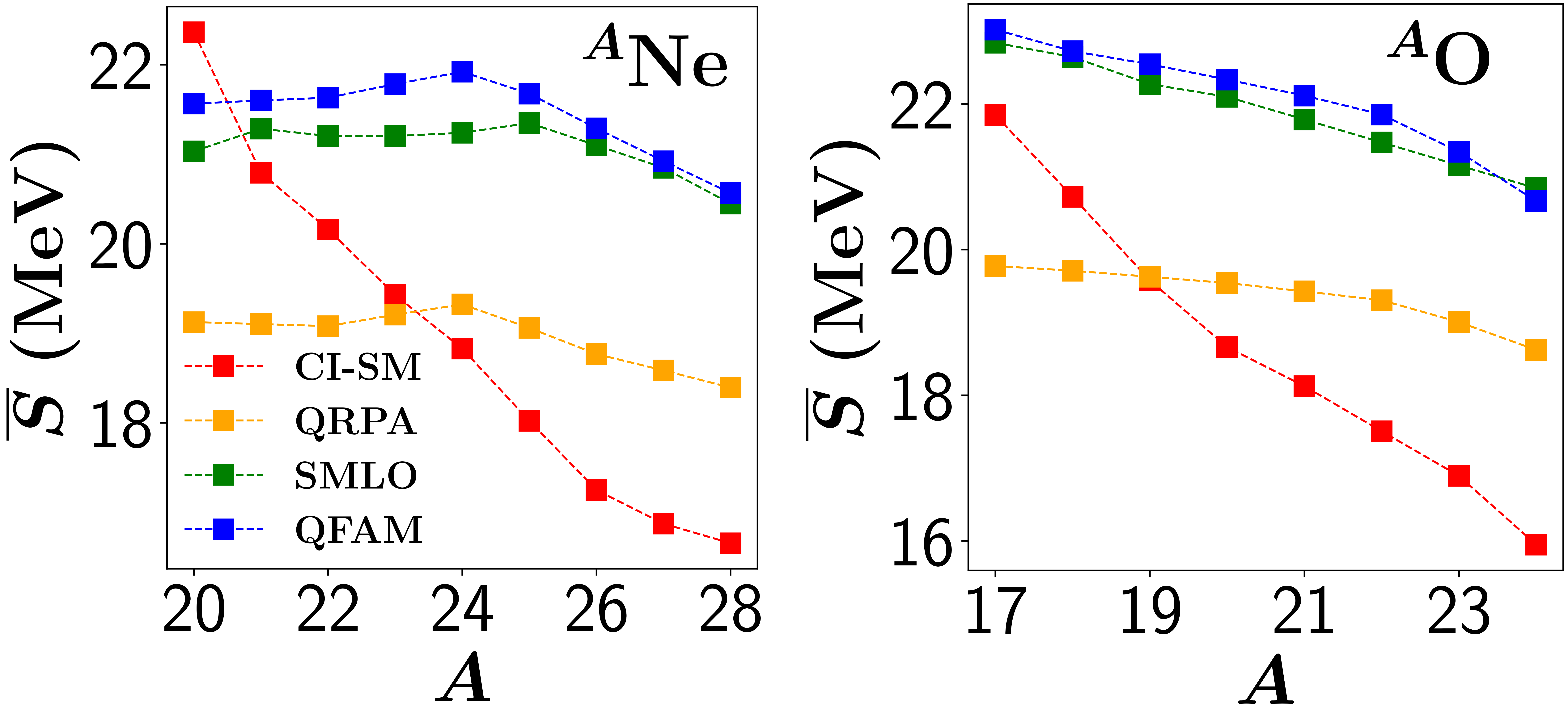




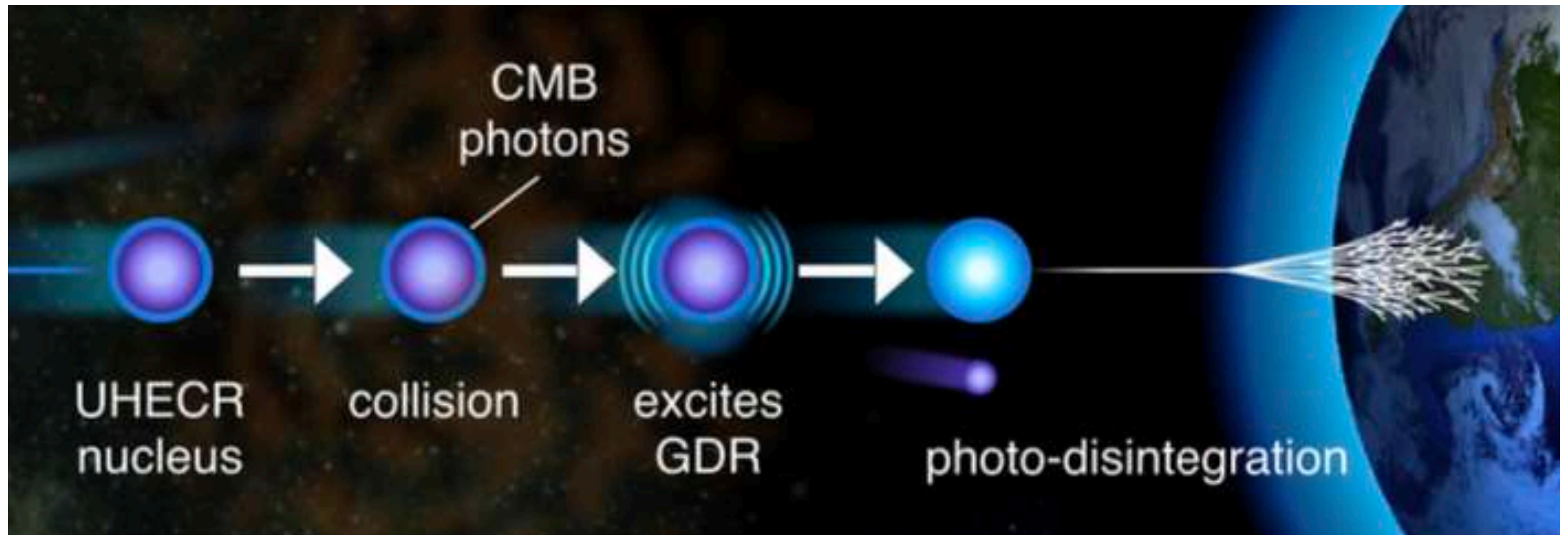
E1 response sd-shell nuclei

Neutron rich nuclei: model comparison

up to 30 MeV



Application to UHECR propagation



$$
\begin{aligned}
\frac{dN_{Z,A}}{dt} = {} & N_{Z+1,A}\lambda_{\beta}^{Z+1,A} + N_{Z-1,A}\lambda_{\beta}^{Z-1,A} \\
& + N_{Z,A+1}\lambda_{\gamma,n}^{Z,A+1} + N_{Z+1,A+1}\lambda_{\gamma,p}^{Z+1,A+1} \\
& + N_{Z+2,A+4}\lambda_{\gamma,\alpha}^{Z+2,A+4} + N_{Z,A+2}\lambda_{\gamma,2n}^{Z,A+2} \\
& + N_{Z+2,A+2}\lambda_{\gamma,2p}^{Z+2,A+2} + N_{Z+4,A+8}\lambda_{\gamma,2\alpha}^{Z+4,A+8} \\
& + N_{Z+1,A+2}\lambda_{\gamma,np}^{Z+1,A+2} + N_{Z+2,A+5}\lambda_{\gamma,n\alpha}^{Z+2,A+5} \\
& + N_{Z+3,A+5}\lambda_{\gamma,p\alpha}^{Z+3,A+5} \\
& - N_{Z,A}\left[\lambda_{\beta}^{Z,A} + \sum_{x}\lambda_{\gamma,x}^{Z,A}\right] ,
\end{aligned}
$$

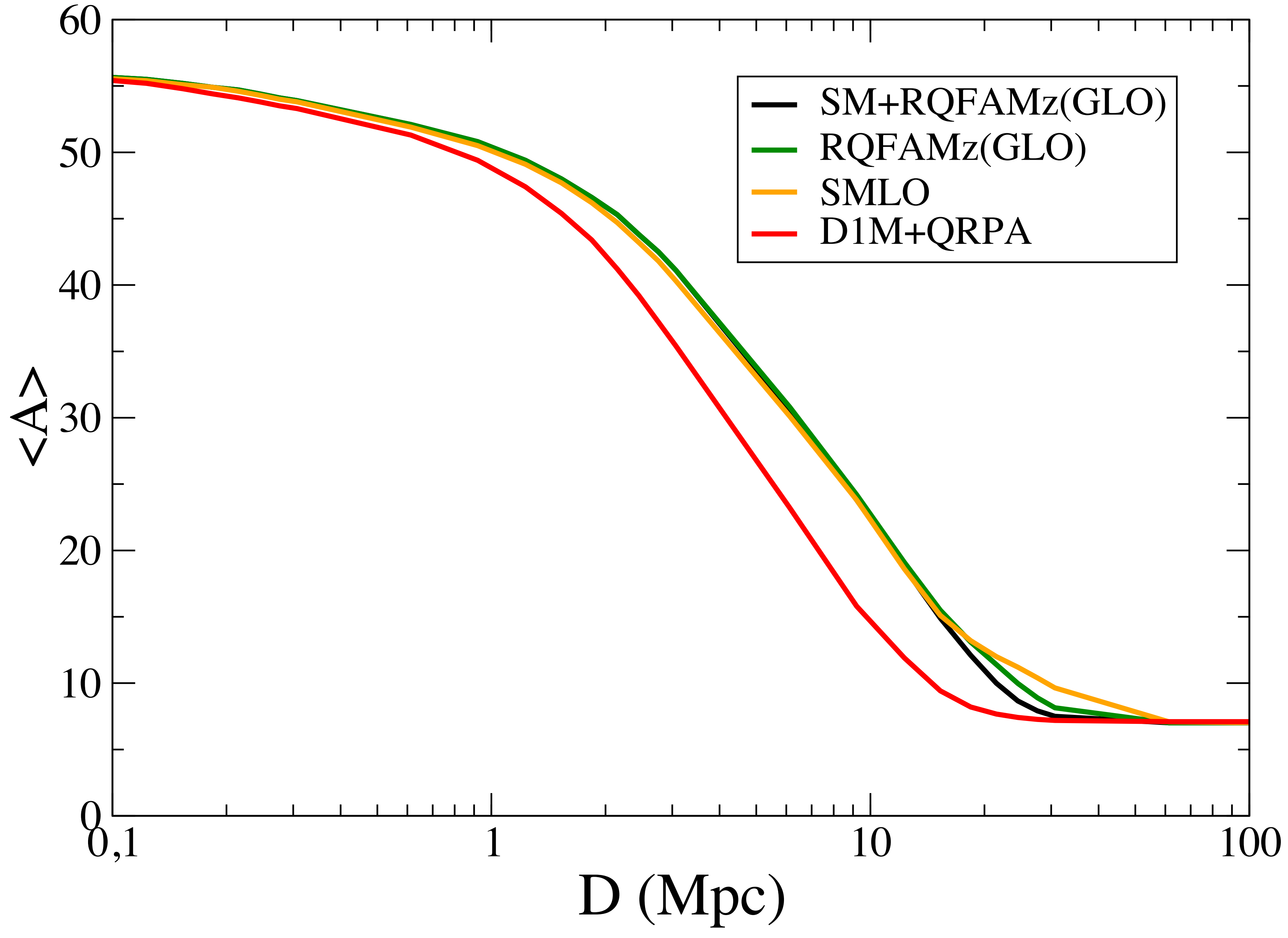
$$\lambda_{\gamma,x} = \int n(\epsilon)\ \sigma_{\gamma,x}(\epsilon)\ c\ d\epsilon$$

