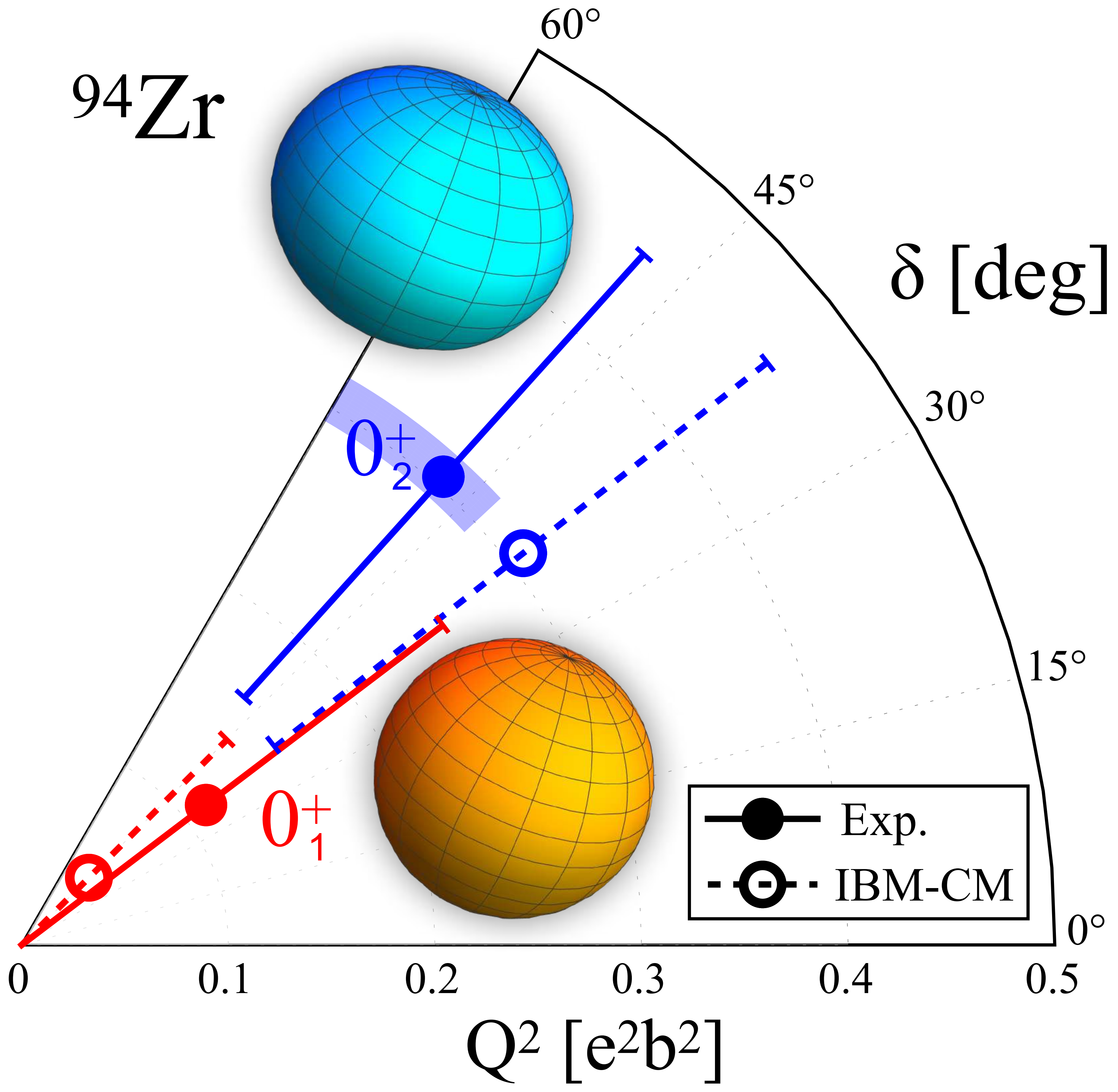
Marco Rocchini
INFN - Istituto Nazionale di Fisica Nucleare
FIRENZE DIVISION

Spherical-Oblate Shape Coexistence in ^{94}Zr and the SPIDER Coulomb-Excitation Campaign at LNL



Outline

1) Excited 0^+ states in nuclei
2) The nuclear shape
3) Coulomb excitation
4) The LNL labs of INFN
5) The Zr isotopes
6) Our latest results on ^{94}Zr

Excited 0⁺ States in Mid-Mass Nuclei

Excited 0⁺ States in Even-Even Mid-Mass Nuclei

Nuclear Shape

Low-Energy Coulomb Excitation

INFN and LNL

SPIDER with GALILEO and AGATA

The Zr Isotopic Chain and QPTs

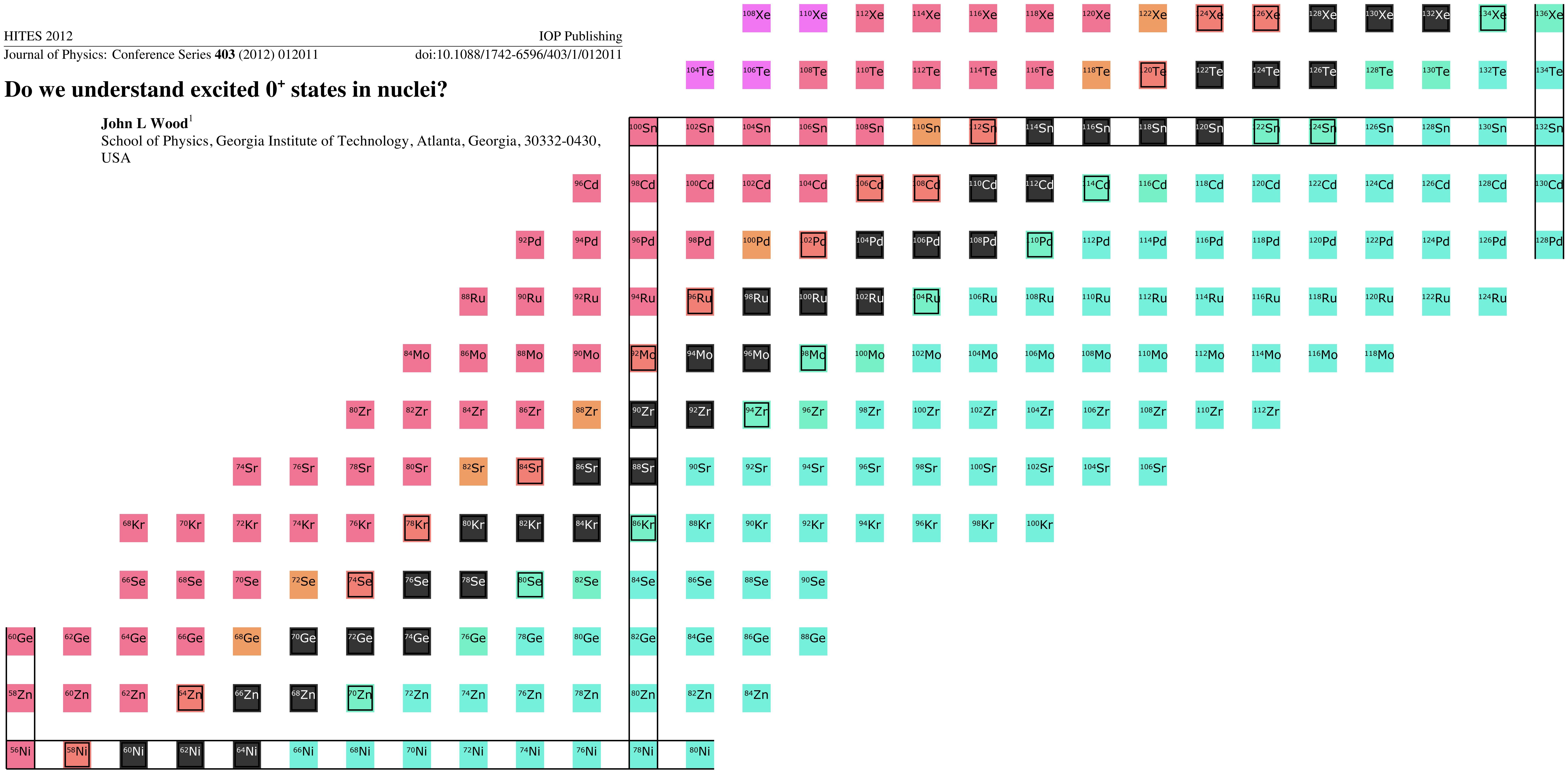
CoulEx Experiment on ^{94}Zr

Experimental Shapes in ^{94}Zr

HITES 2012 IOP Publishing
Journal of Physics: Conference Series **403** (2012) 012011 doi:10.1088/1742-6596/403/1/012011

Do we understand excited 0⁺ states in nuclei?

John L Wood[1]
School of Physics, Georgia Institute of Technology, Atlanta, Georgia, 30332-0430, USA