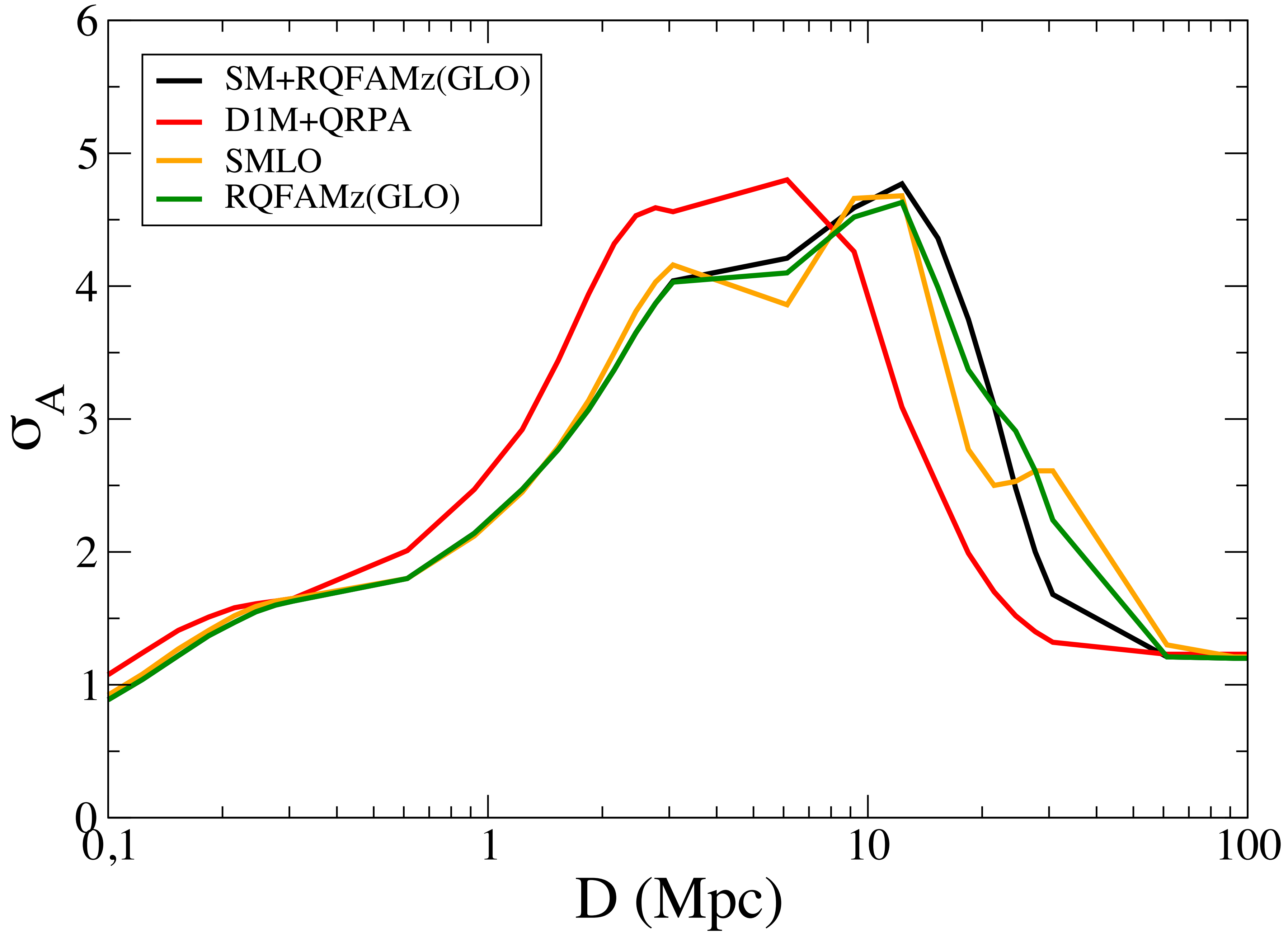
$\uparrow$

TALYS

$\uparrow$

CI-SM
PSFs

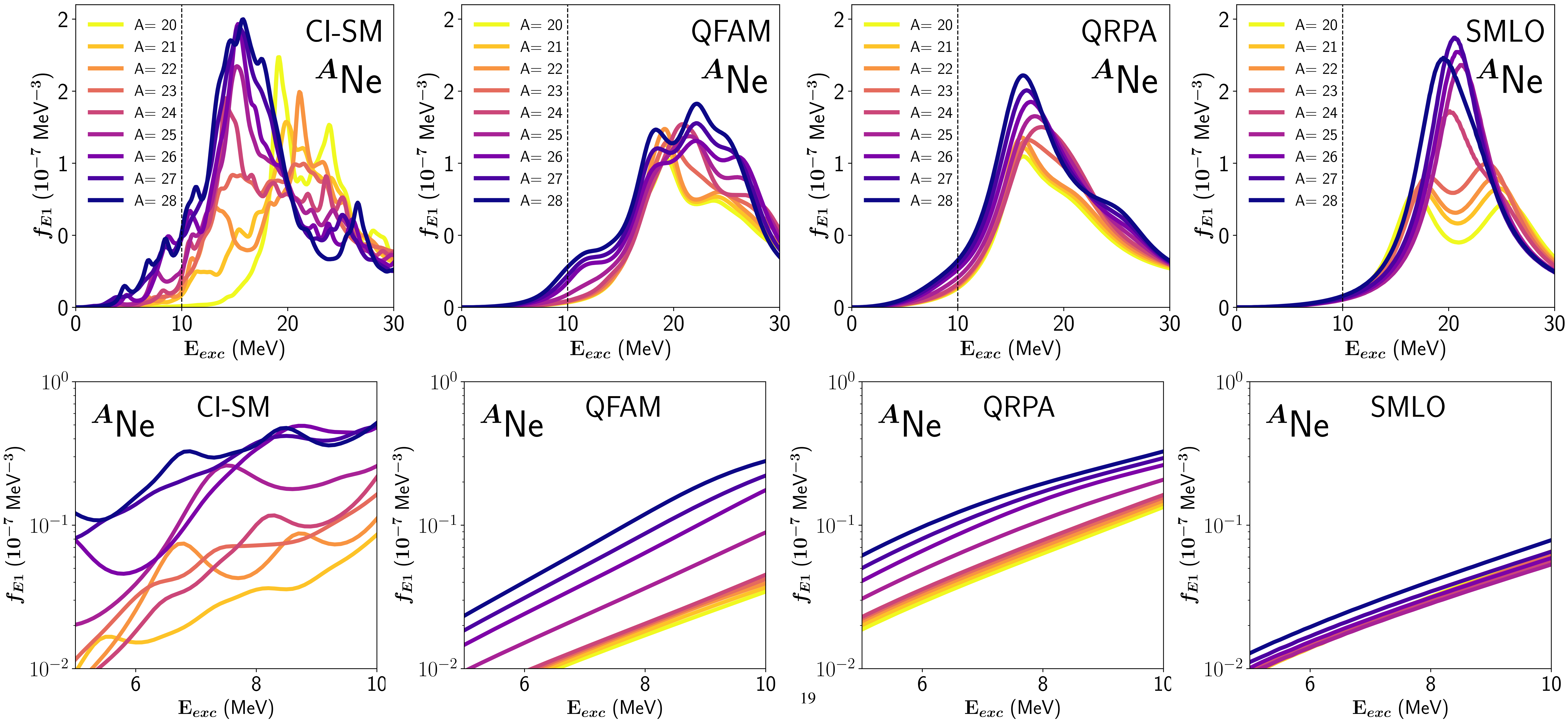




O. Le Noan, E. Khan, K. Sieja, S. Goriely, in preparation

Study of the Pygmy dipole resonance

Neon isotopic chain

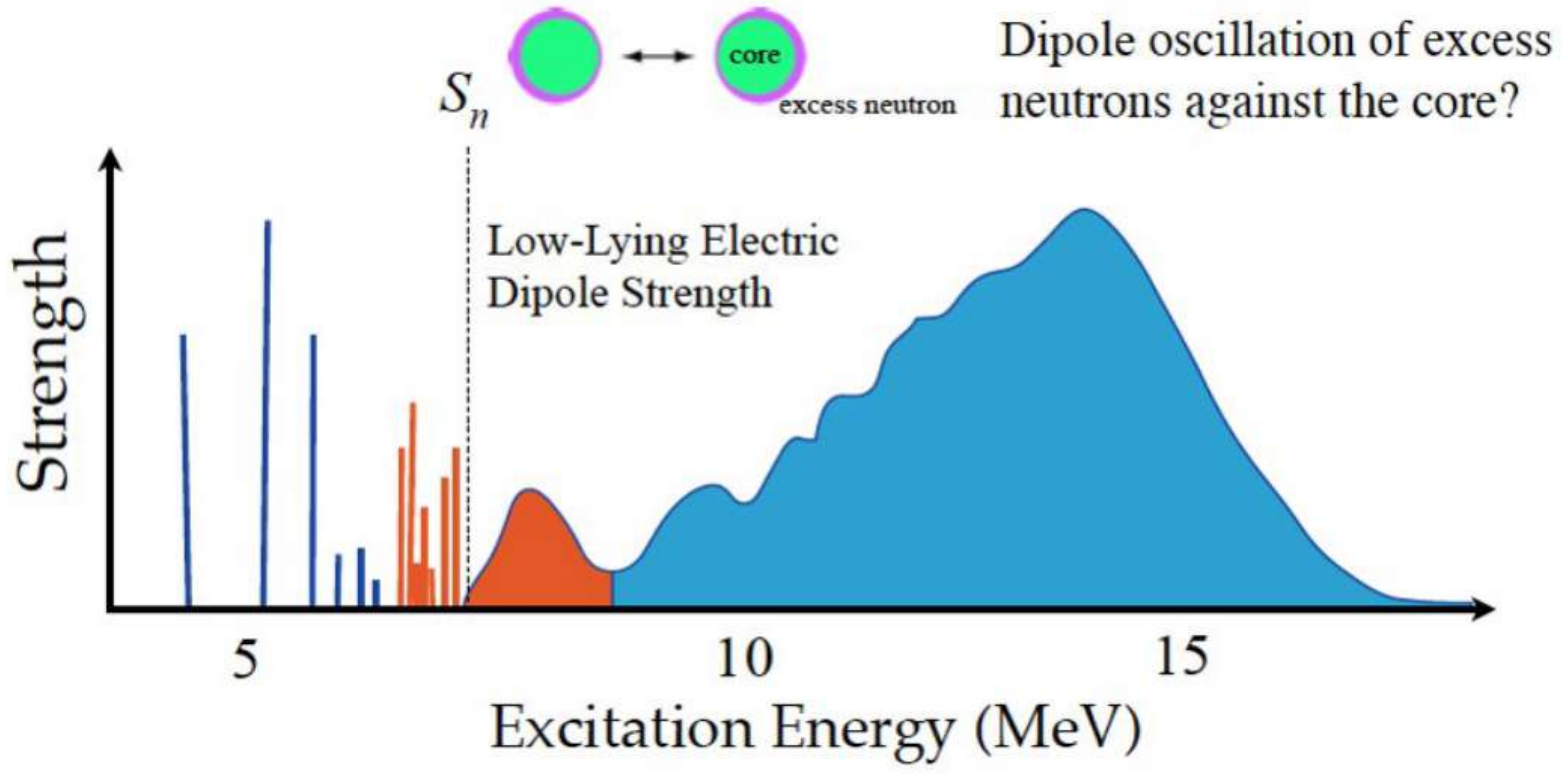


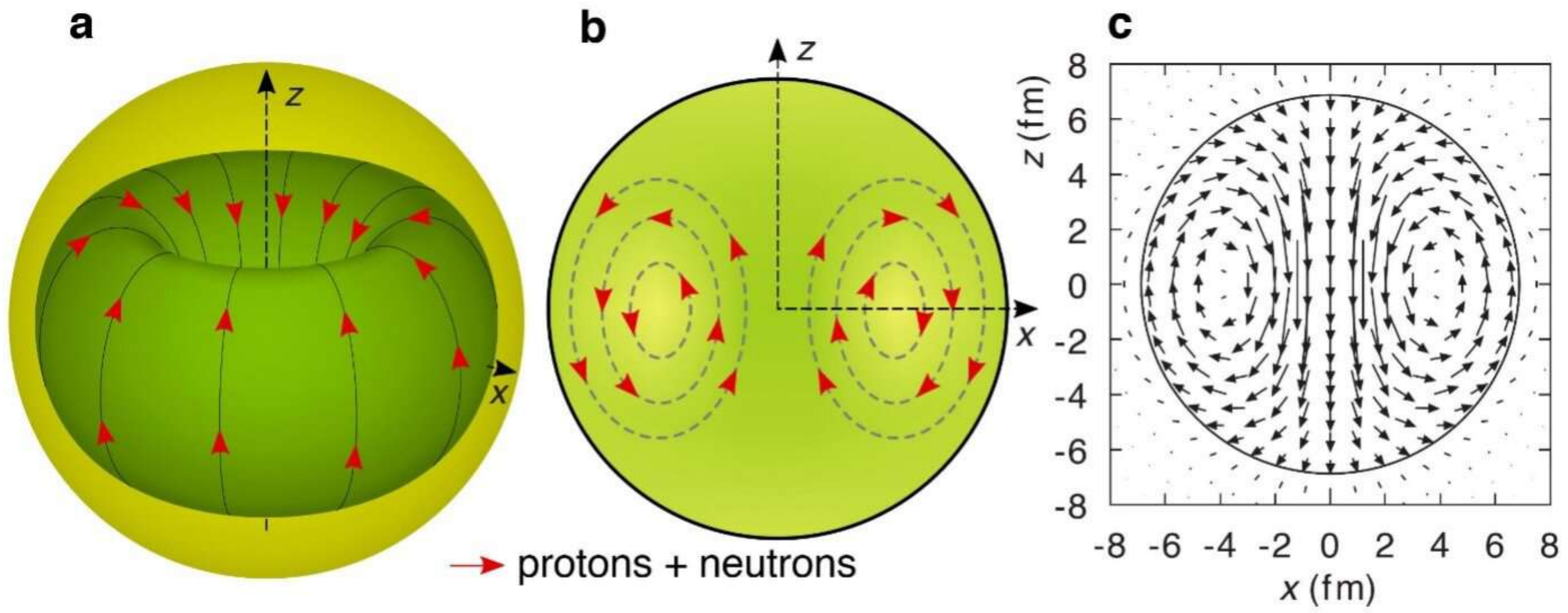
Study of the Pygmy dipole resonance

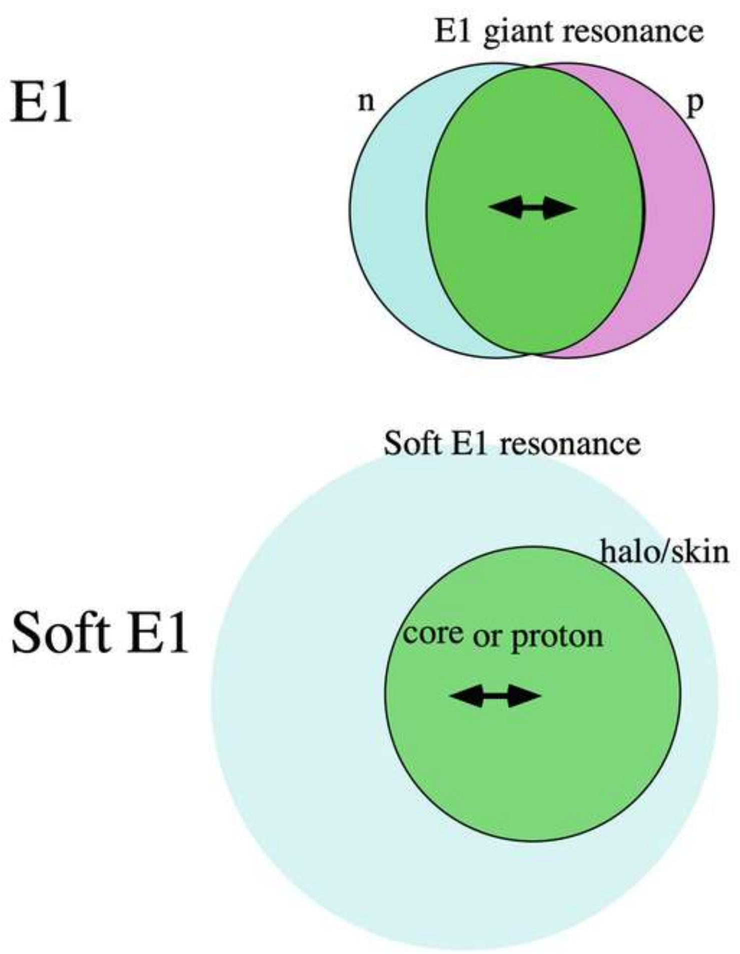
Open questions:

- Collective resonance or single-particle like excitation ?
- Tail of the GDR ?
- Isospin mixing beyond S_n ?
- Classical representation : neutron excess oscillation ?

  Toroidal mode ?

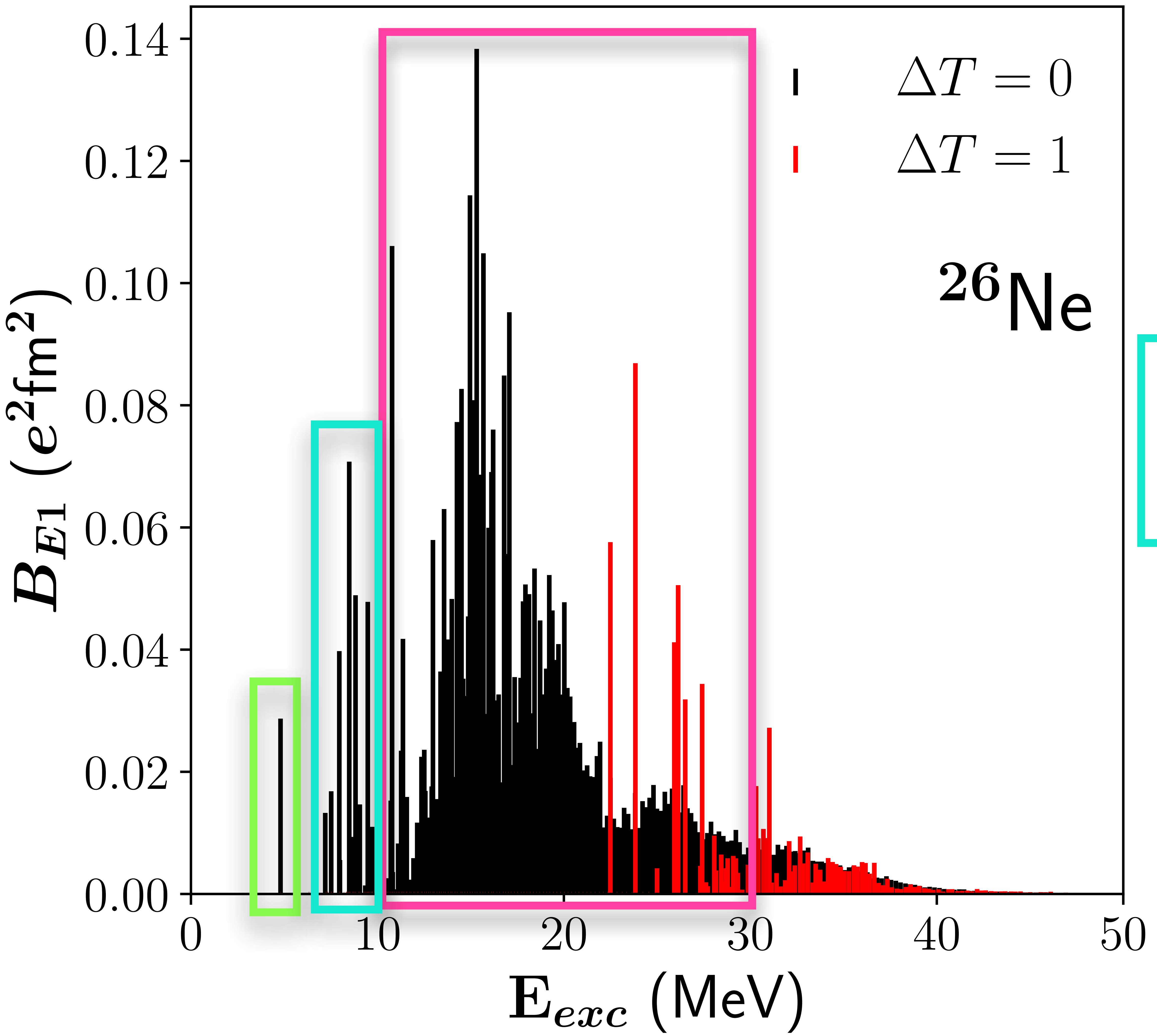






Study of the Pygmy dipole resonance ^{26}Ne

Comparison to experiment



CISM

$$\boxed{\begin{aligned}\bar{S}_{\mathrm{PDR}} &= 8.63\ \mathrm{MeV}\\ S_0^{\mathrm{PDR}} &= 0.33\ e^2\mathrm{fm}^2\end{aligned}}$$

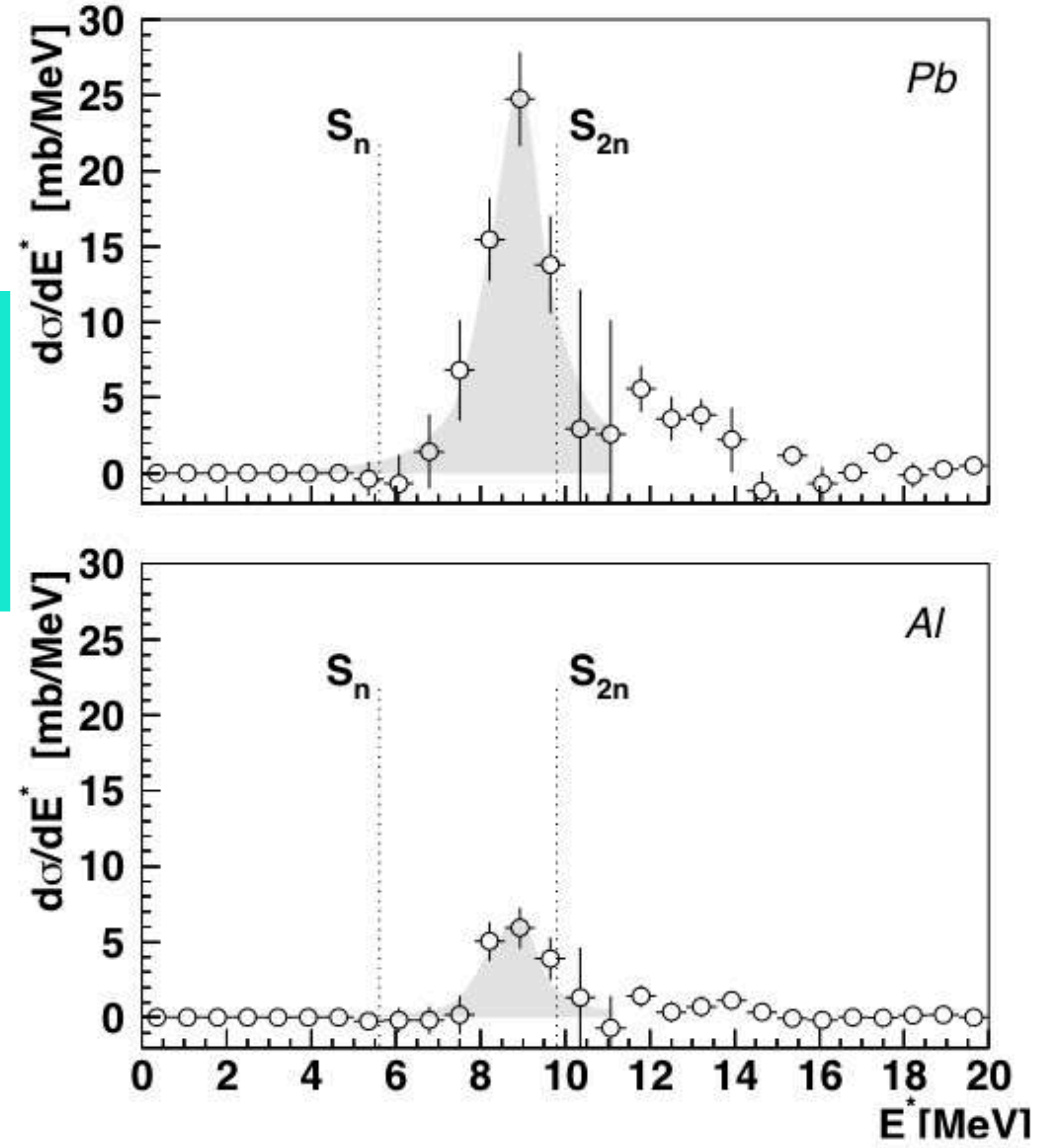
$\sim 4\%$ TRK

EXP

$$\bar{S}_{\mathrm{PDR}} \approx 9\ \mathrm{MeV}$$

$$S_0^{\mathrm{PDR}} = 0.49 \pm 0.16\ e^2\mathrm{fm}^2$$

$^{26}\mathrm{Ne} + \mathrm{Pb} \longrightarrow {}^{26}\mathrm{Ne}^*$



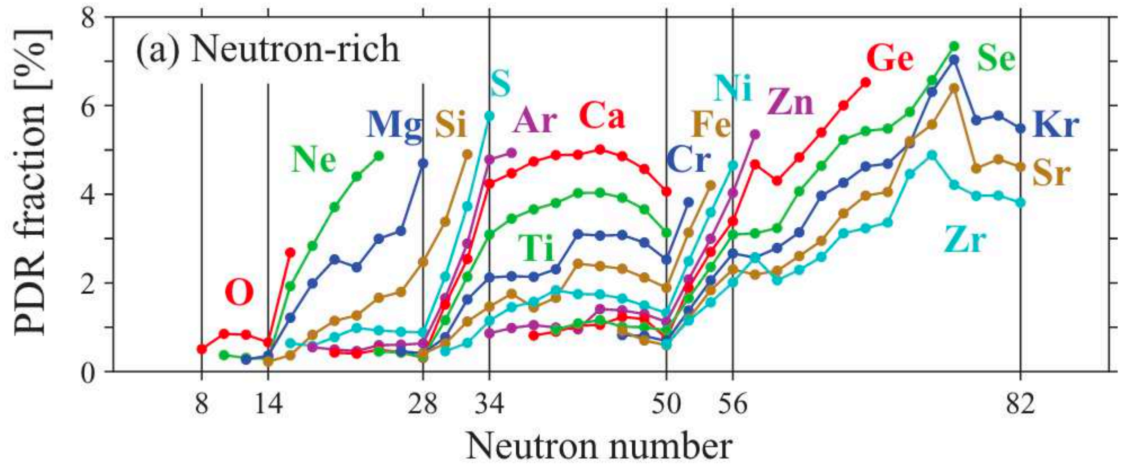
Gibelin, J. et al. Nuclear Physics A 2007, 788, 153–158.

Study of the Pygmy dipole resonance ^{26}Ne

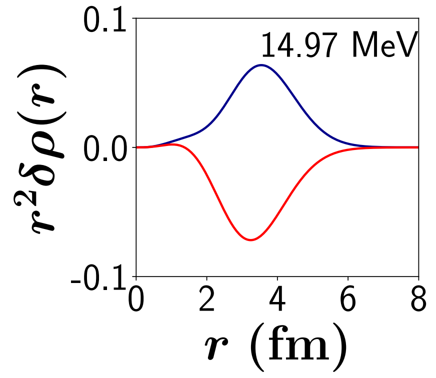
Degree of collectivity and classical interpretation

Collectivity indicators:

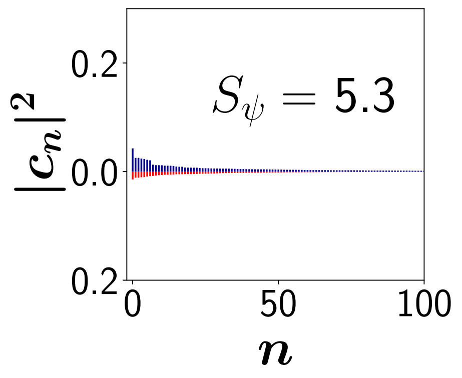
- Great regularity of the PDR in neutron-rich nuclei



- Similar structure of the Pygmy states $\longrightarrow$ $\delta\rho(r)$

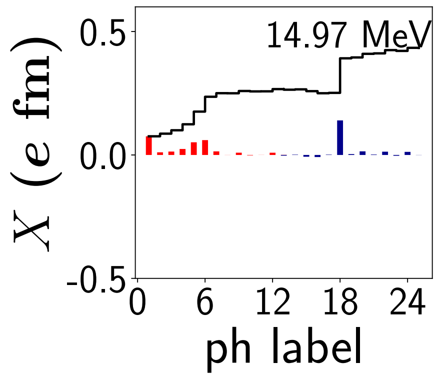


- Imply many nucleons ~ many single-particle states $\longrightarrow$ S_ψ



%TRK

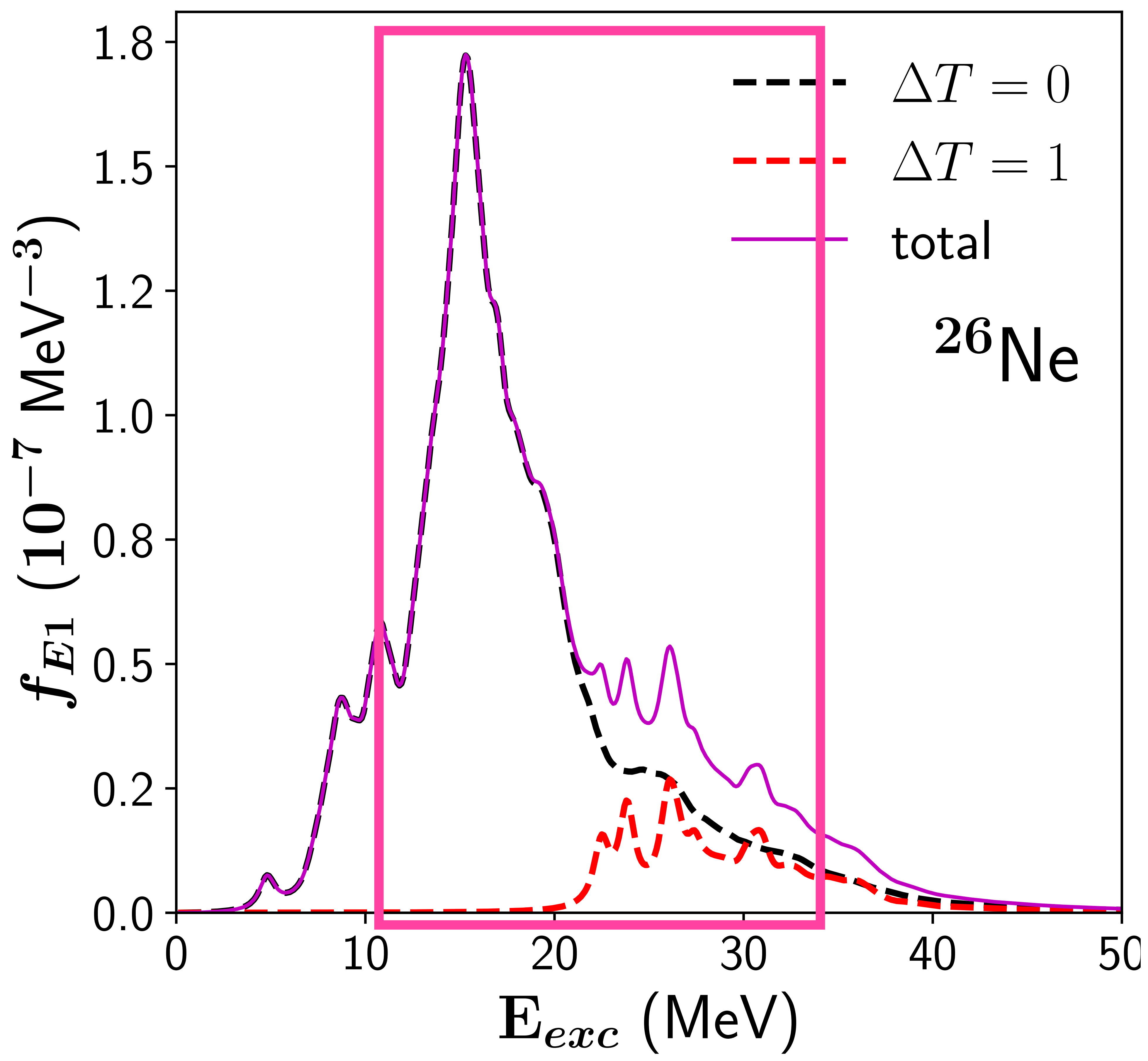
- Nucleons respond coherently to the perturbation $\longrightarrow$ X



Study of the Pygmy dipole resonance ^{26}Ne

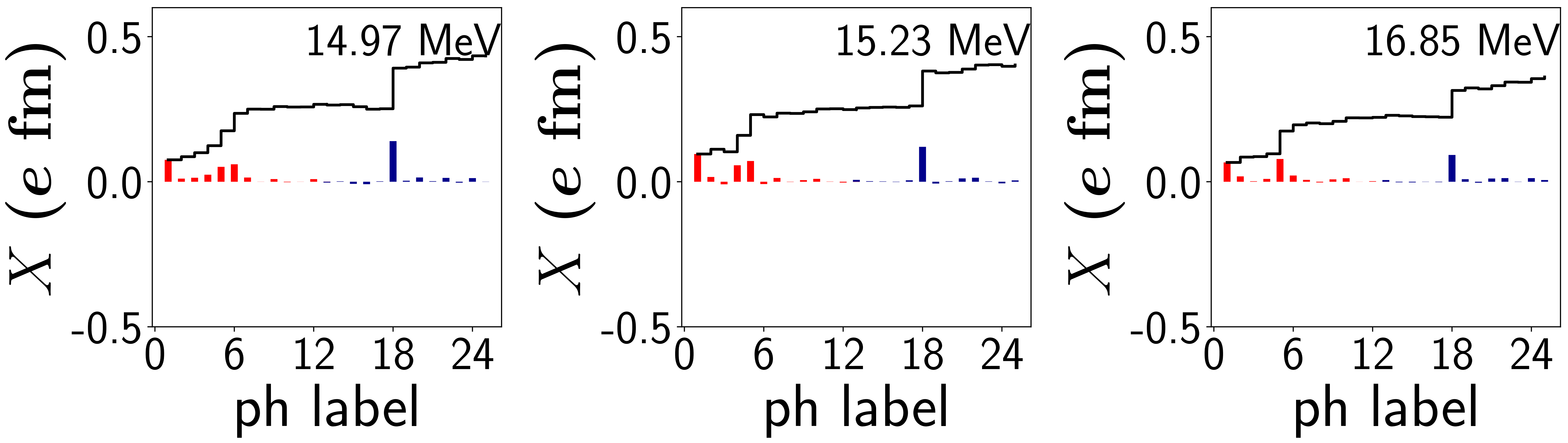
Degree of collectivity and classical interpretation

Paradigmatic example of a collective resonance: GDR



Fraction of TRK $\quad S_1^{\mathrm{GDR}} \sim 100\%\mathrm{TRK}$

Cumulative sum $\quad B_{\nu 0} = \left| \sum_{k_\alpha k_\beta} \left(X^p_{k_\alpha k_\beta} + X^n_{k_\alpha k_\beta} \right) \right|^2$



Study of the Pygmy dipole resonance ^{26}Ne

Degree of collectivity and classical interpretation

Paradigmatic example of a collective resonance: GDR

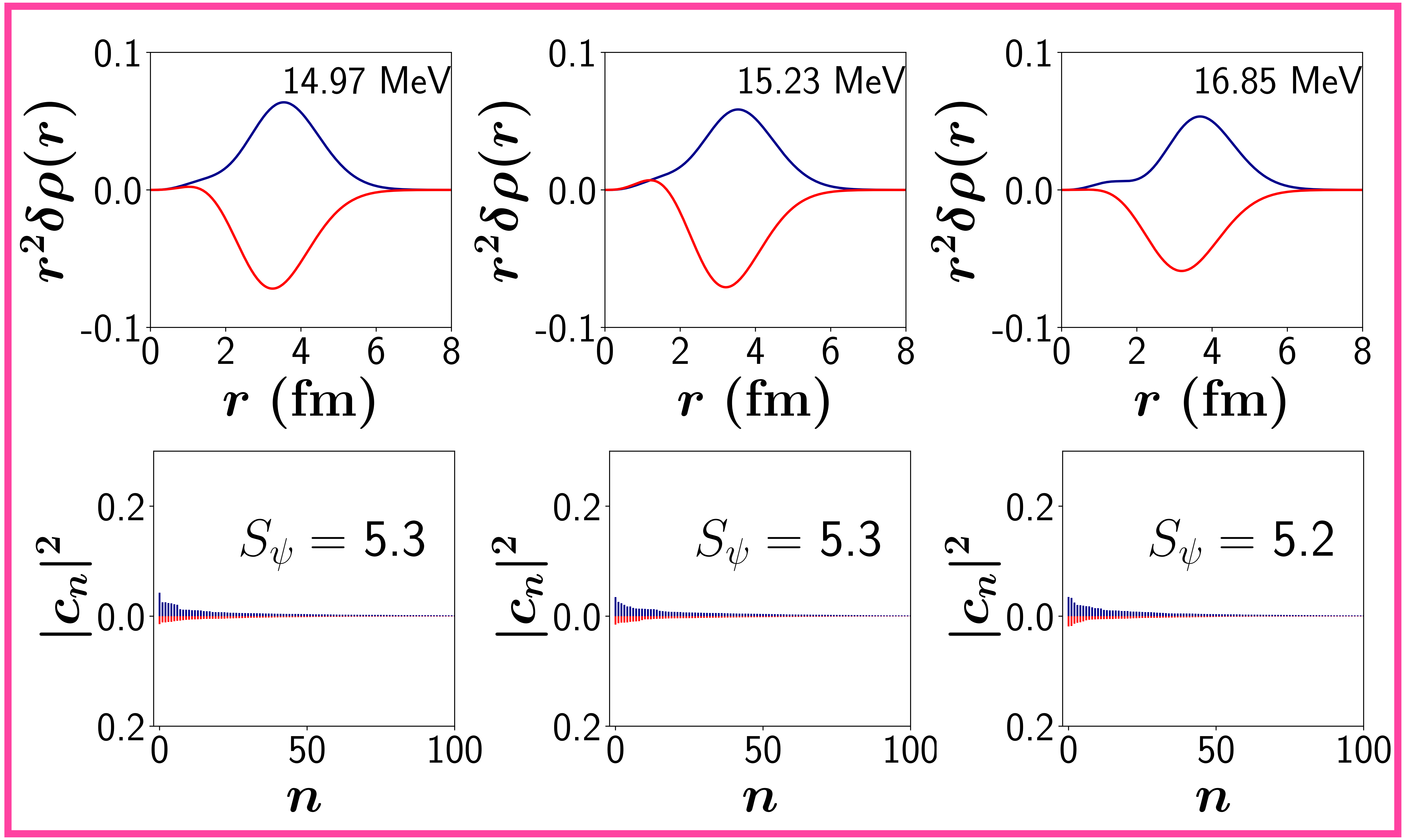
Transition density = Fourier mode of vibration

$$\delta\rho_\psi(\mathbf{r},t) = \rho_\psi(\mathbf{r},t) - \rho_0(\mathbf{r}) = \sum_{\nu\neq 0} b_\nu \langle 0|\,\hat{\rho}(\mathbf{r})\,|\nu\rangle\, e^{-\frac{i}{\hbar}\varepsilon_\nu t}$$

$$\delta\rho_{0\nu}(\mathbf{r}) = \langle 0|\,\hat{\rho}(\mathbf{r})\,|\nu\rangle$$