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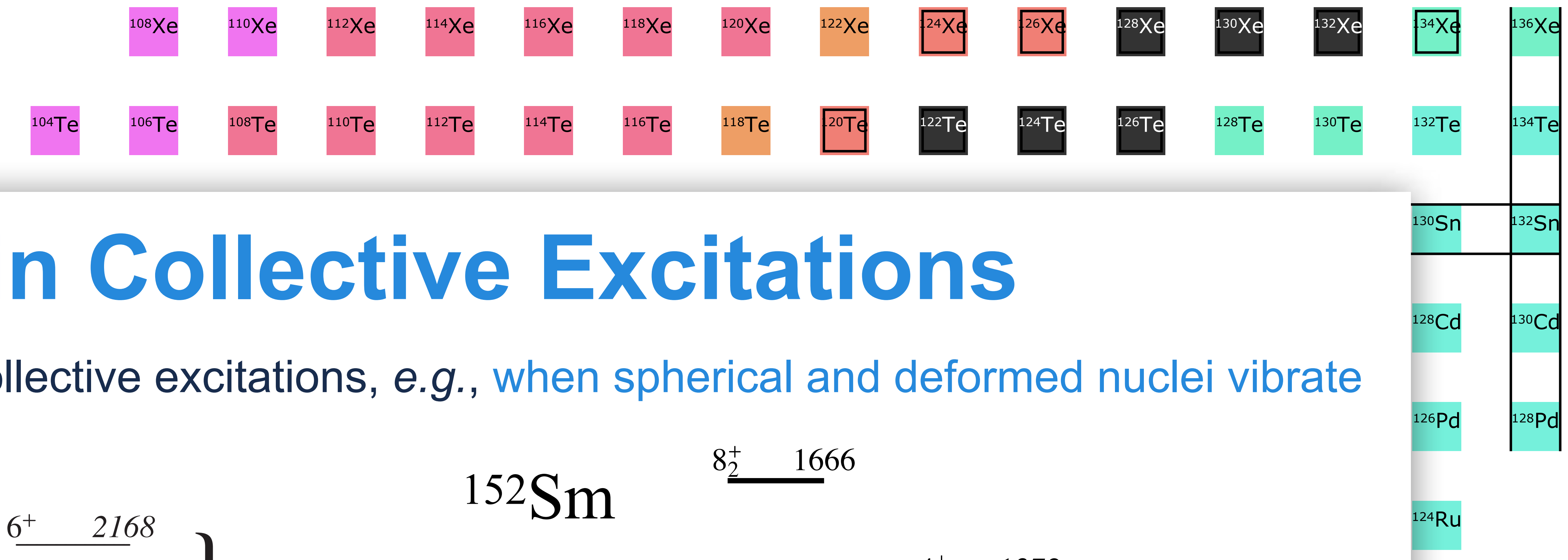
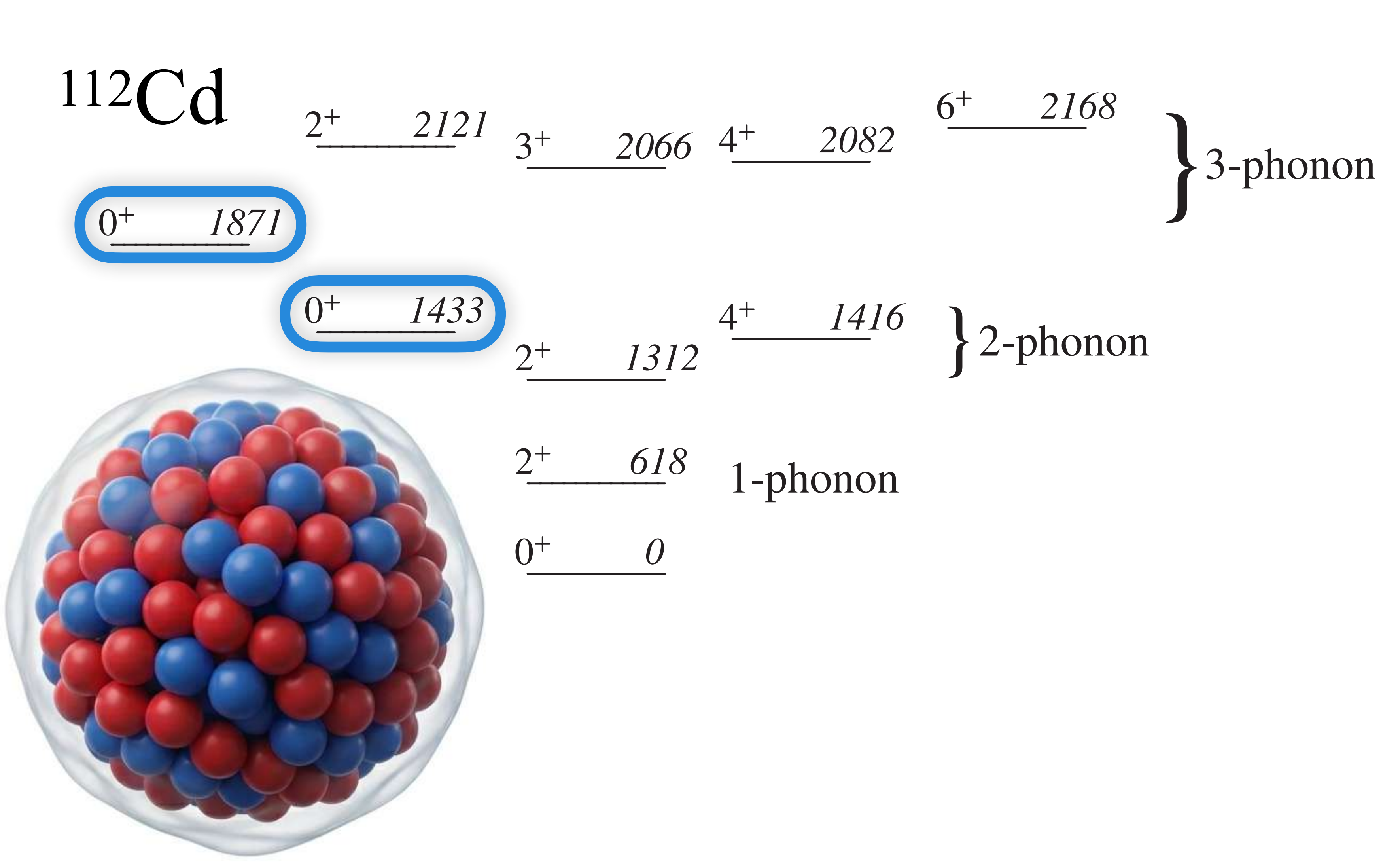
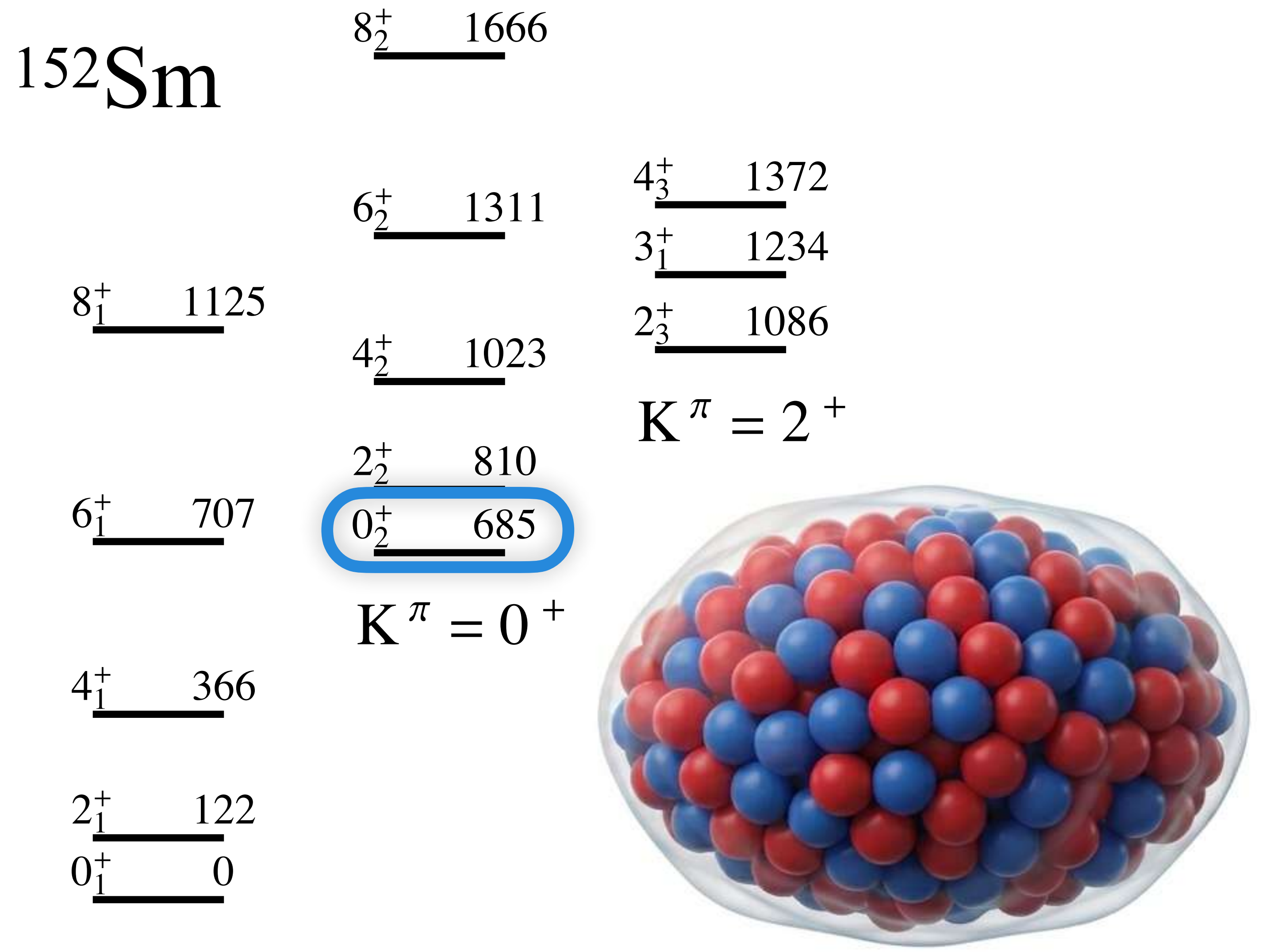
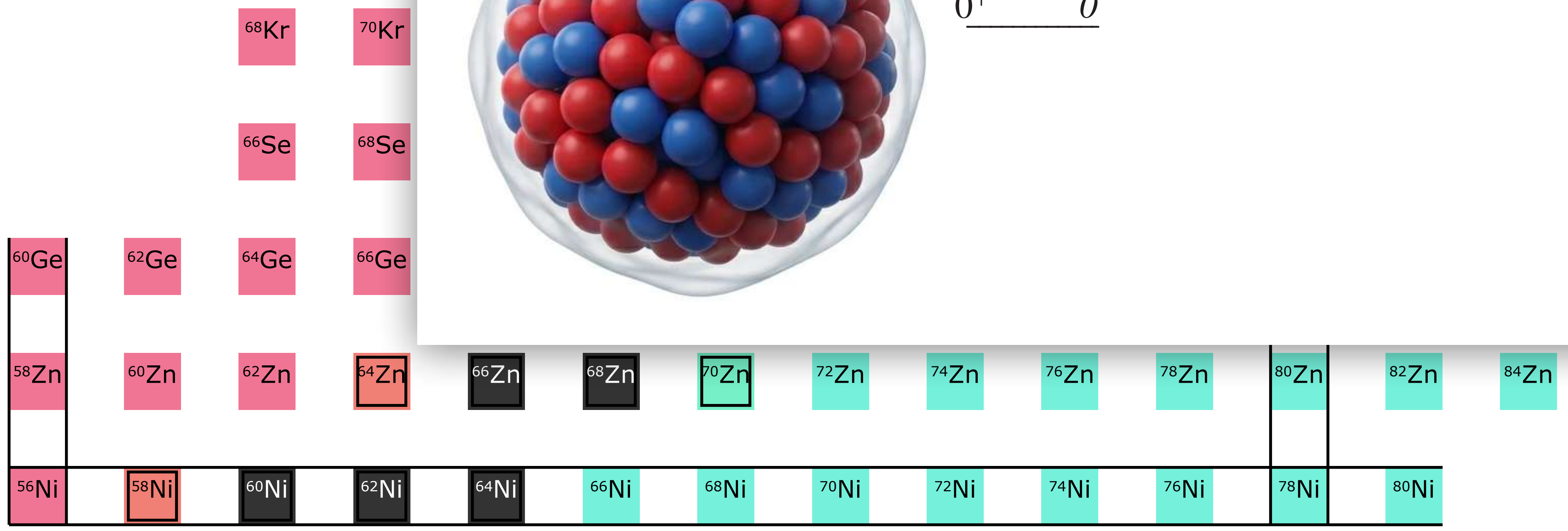


Excited 0⁺ States in Collective Excitations

- Excited 0⁺ states can originate in collective excitations, *e.g.*, when spherical and deformed nuclei vibrate