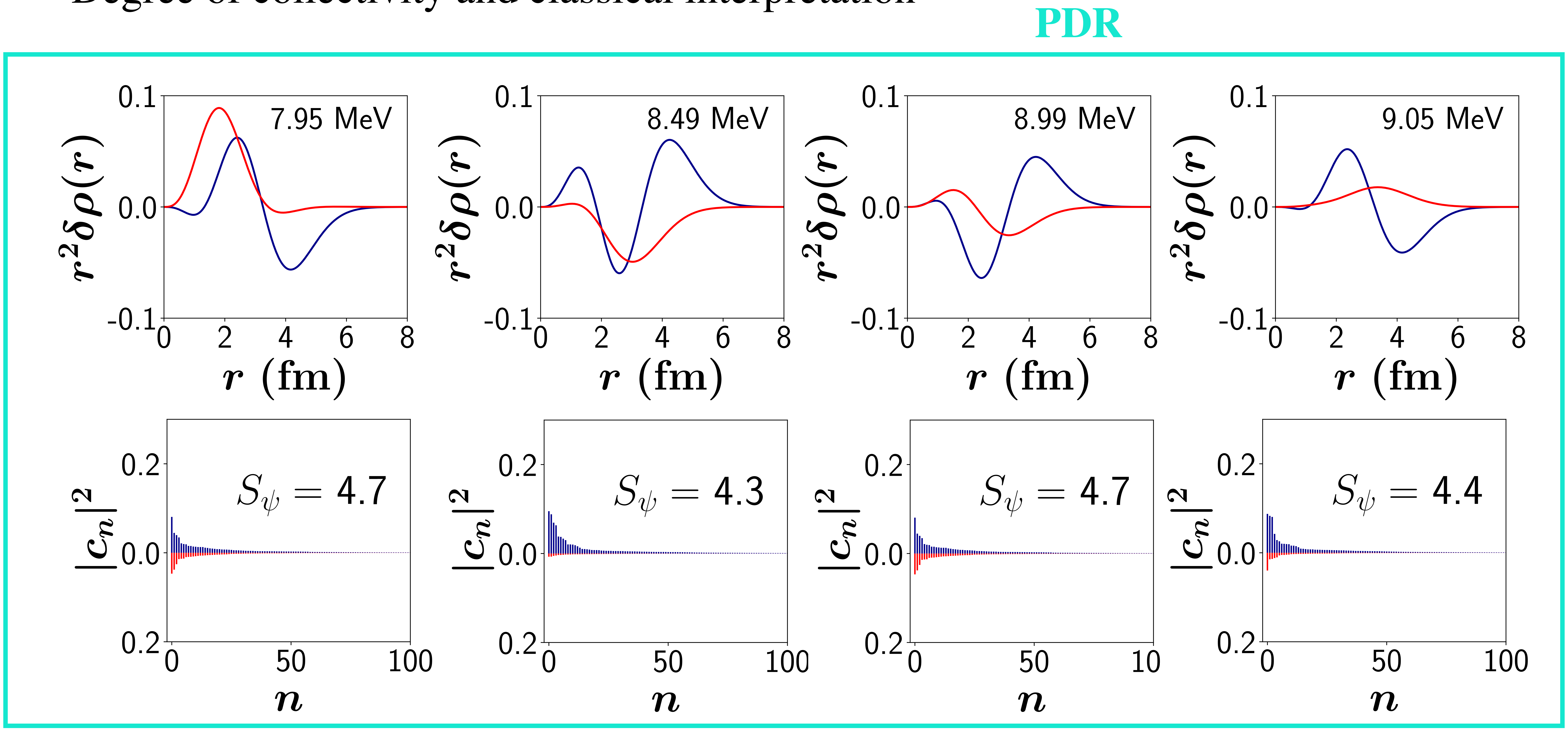
Configuration entropy = measure of the dispersion over single-particle states

$$S_\psi = -\sum_n |c_n^\psi|^2 \ln |c_n^\psi|^2$$

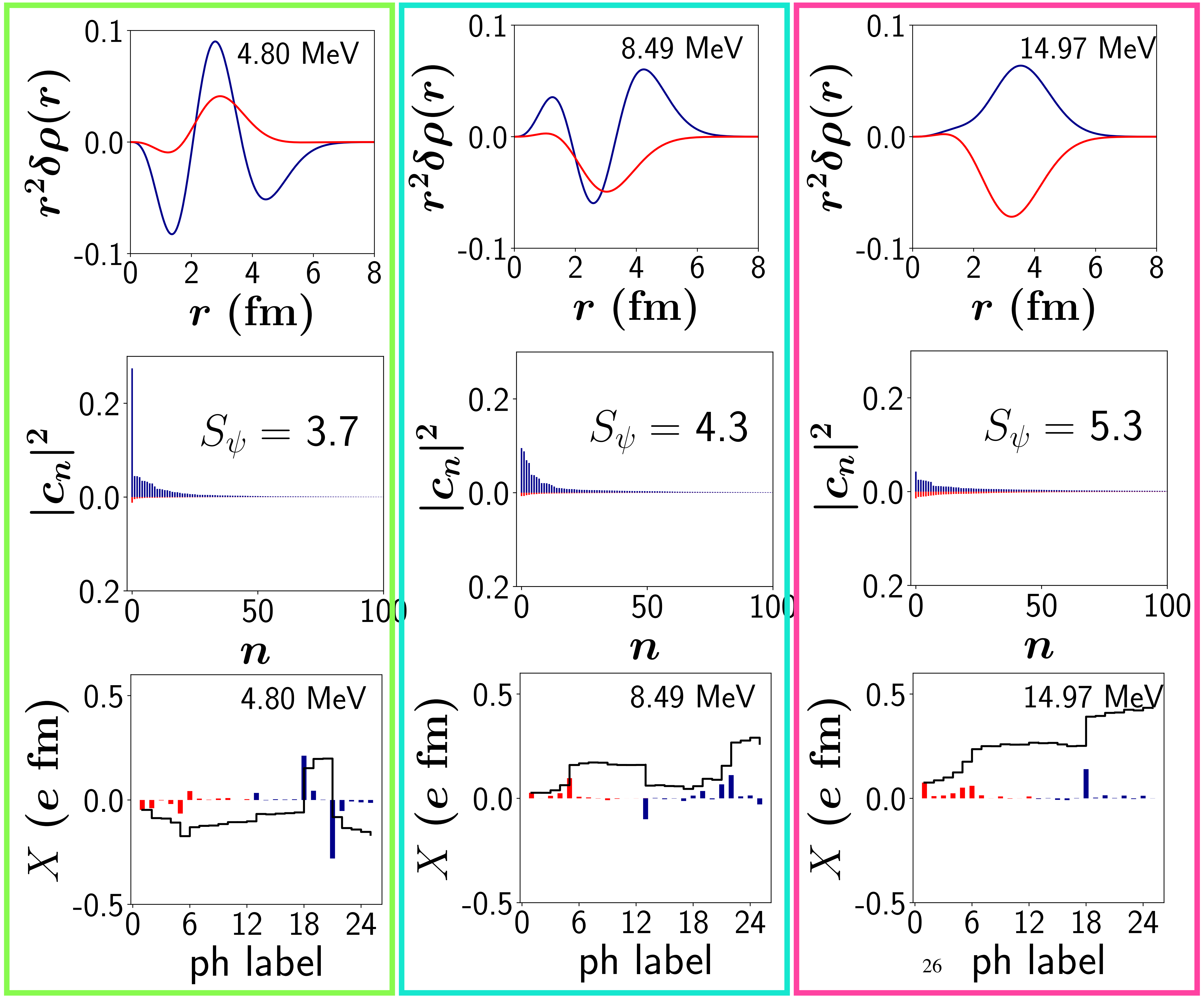


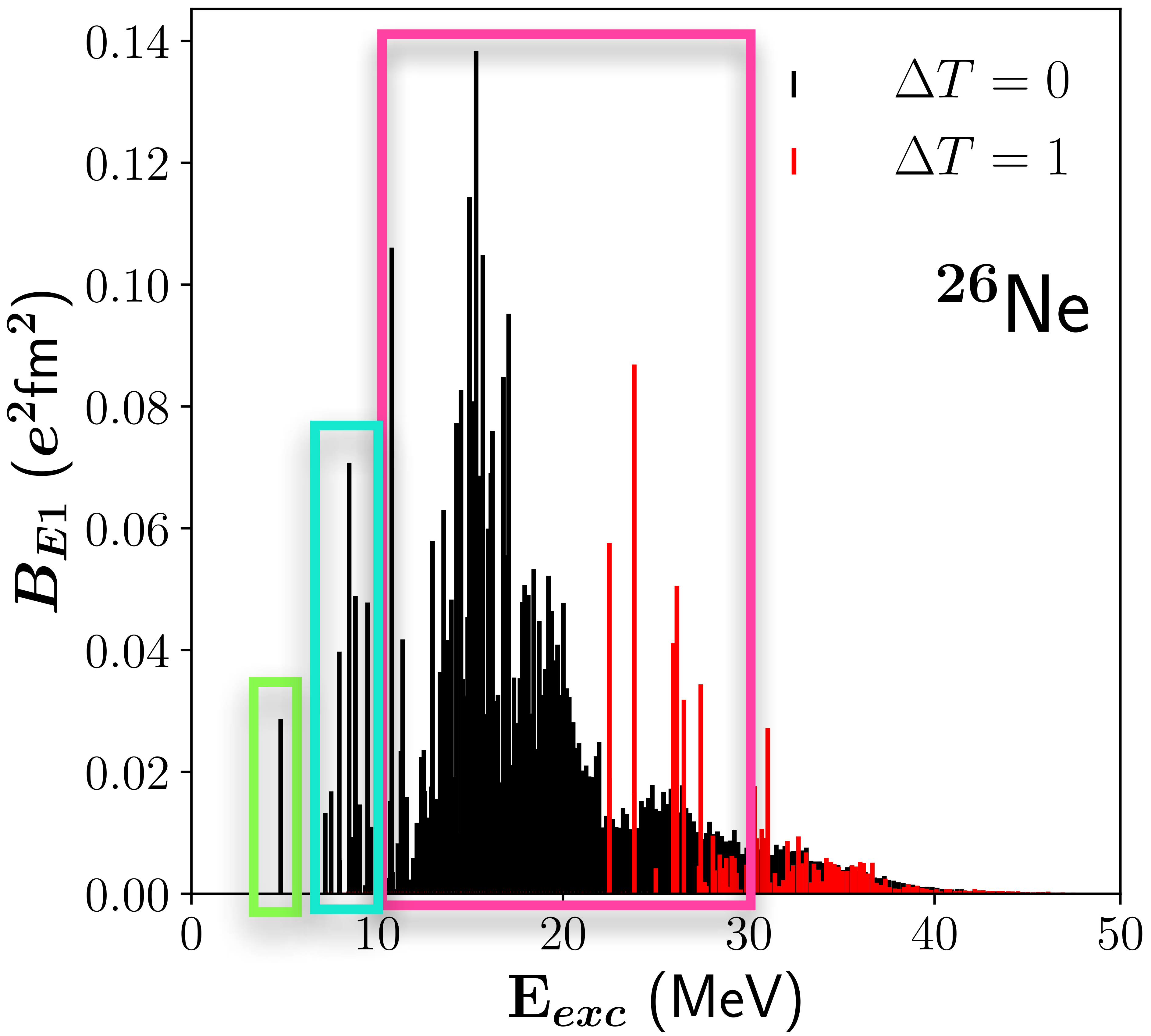
Study of the Pygmy dipole resonance ^{26}Ne

Degree of collectivity and classical interpretation



Study of the Pygmy dipole resonance ^{26}Ne





O. Le Noan and K. Sieja, Physical Review C 111, 064308 (2025)

Study of the Pygmy dipole resonance $^{26}\mathrm{Ne}$

Degree of collectivity and classical interpretation

Collectivity indicators:

- Great regularity of the PDR in neutron-rich nuclei
- Similar structure of the Pygmy states $\longrightarrow$ $\delta\rho(r)$
- Imply many nucleons ~ many single-particle states $\longrightarrow$ S_ψ
- Nucleons respond coherently to the perturbation $\longrightarrow$ $\%\mathrm{TRK}$, X

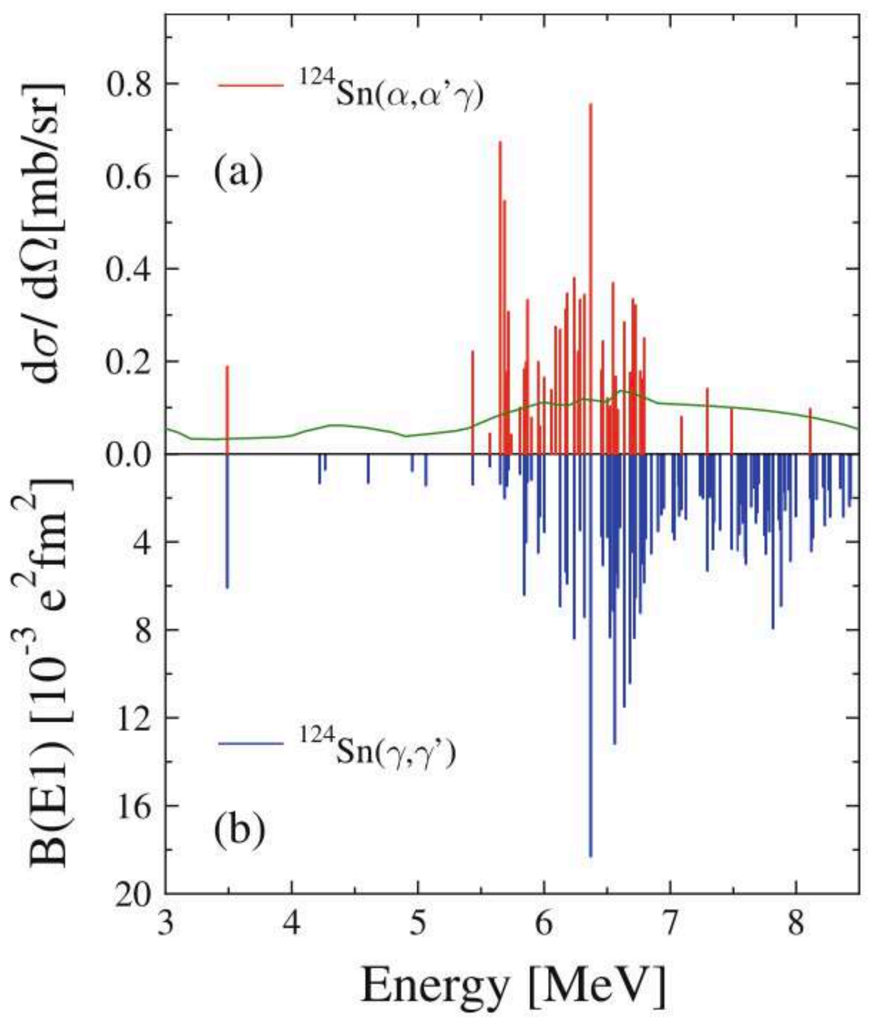
Study of the Pygmy dipole resonance $^{26}\mathrm{Ne}$