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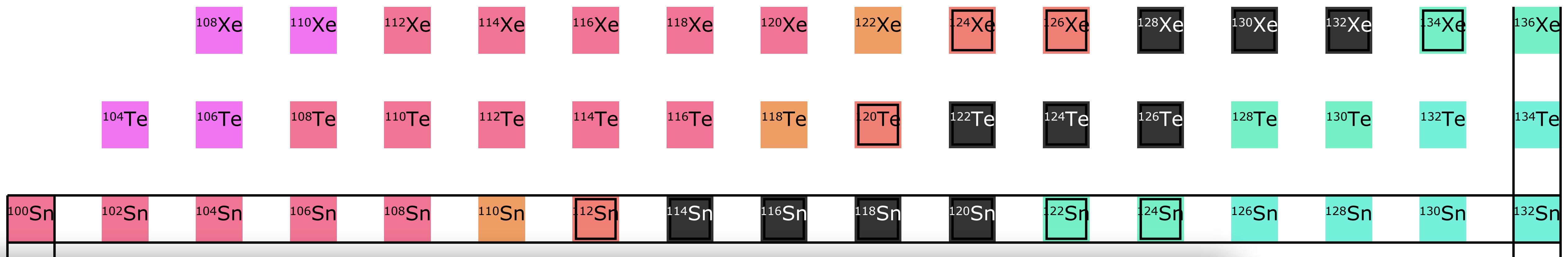
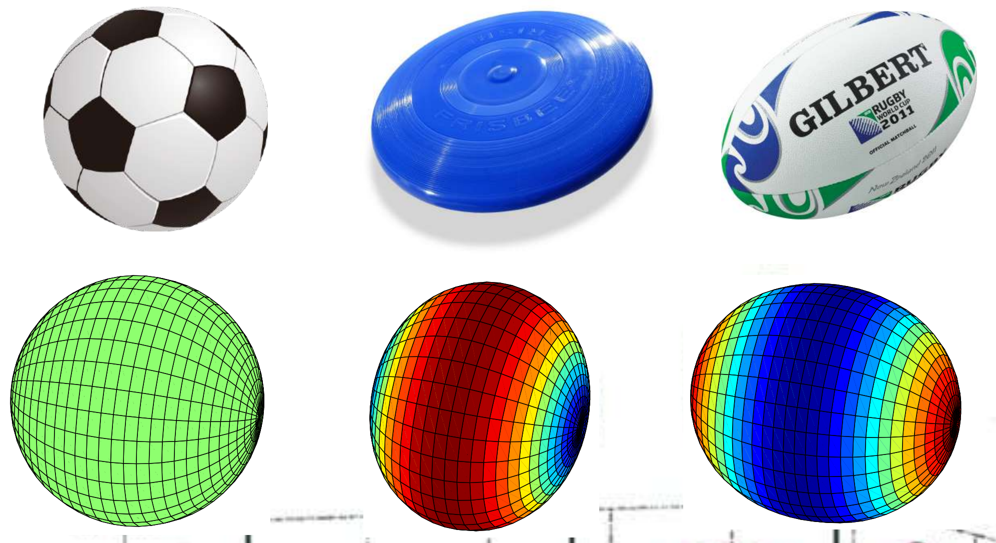
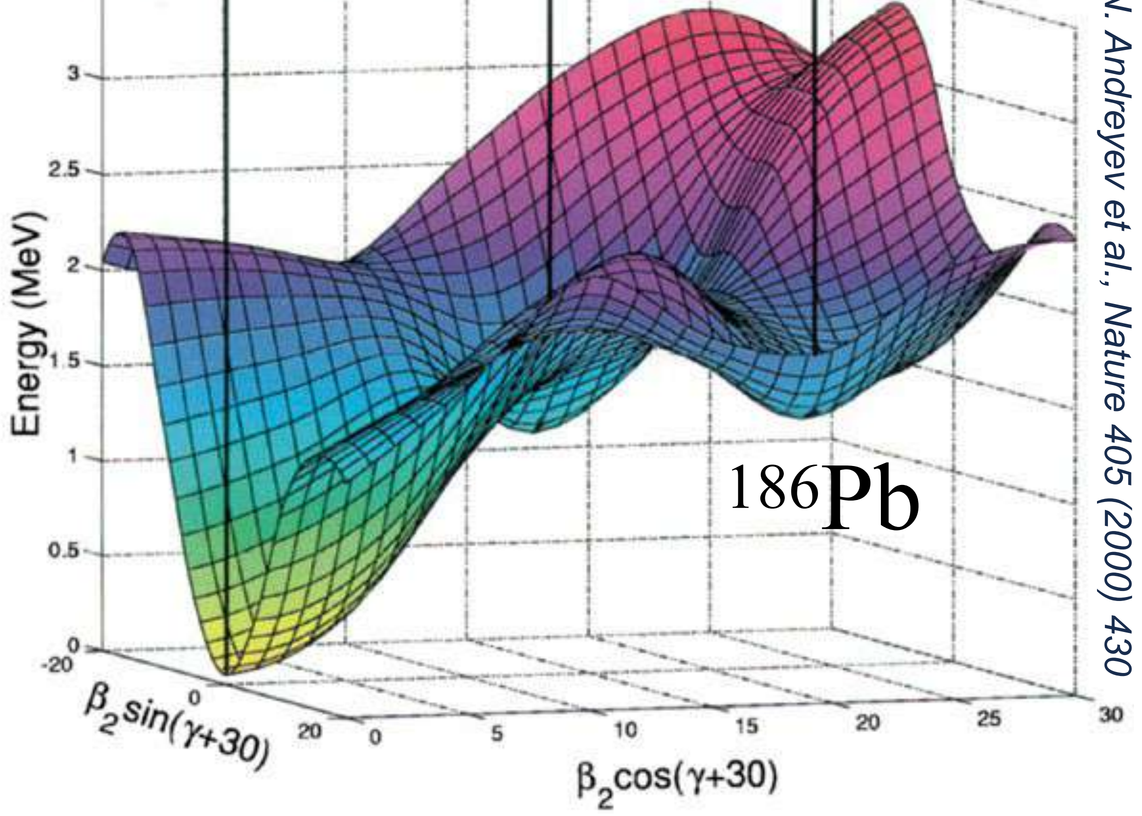