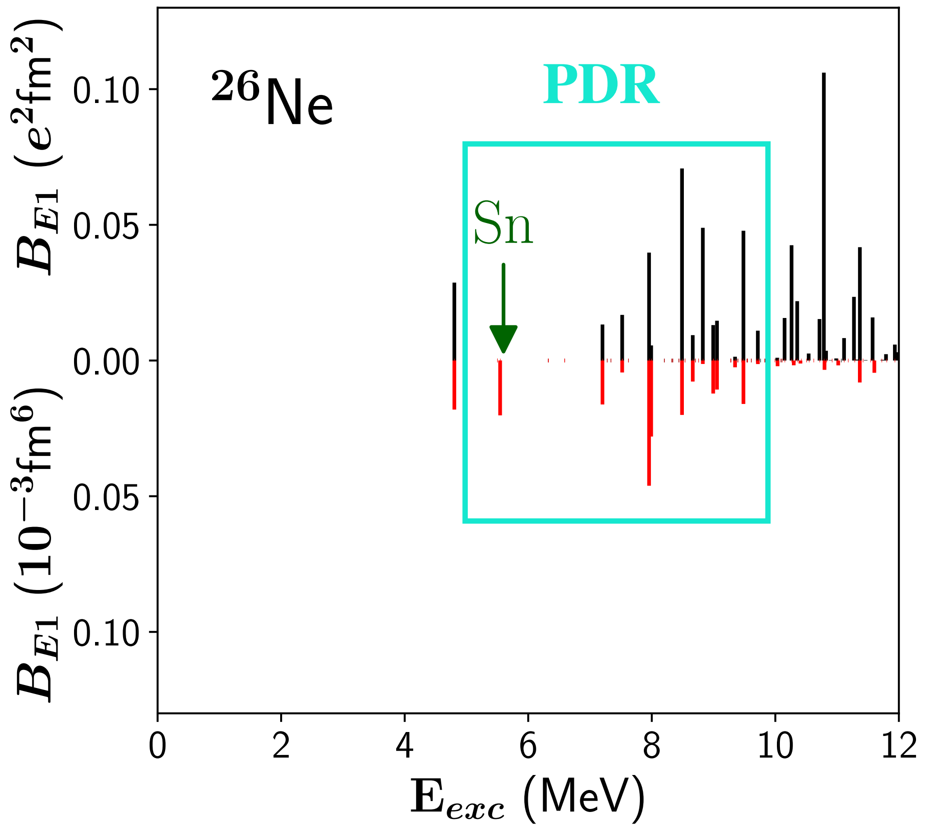
Isospin mixing

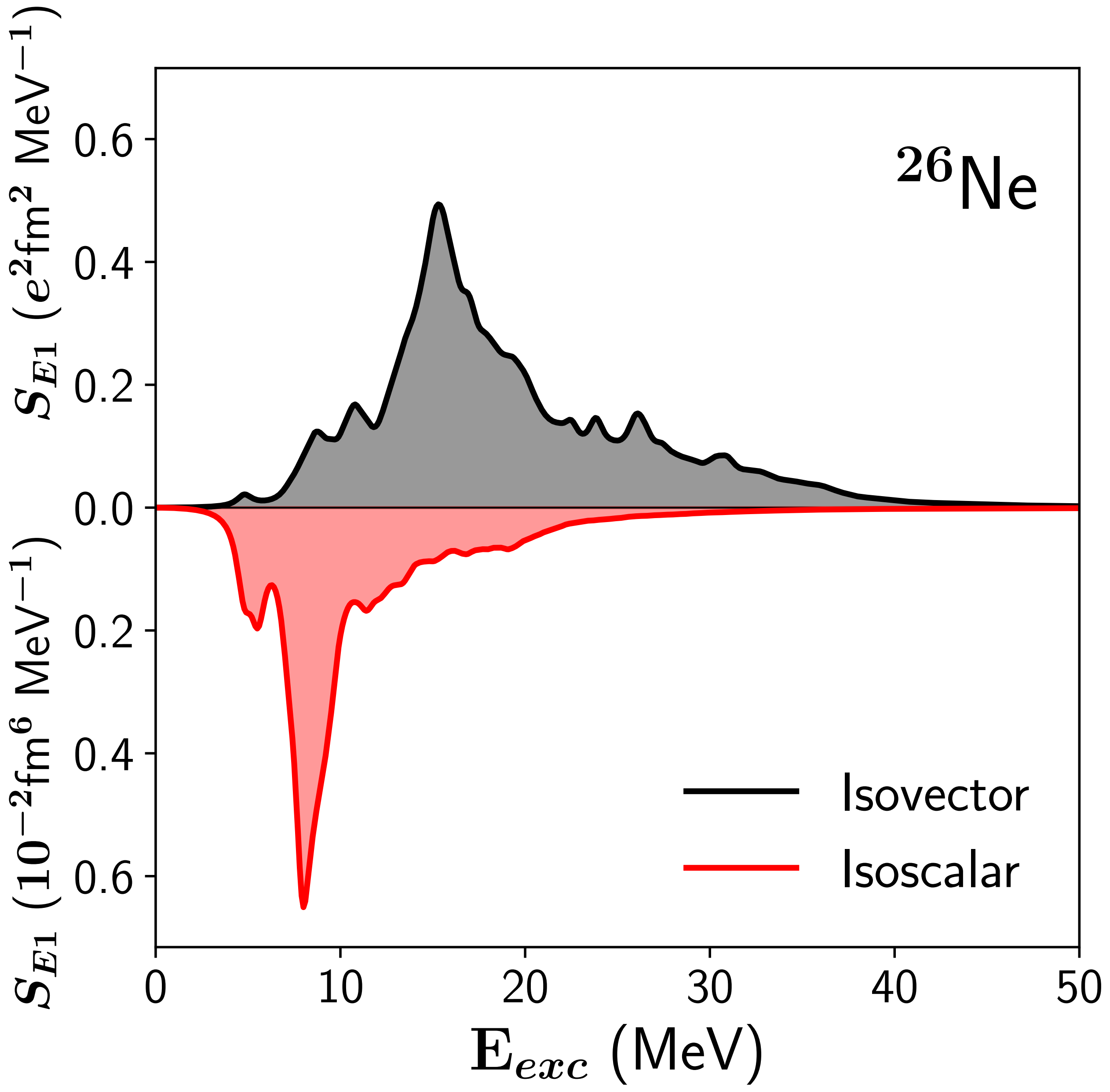
$$O_{\mathrm{E1}} = O_{\mathrm{E1}}^{(0)} + O_{\mathrm{E1}}^{(1)}$$

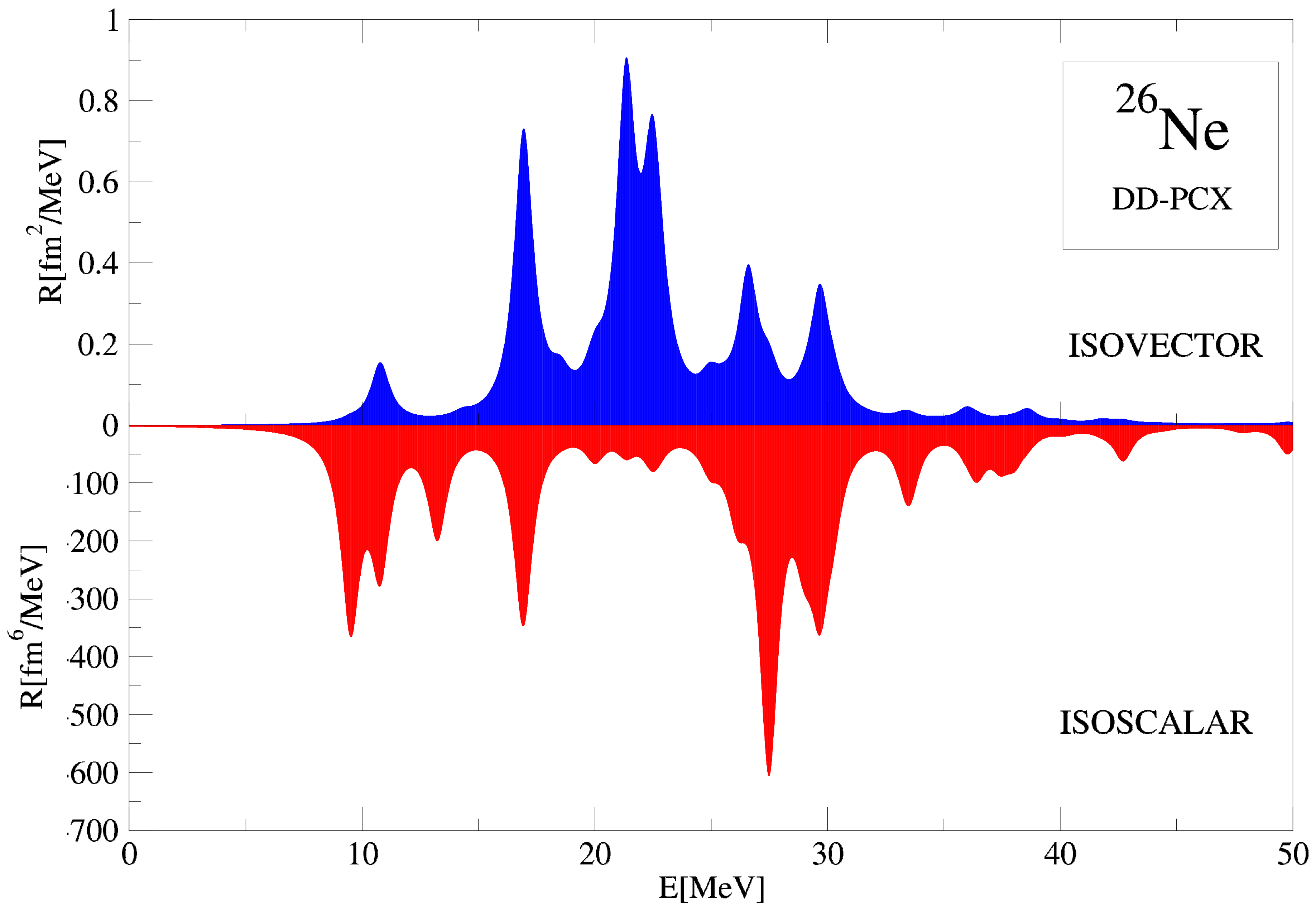
Isoscalar $\sim r^3 Y_1$ Isovector $\sim r Y_1 \tau_z$

Experimentally







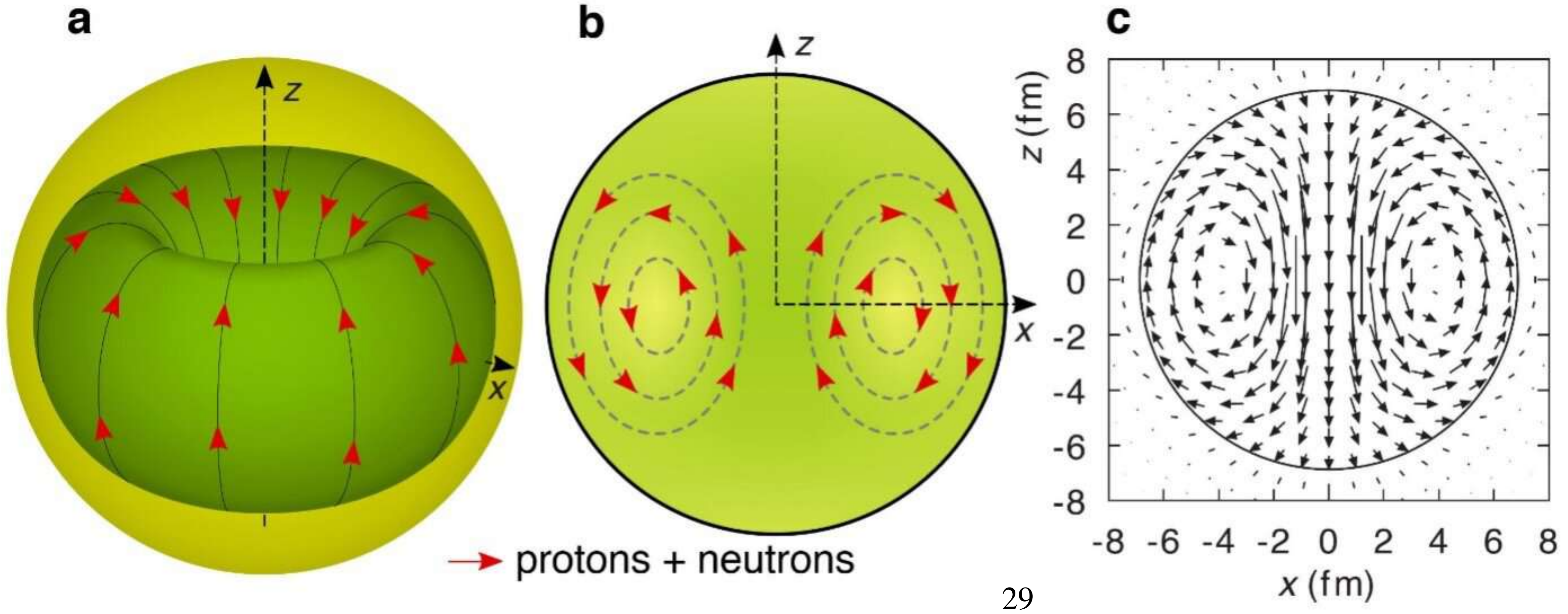


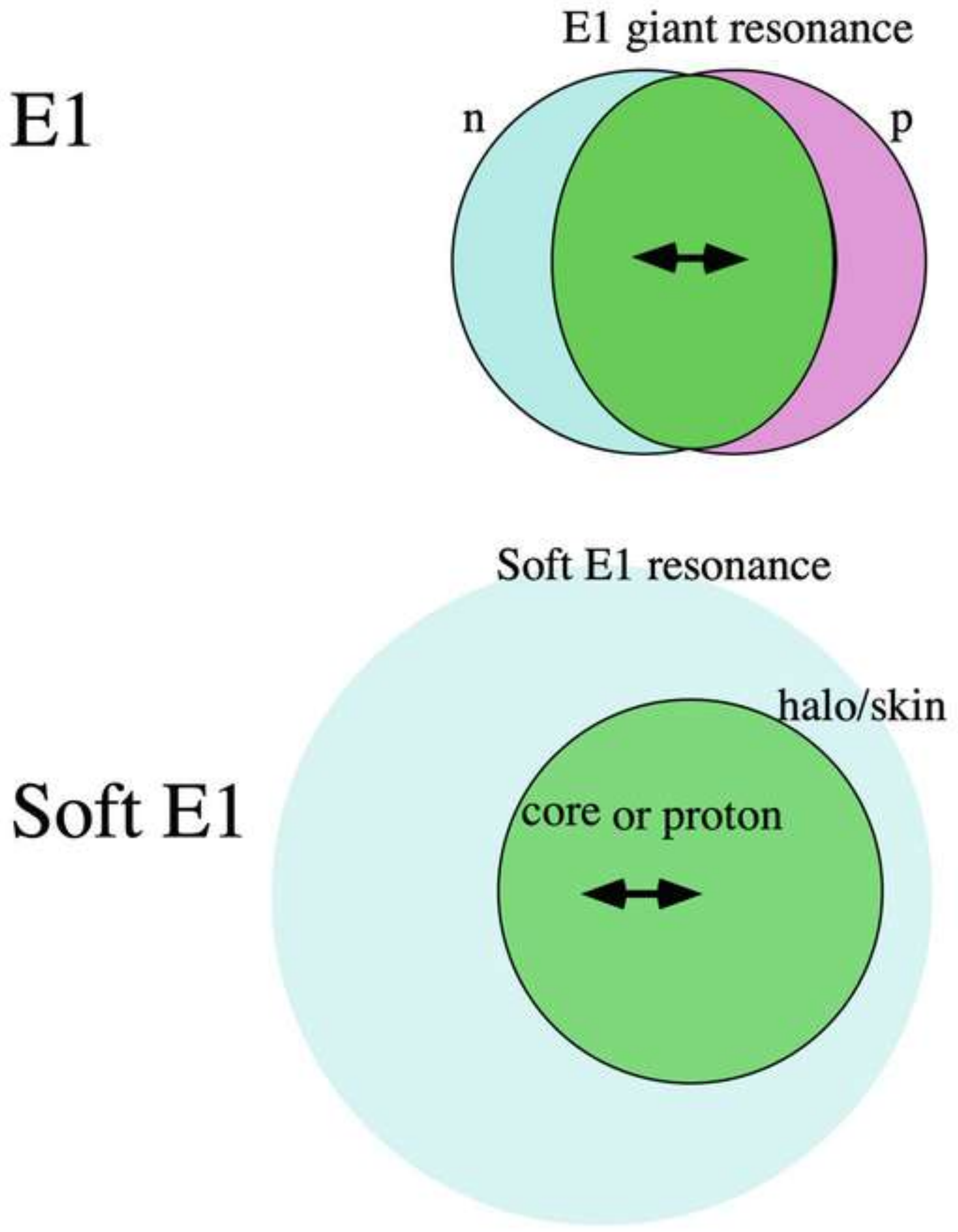
Nils Paar RQRPA, unpublished

Study of the Pygmy dipole resonance

Conclusion

- CI-SM provides good prediction of E1 response for $A < 40$.
- The PDR is collective in a shell-model picture (< GDR).
- PDR states are not tail of GDR.
- The isospin mixing occurs also after S_n.
- The classical representation of the PDR as neutron skin oscillation holds. The possibility of a toroidal mode still requires further study.