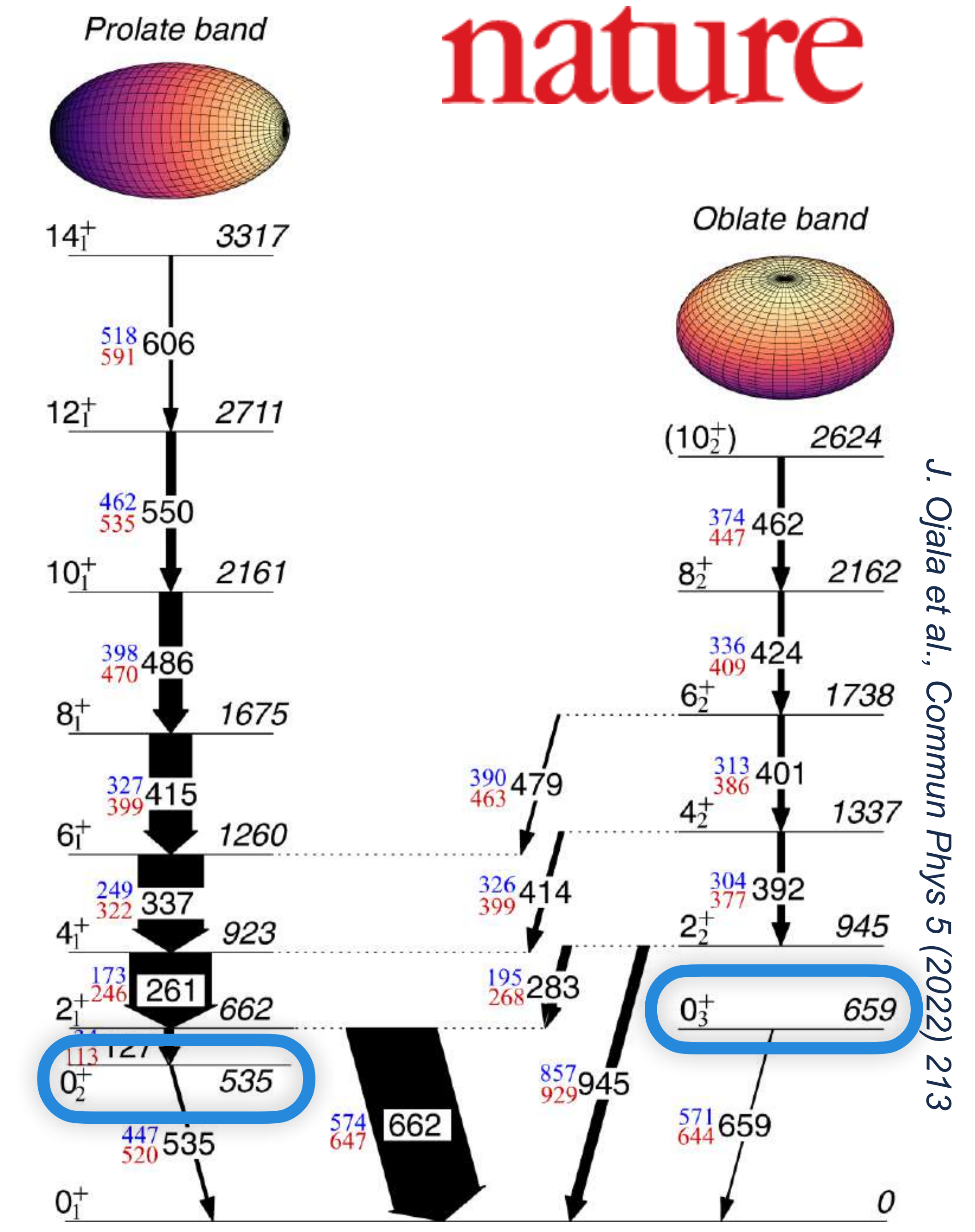
Shape Coexistence

- Presence of low-lying excited states with different shapes in the same nucleus
- In even-even nuclei, each coexisting band with a different shape has a 0⁺ band-head



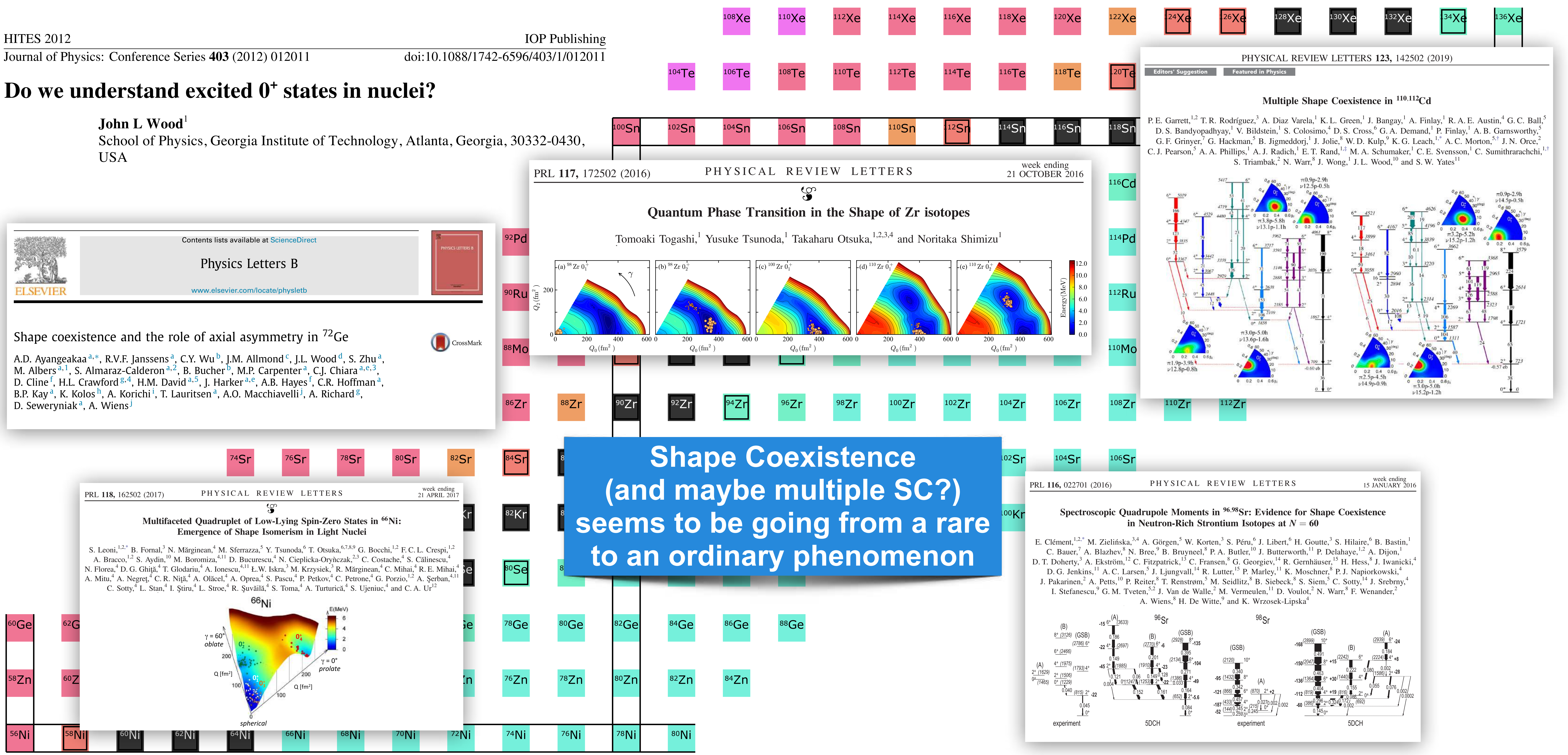


A. N. Andreyev et al., Nature 405 (2000) 430



J. Ojala et al., Commun Phys 5 (2022) 213

Excited 0+ States in Mid-Mass Nuclei

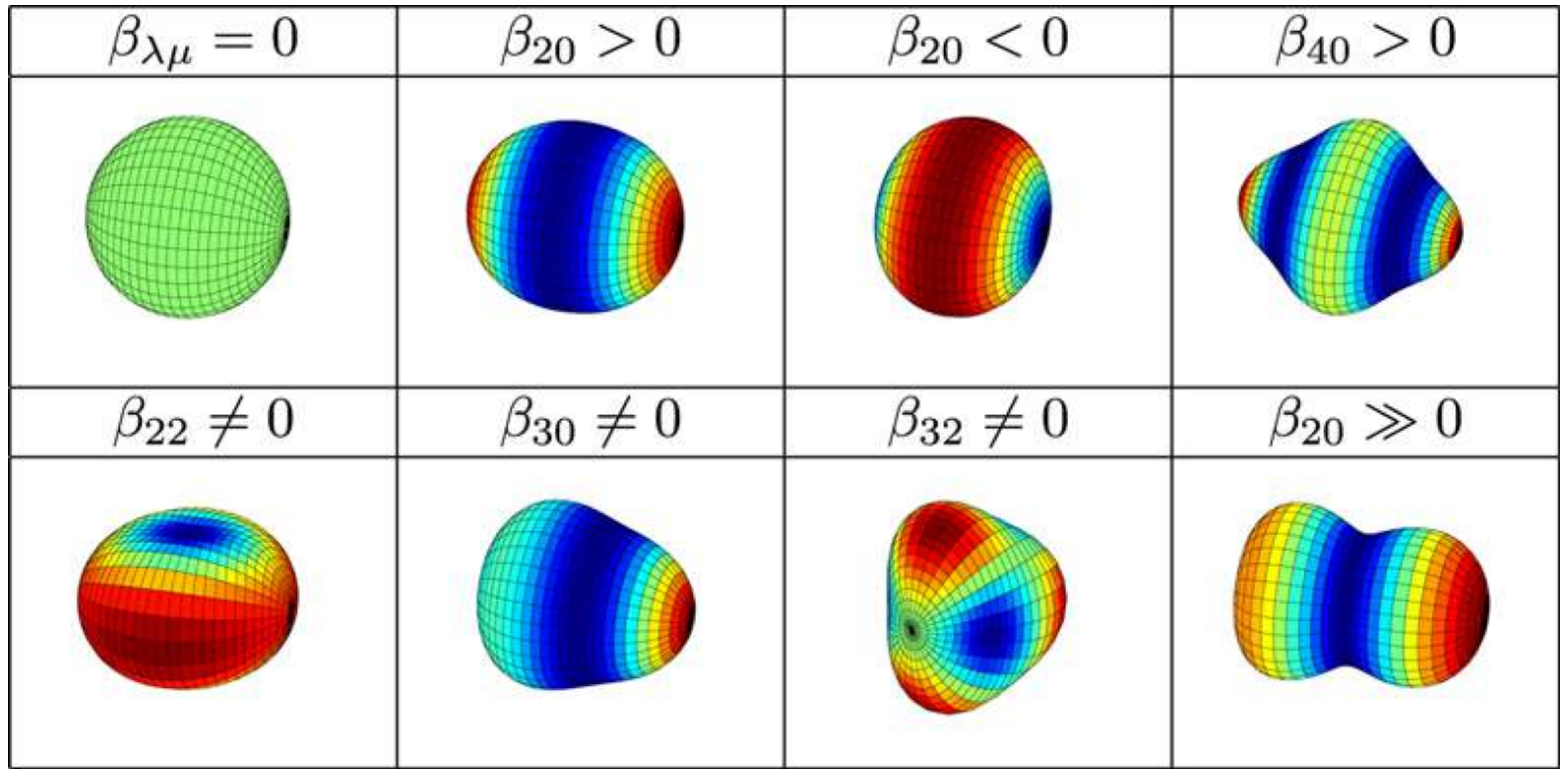


Shape Coexistence (and maybe multiple SC?) seems to be going from a rare to an ordinary phenomenon

Nuclear Shape

- Generic shape:

$$R(\theta,\phi;t) = R_0\left[1+\sum \alpha_{\lambda\mu}(t)\,Y_{\lambda\mu}(\theta,\phi)\right]$$



Bing-Nan Lu, Jie Zhao, En-Guang Zhao and Shan-Gui Zhou, Journal of Physics: Conference Series 492 (2014) 012014

λ	2^λ — pole
2	**Quadrupole**
3	**Octupole**
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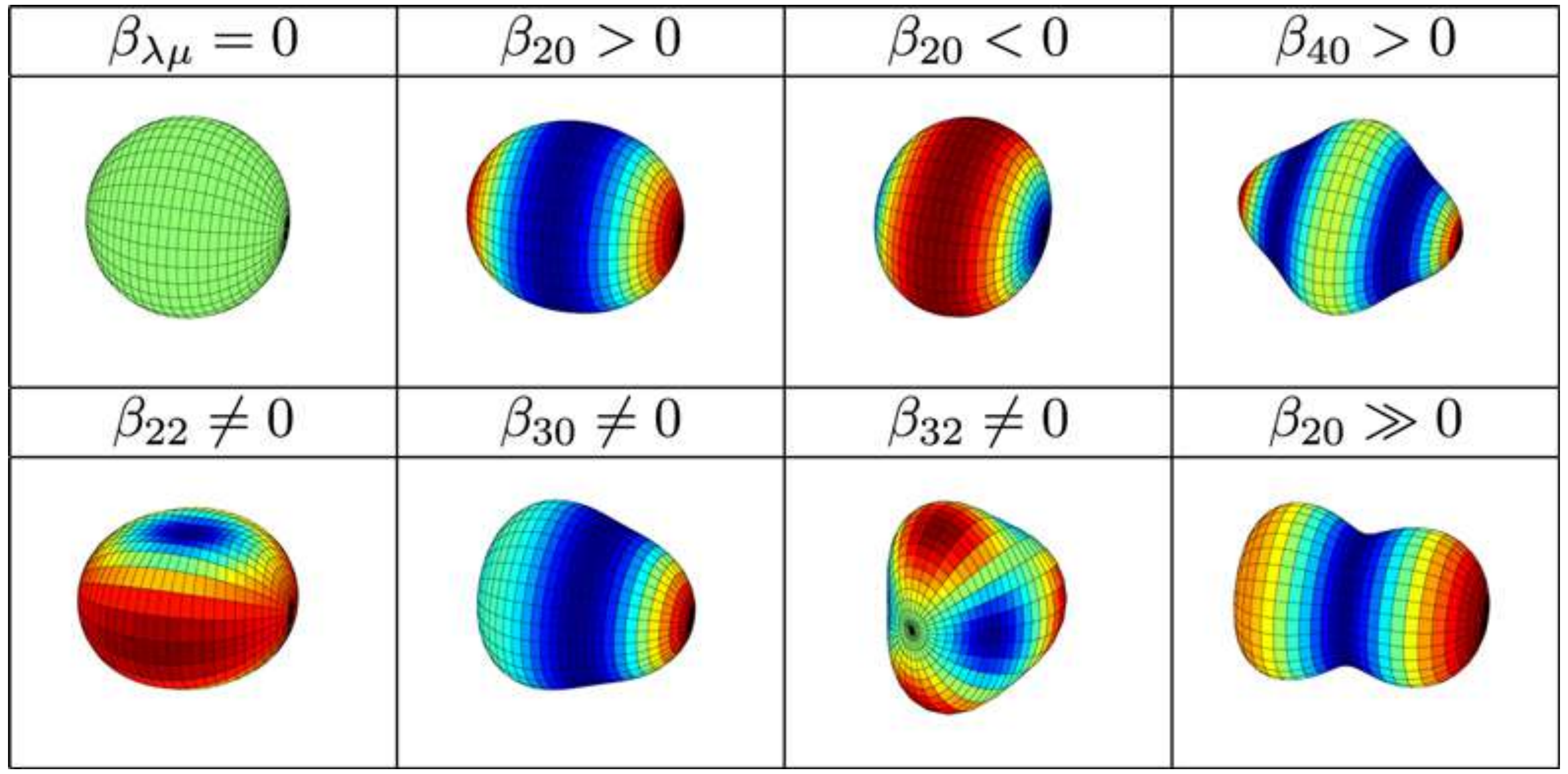