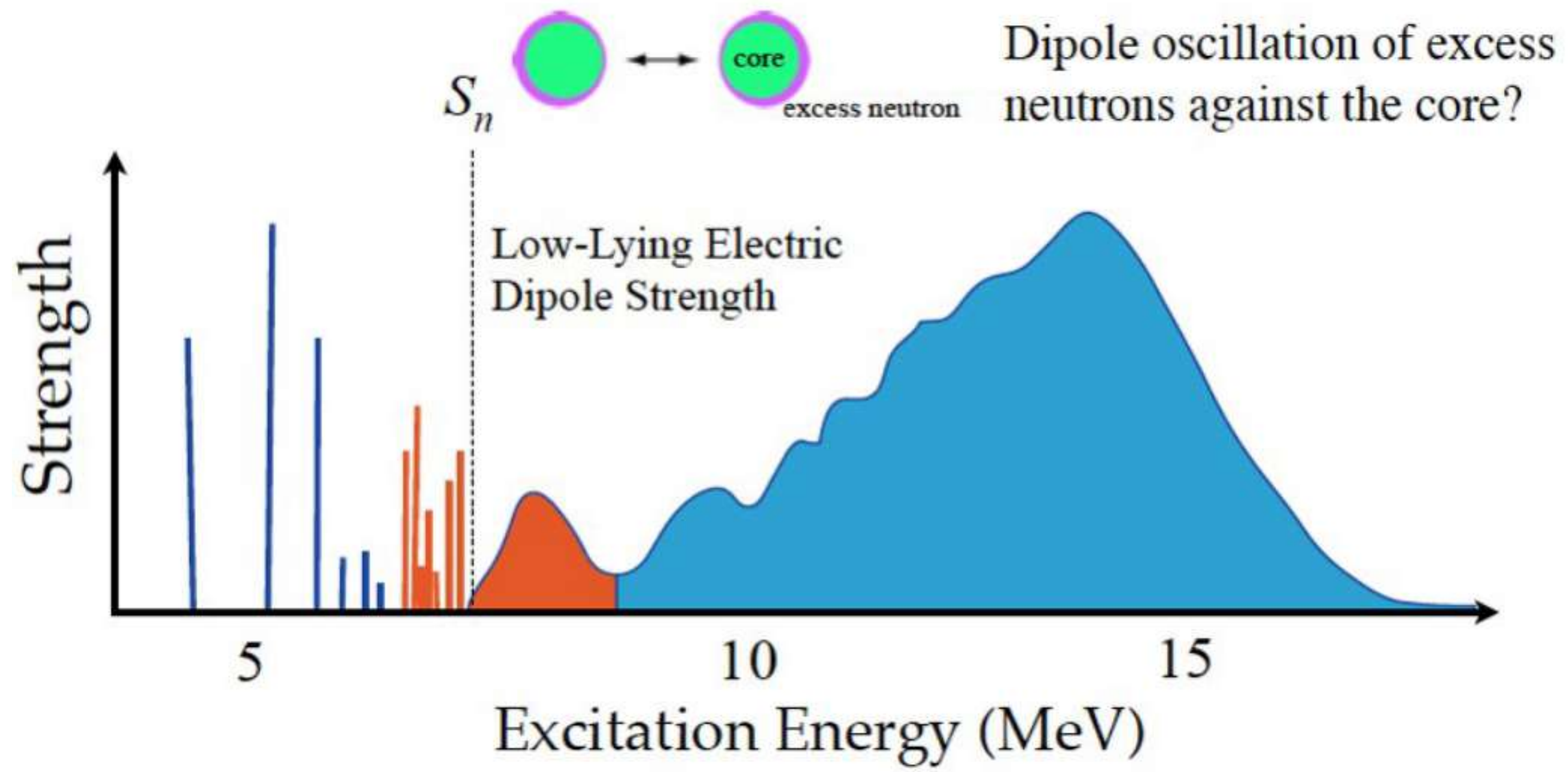


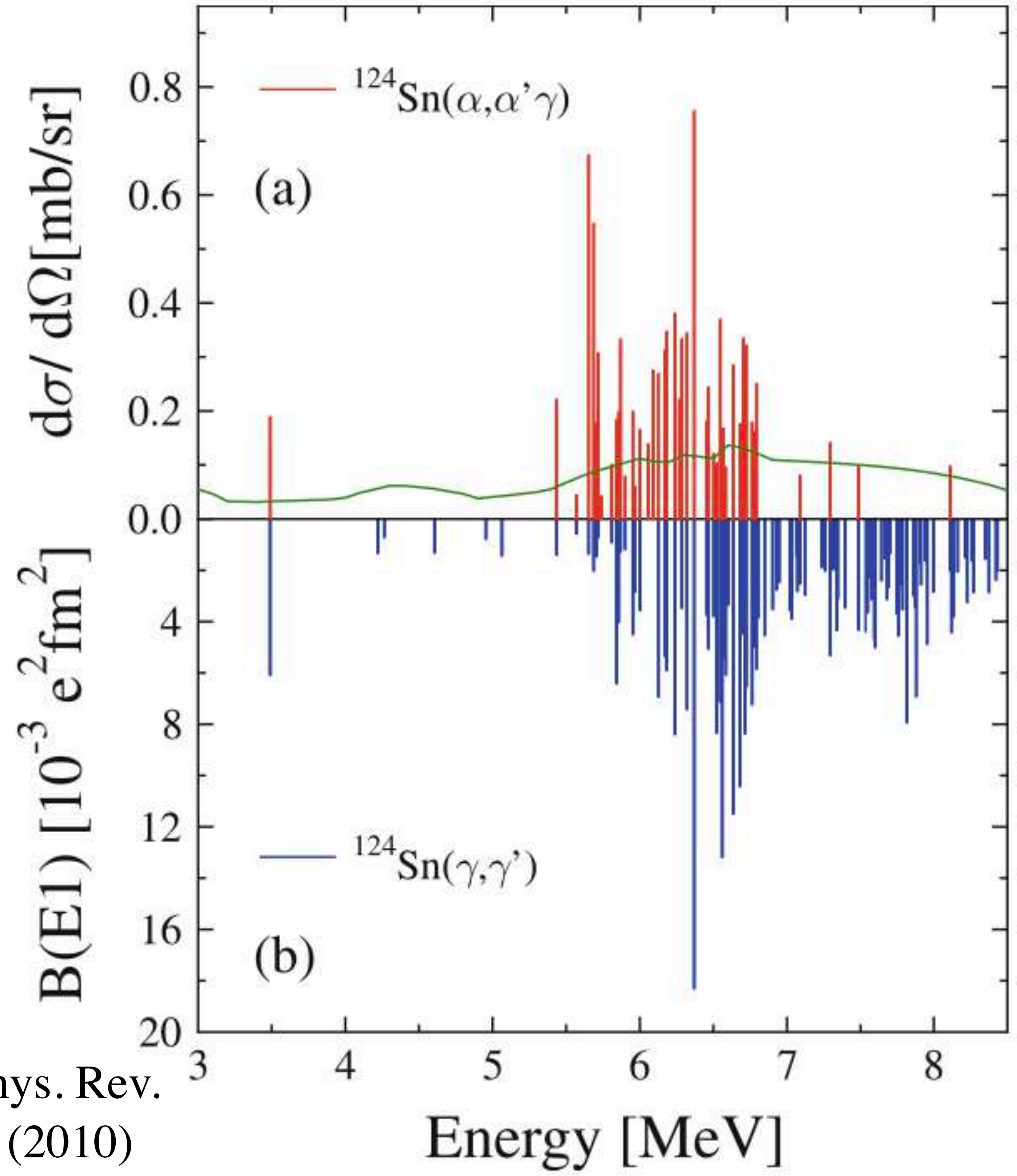
Back up

Study of the Pygmy dipole resonance (PDR)

Experimental side:

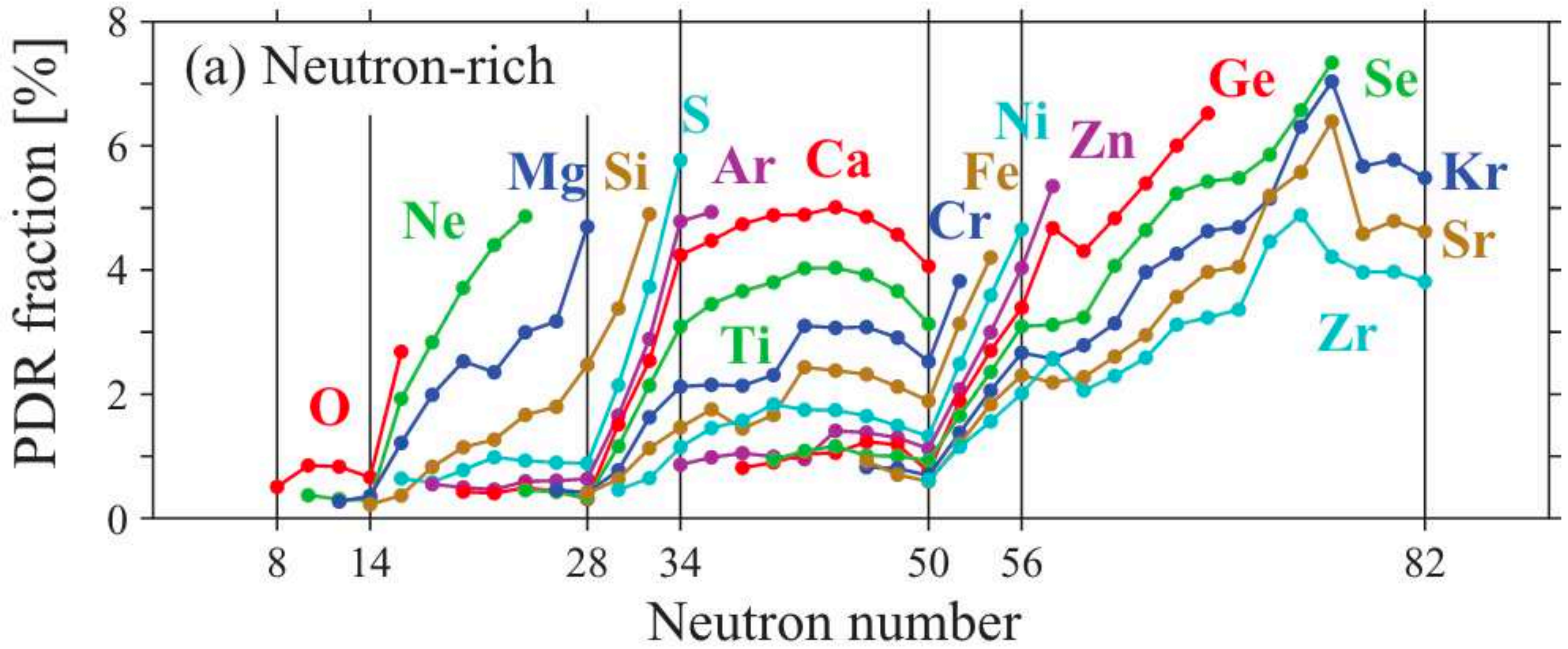
- Appears in all neutron-rich nuclei
- Appears before and after S_n
- Has some degree of isospin mixing





J. Endres et al., Phys. Rev. Lett. 105, 212503 (2010)

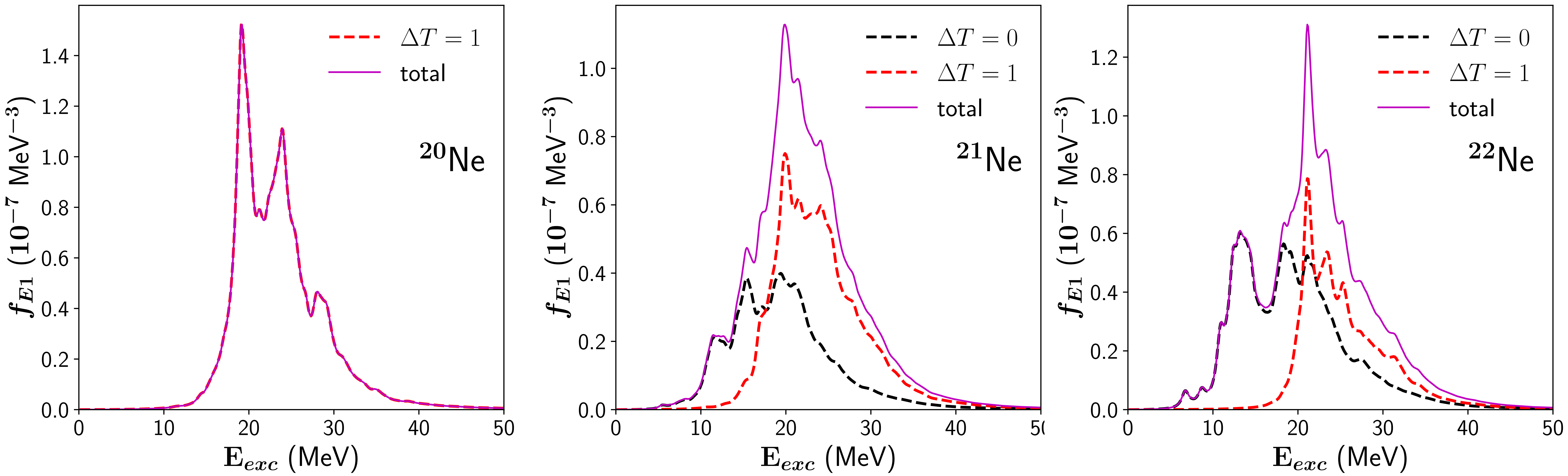
Theory side (QRPA):



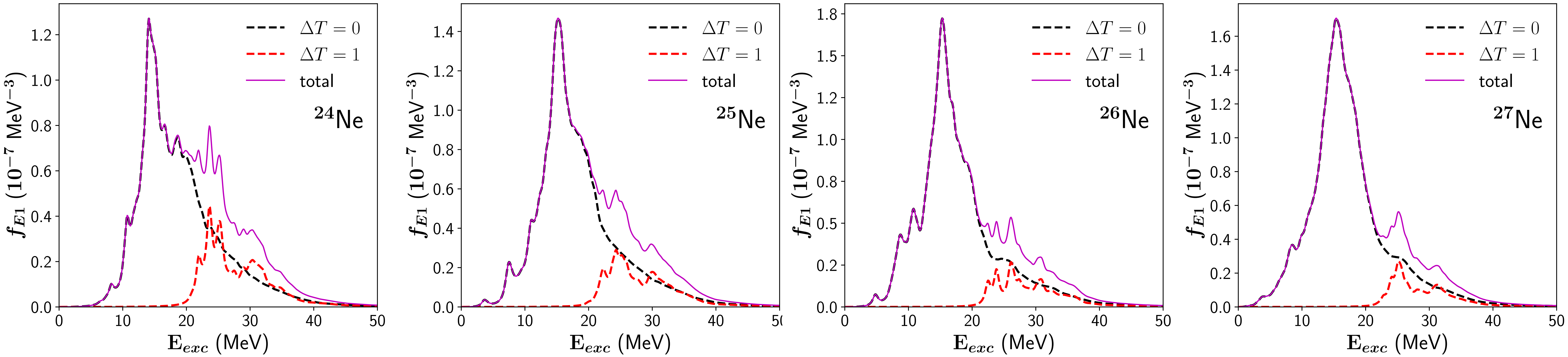
T. Inakura, T. Nakatsukasa, and K. Yabana, Physical Review C 84, 021302 (2011)

Study of the Pygmy dipole resonance

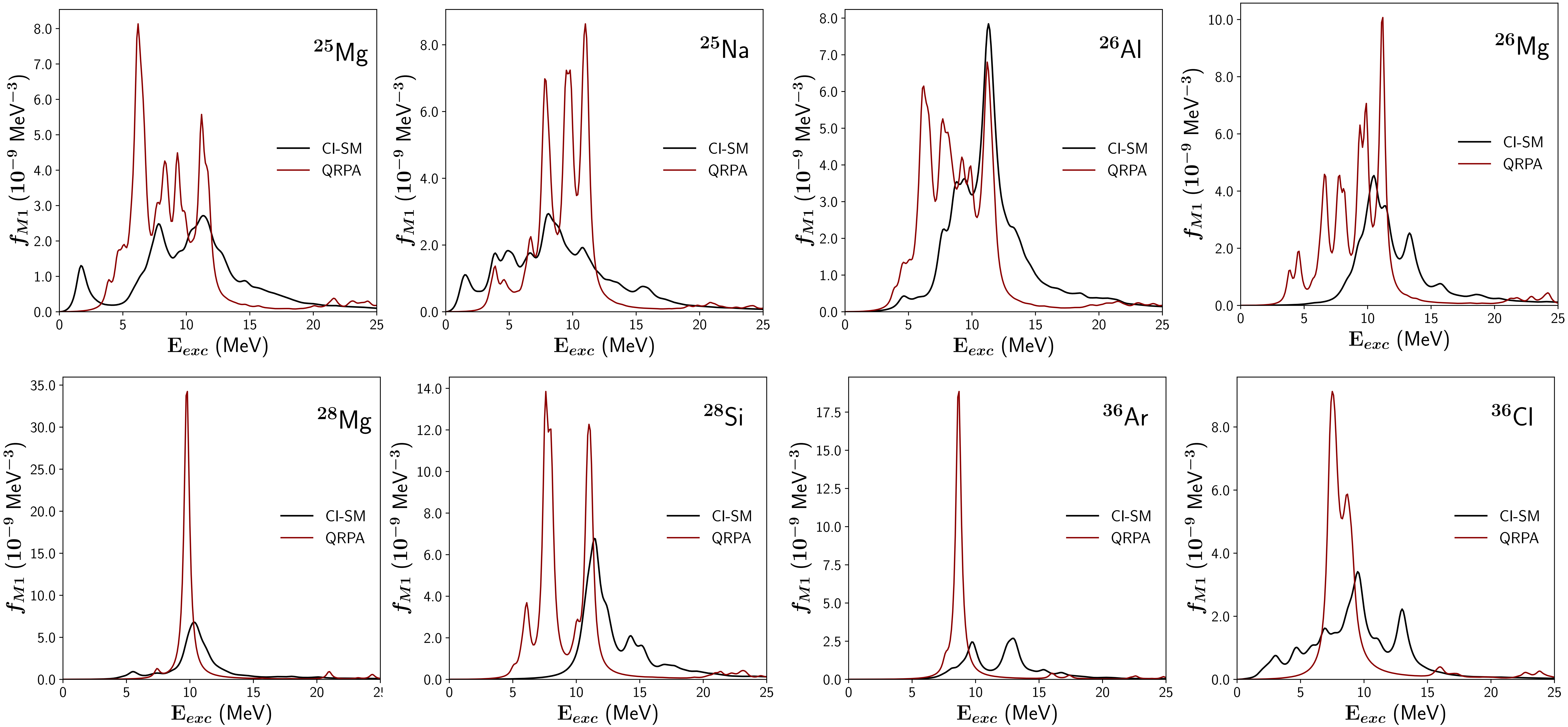
Neon isotopic chain: isospin splitting



$$\langle J_f M_f; T_f M_T | \hat{O}_{LM} | J_i M_i; T_i M_T \rangle = \delta_{T_i T_f} \langle J_f M_f | \hat{O}^{(0)}_{LM} | J_i M_i \rangle + \frac{(T_i M_T 1 0 | T_f M_T)}{\sqrt{2T_f + 1}} \langle J_f M_f; T_f || \hat{O}^{(1)}_{LM} || J_i M_i; T_i \rangle \,,$$