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- Quadrupole shape, periodic for γ + 60°:

$$R(\theta, \phi) = R_0 \left[1 + \beta \sqrt{\frac{5}{16\pi}} \left(\cos\gamma \left(3\cos^2\theta - 1\right) + \sqrt{3}\sin\gamma \sin^2\theta \cos 2\phi\right)\right]$$

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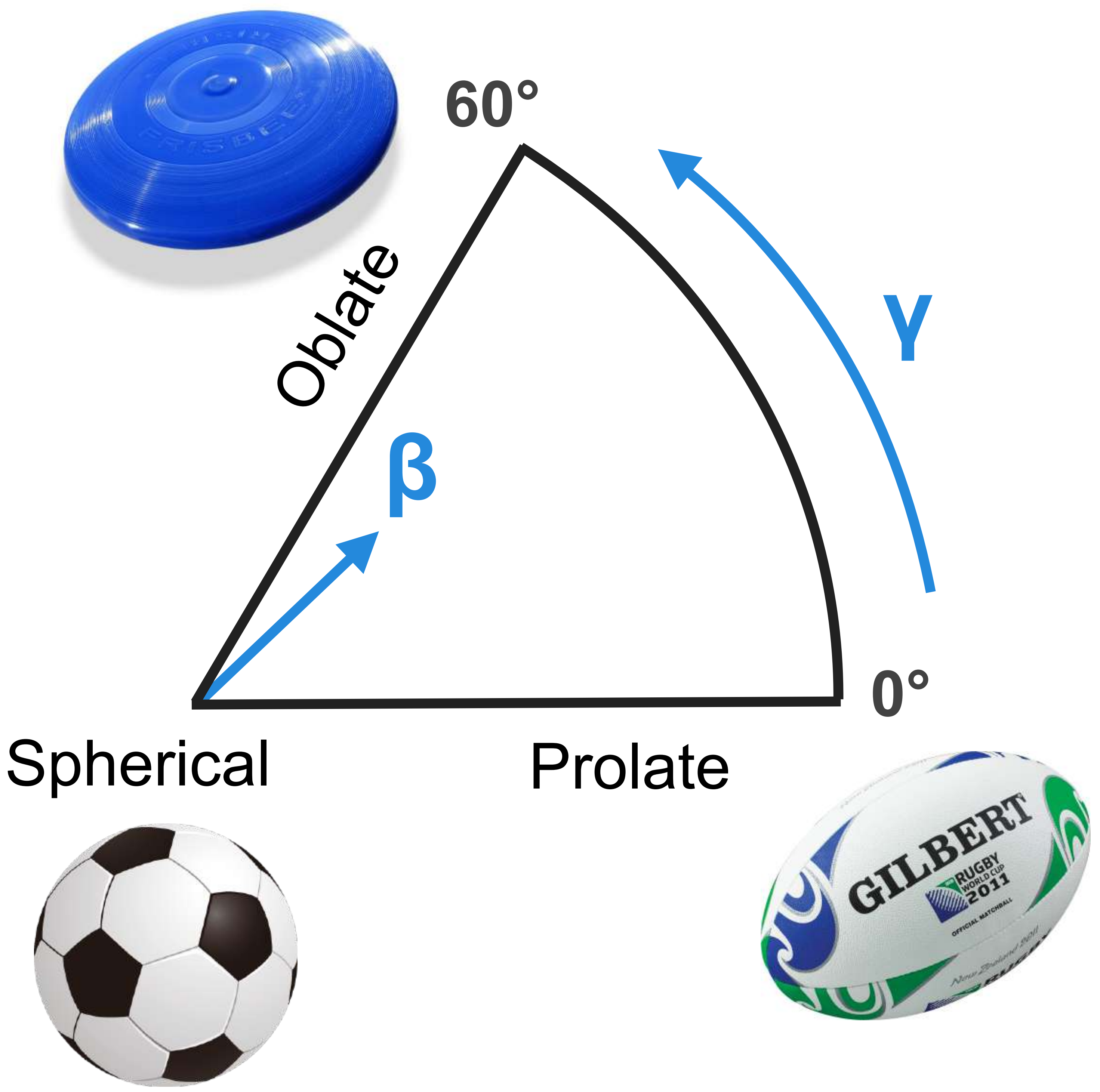

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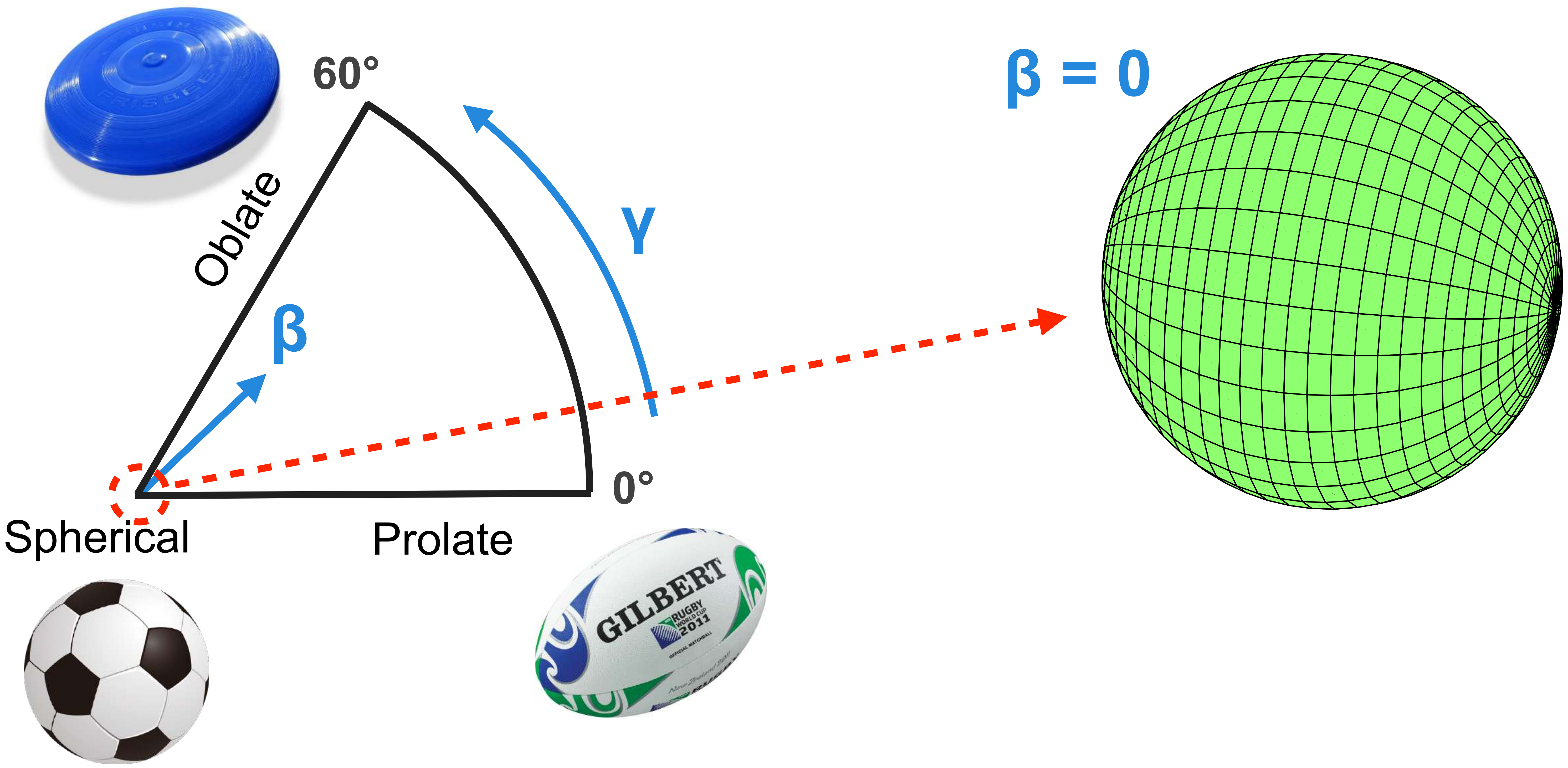