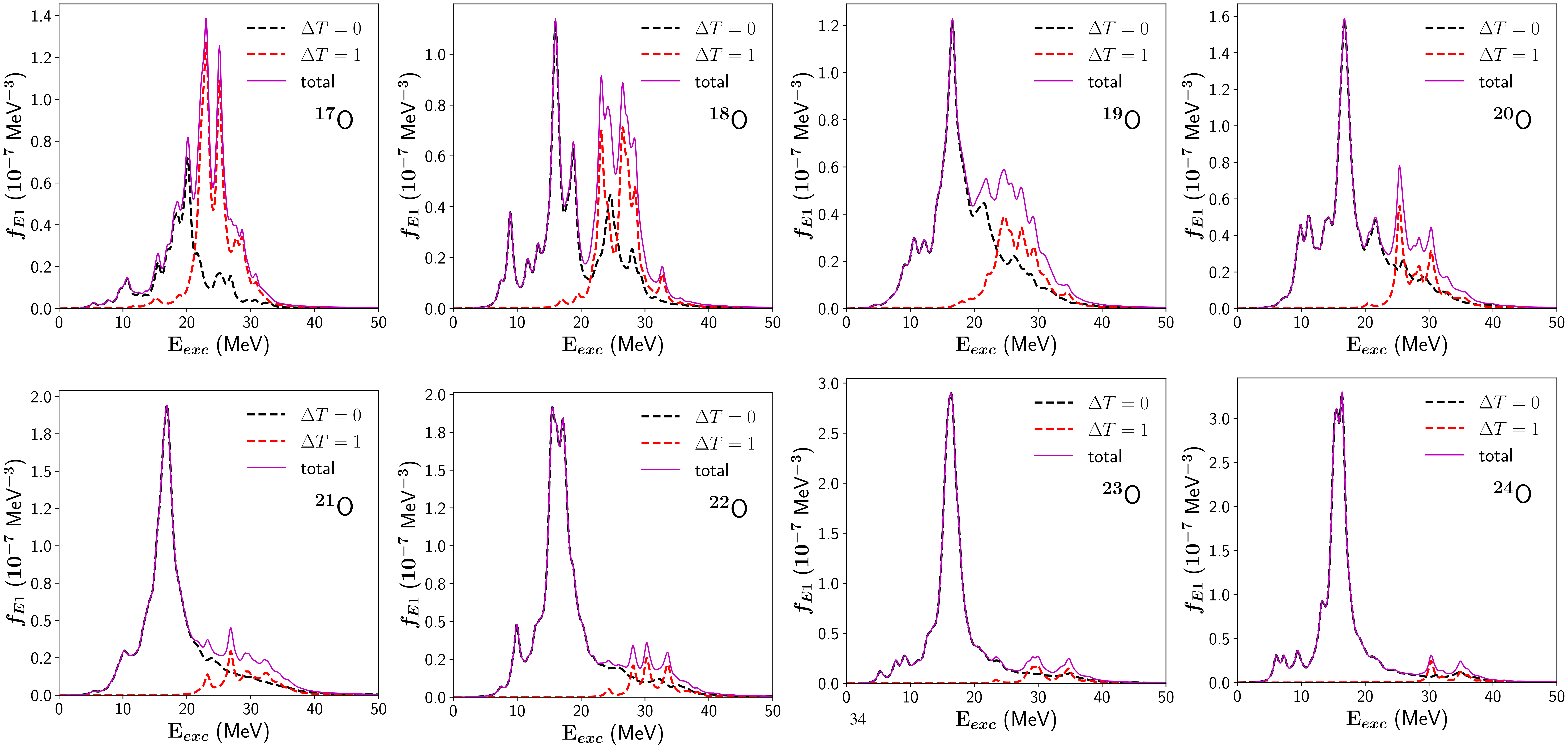


M1 p, and sd-shell nuclei



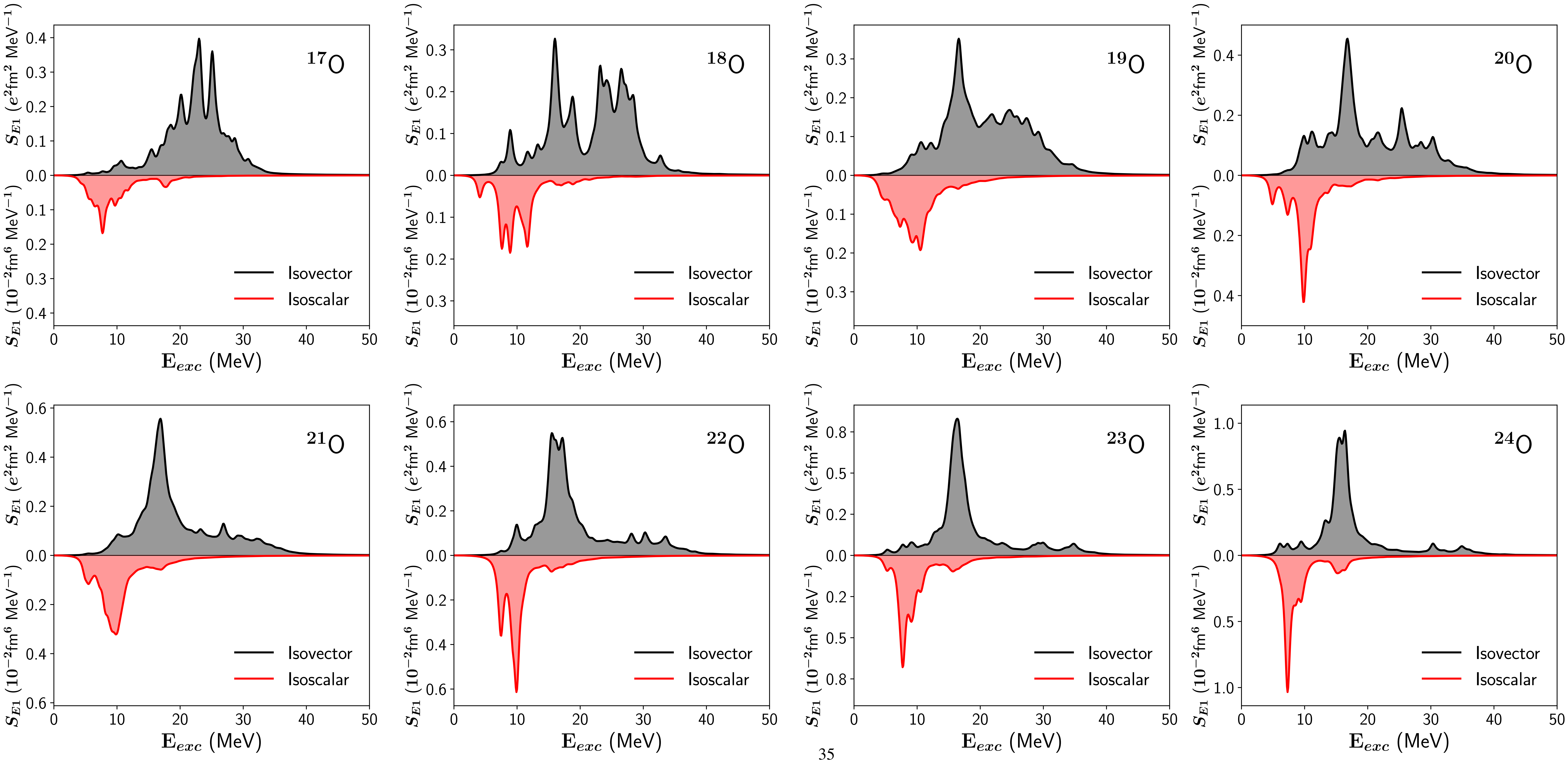
Study of the Pygmy dipole resonance

Isospin splitting in the oxygen chain



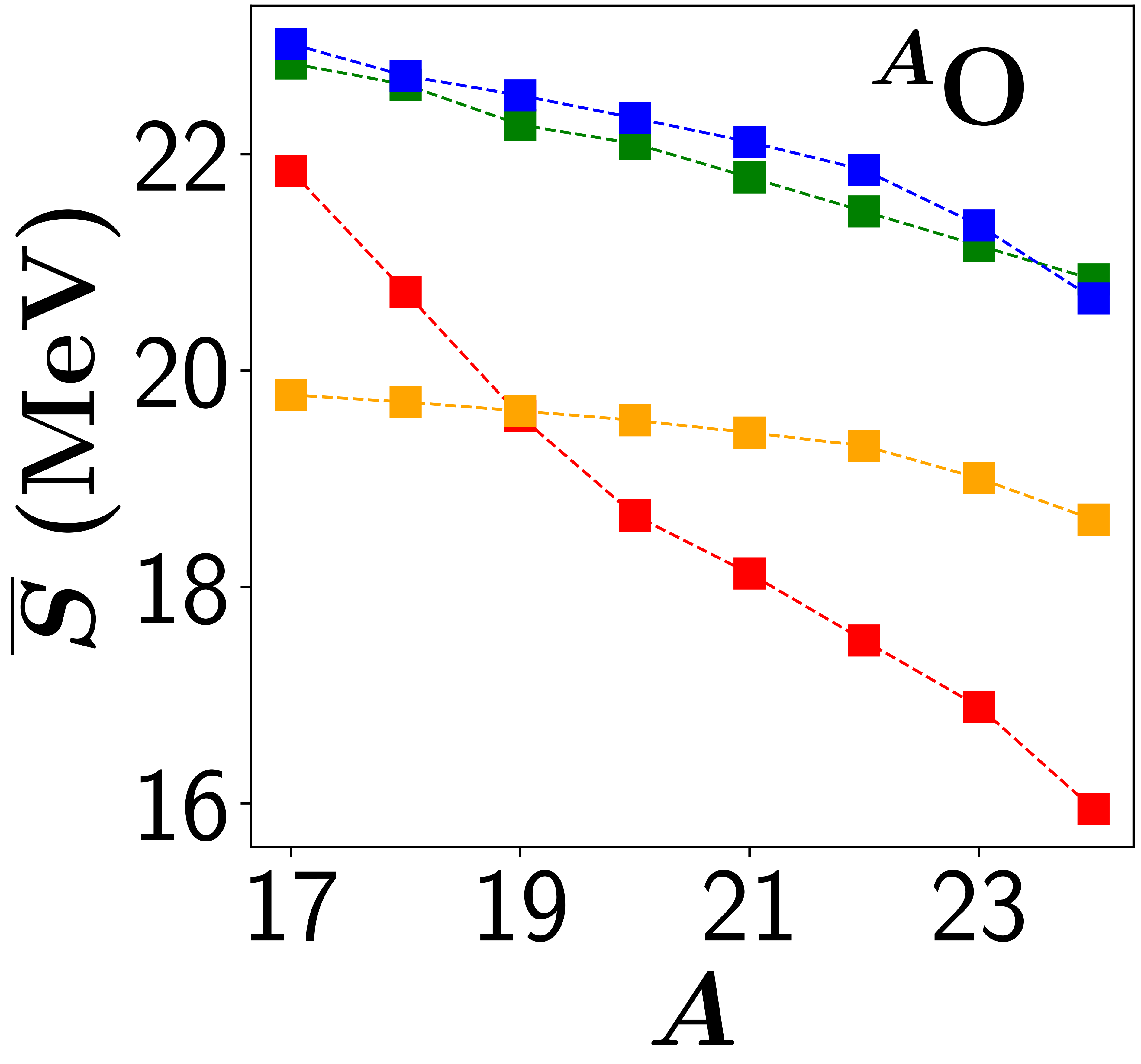
Study of the Pygmy dipole resonance

Isospin mixing in the oxygen chain

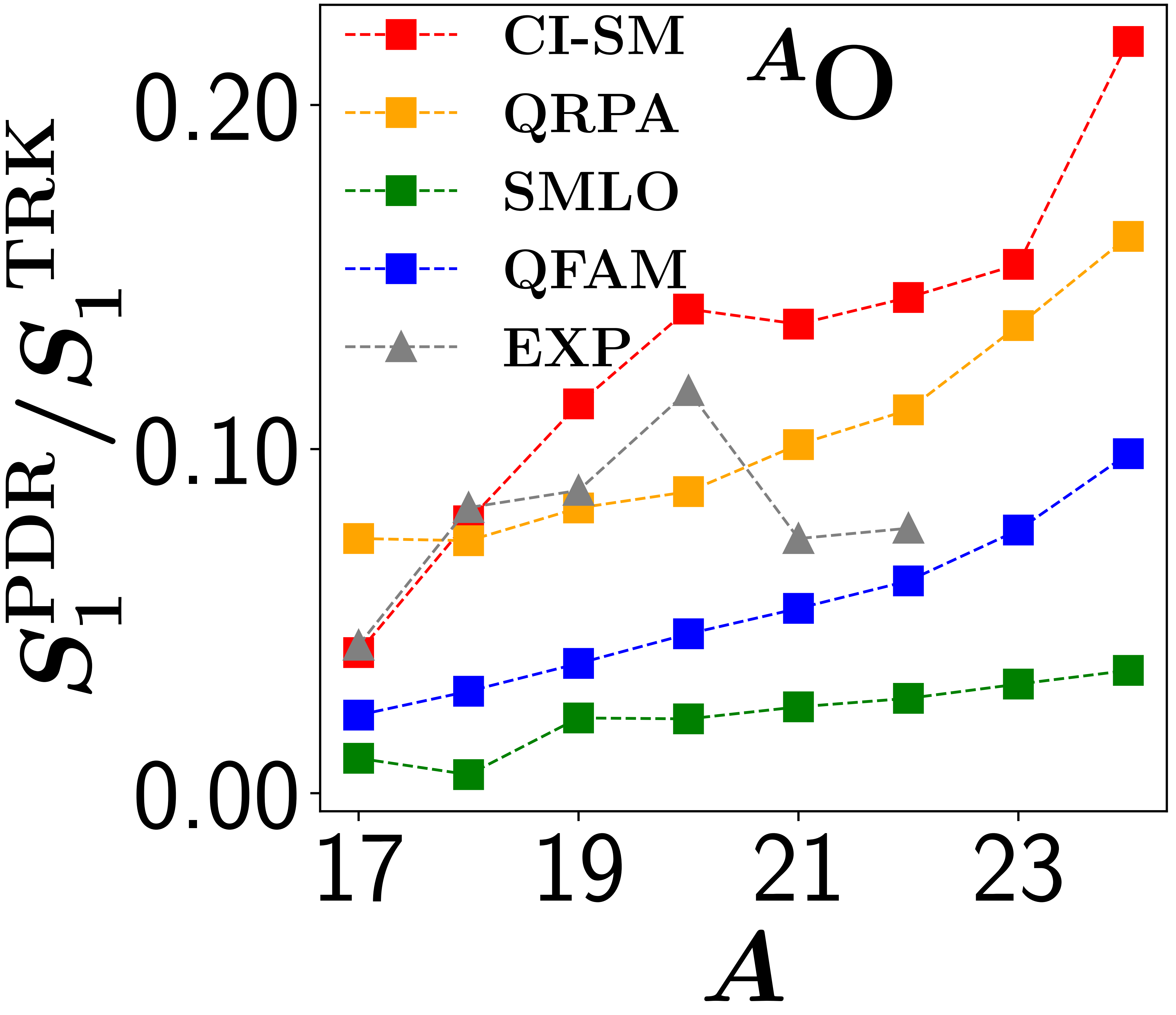


Study of the Pygmy dipole resonance

Oxygen chain: model comparison



up to 30 MeV

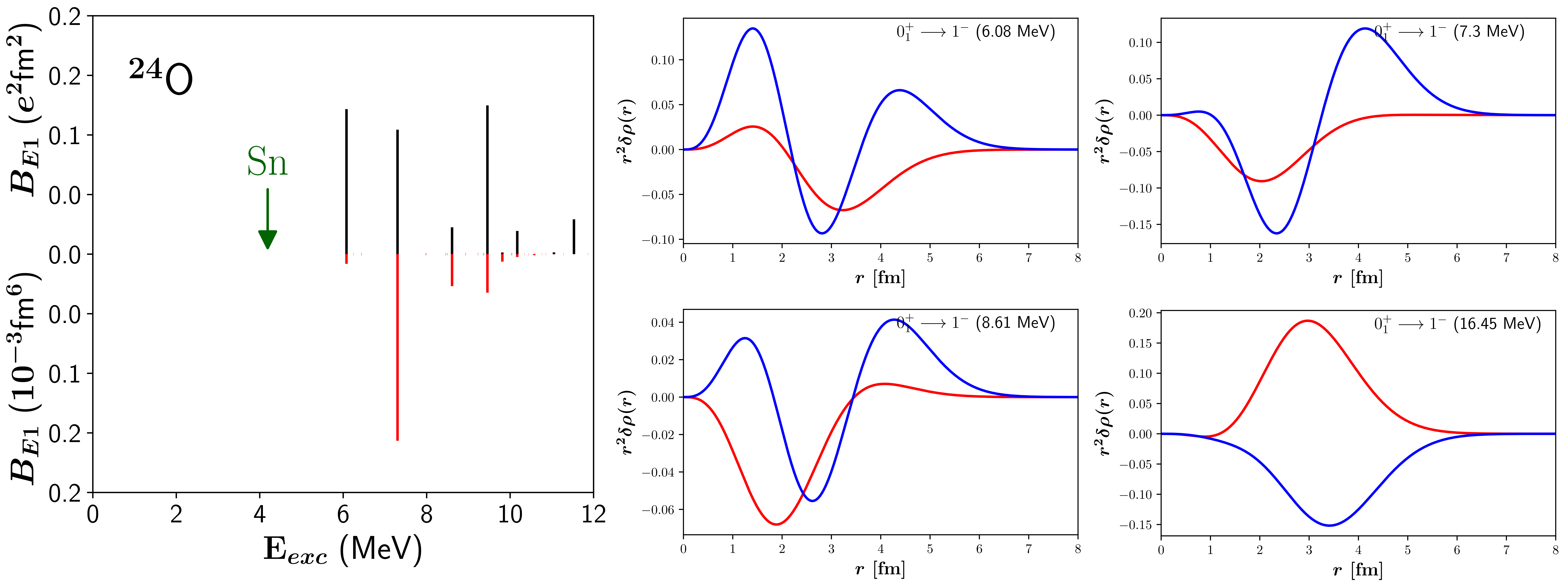


from S_n to 15 MeV

Leistenschneider, A. et al. Physical Review Letters 2001, 86, Publisher: American Physical Society, 5442–5445.

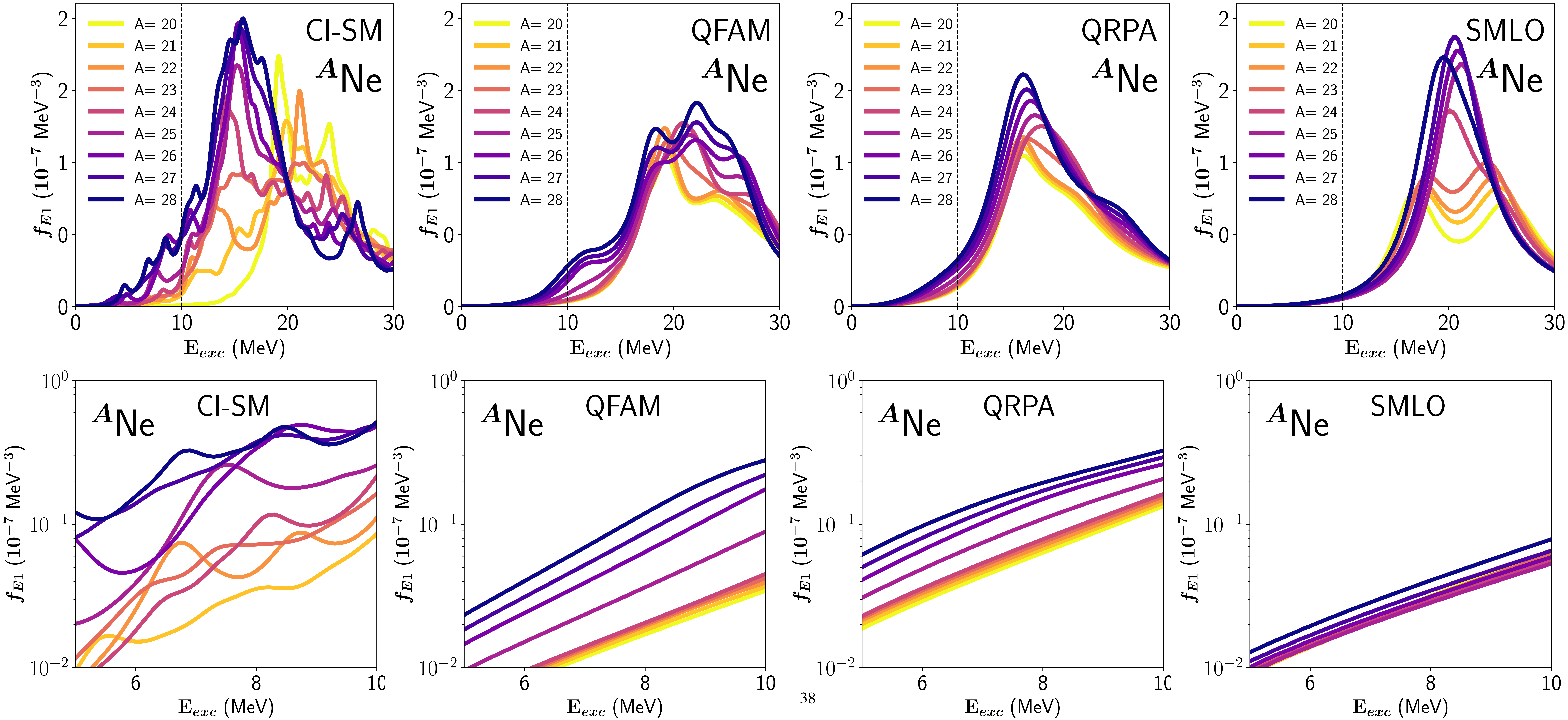
Study of the Pygmy dipole resonance ^{24}O

Oxygen chain: model comparison



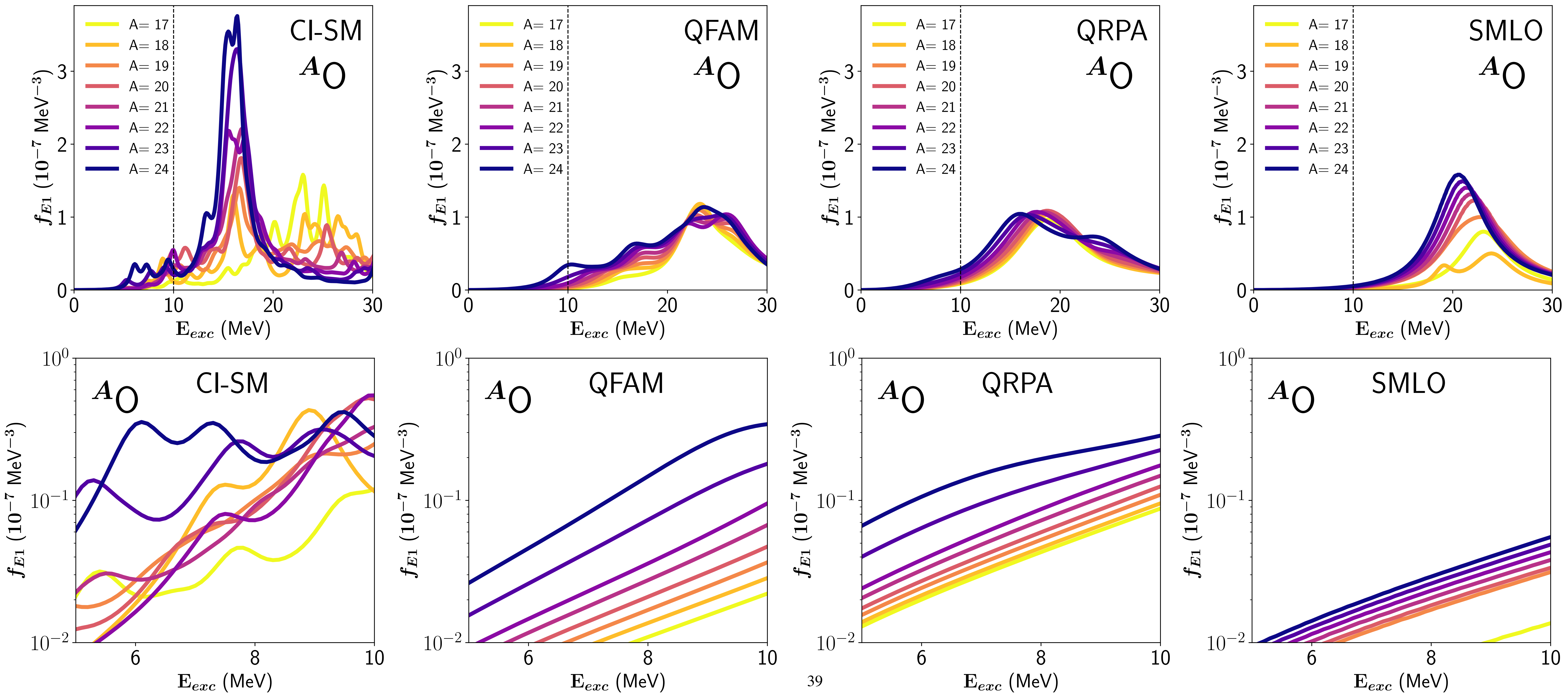
E1 response sd-shell nuclei

Neon isotopic chain: model comparison



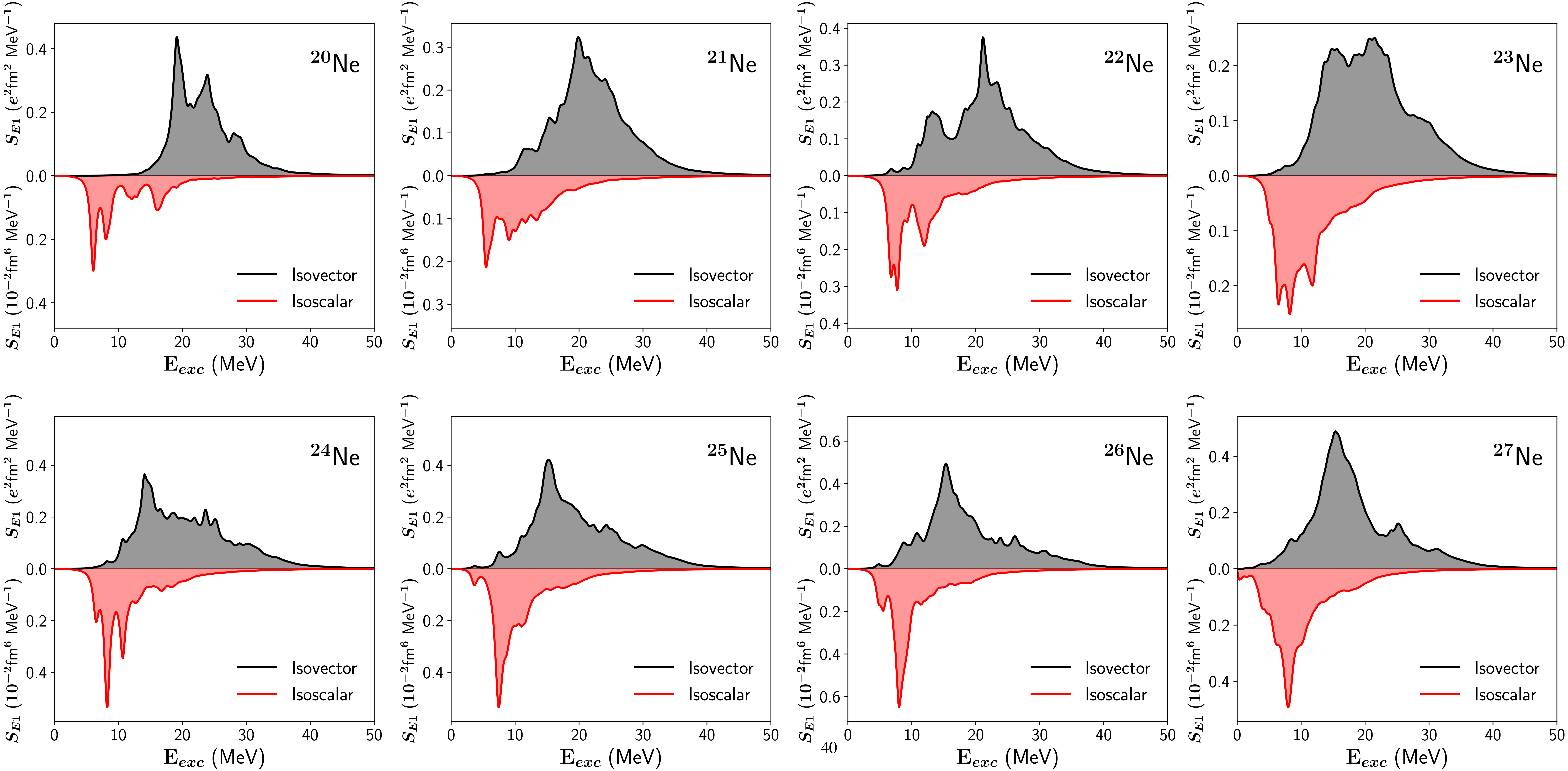
Study of the Pygmy dipole resonance

Oxygen chain: model comparison



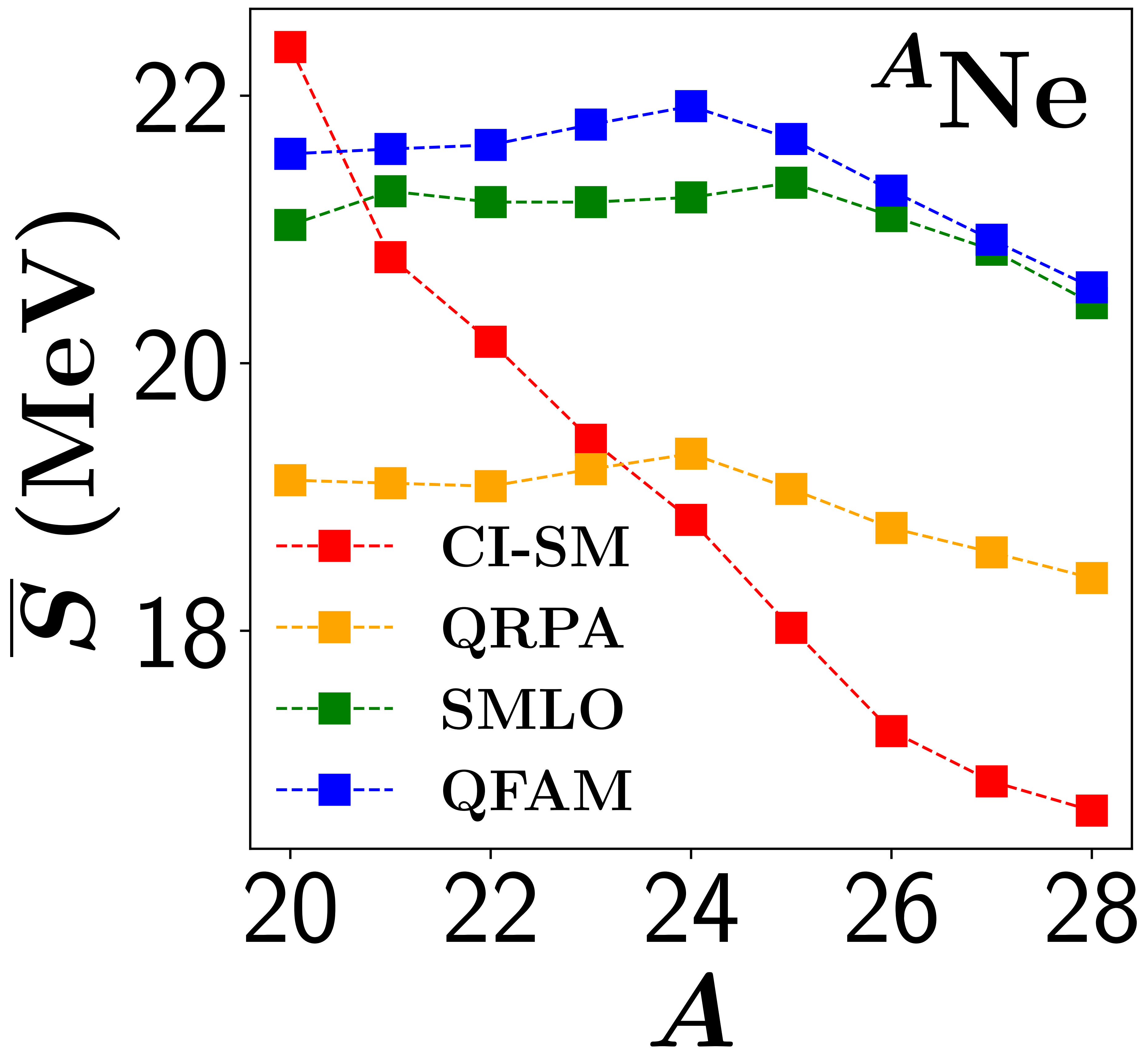
Study of the Pygmy dipole resonance

Isospin mixing in the neon chain

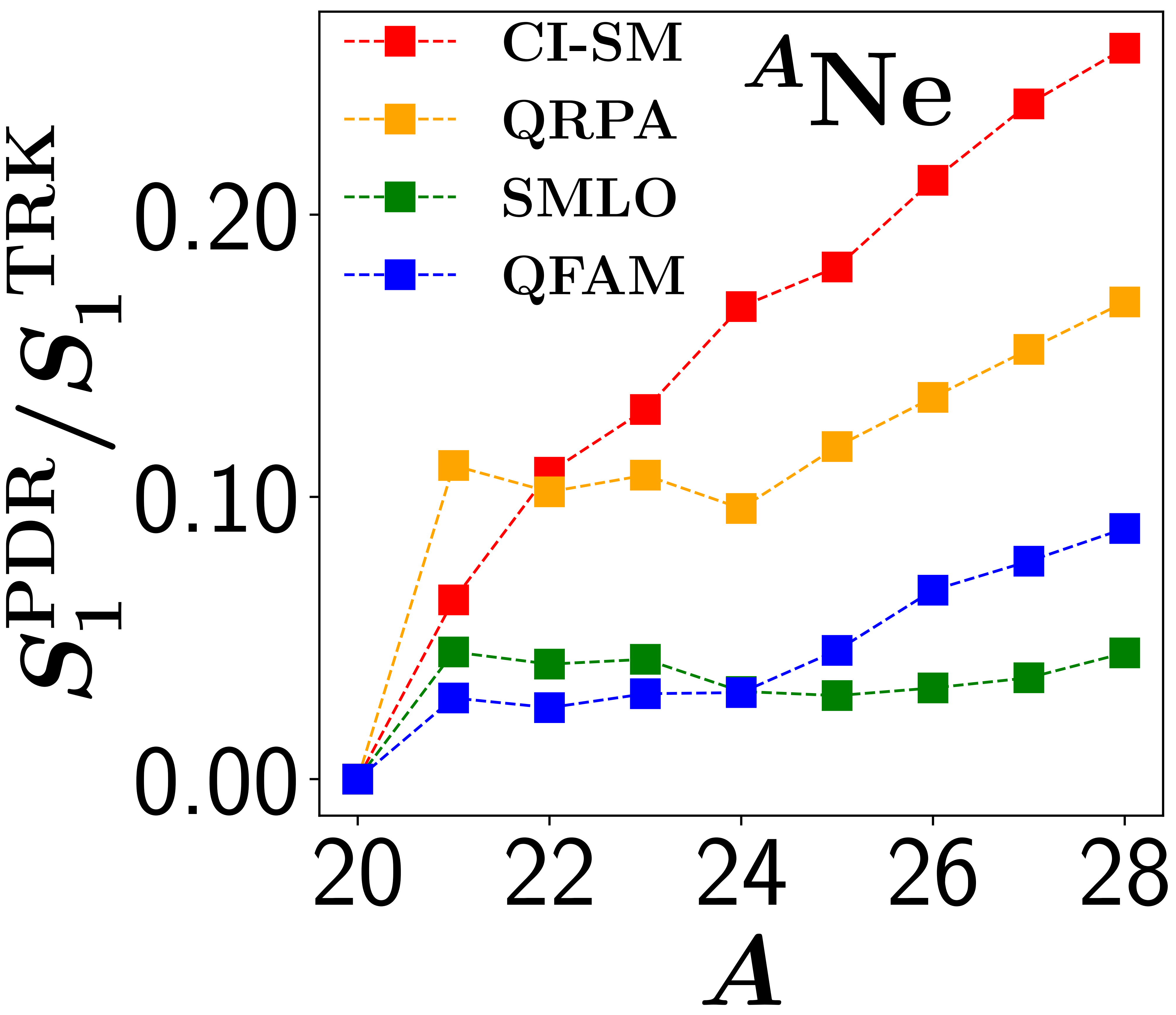


E1 response sd-shell nuclei

Neon isotopic chain: model comparison



up to 30 MeV



from S_n to 15 MeV