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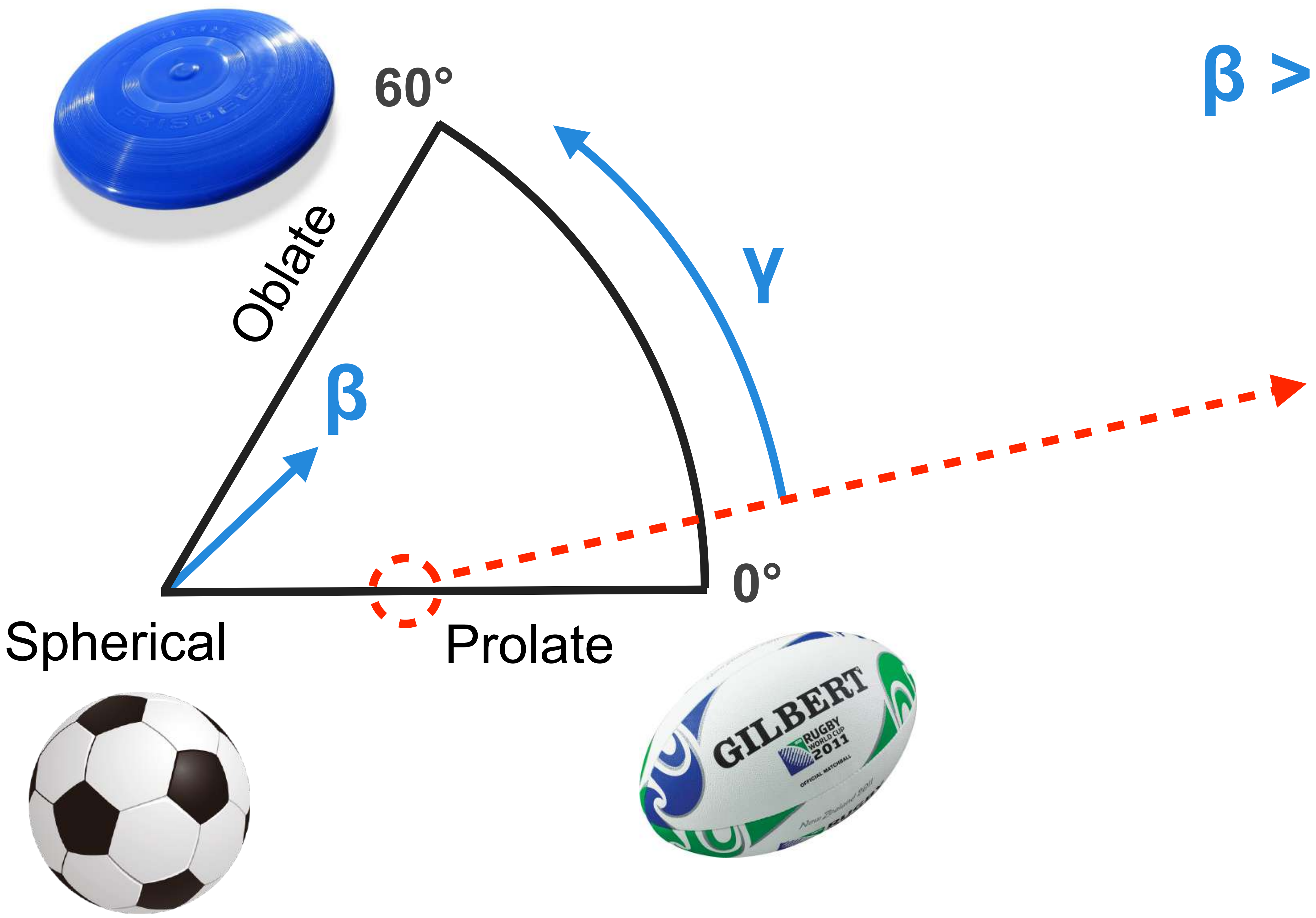
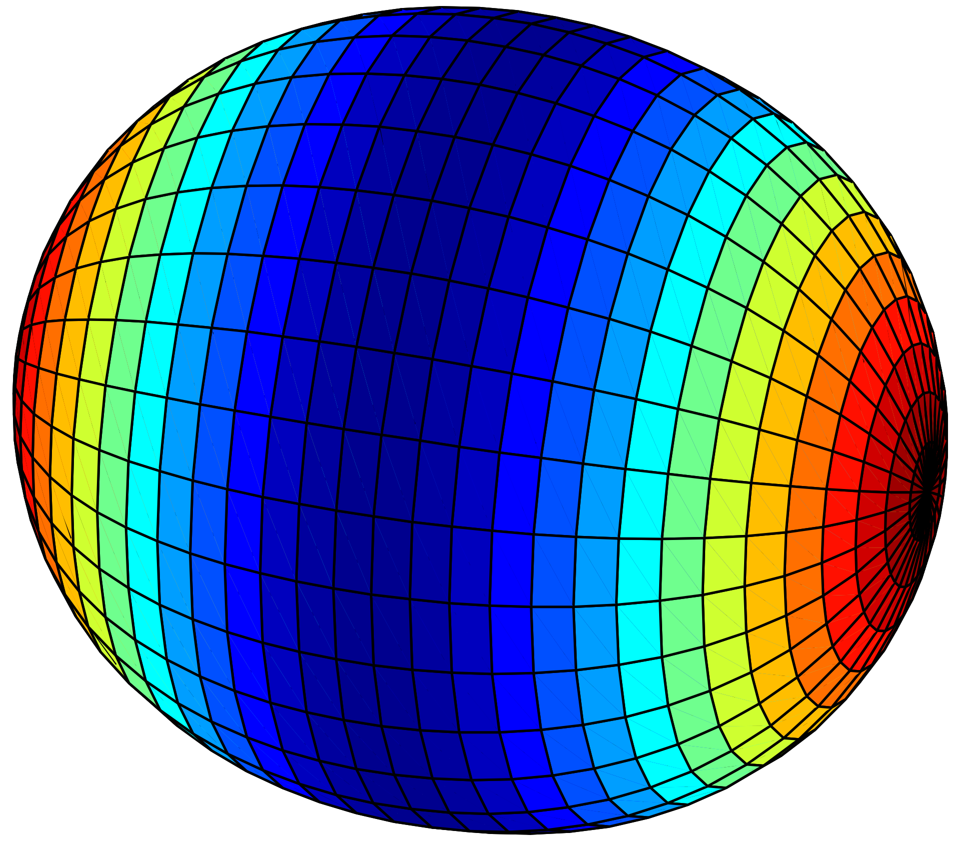


β > 0, γ = 0°



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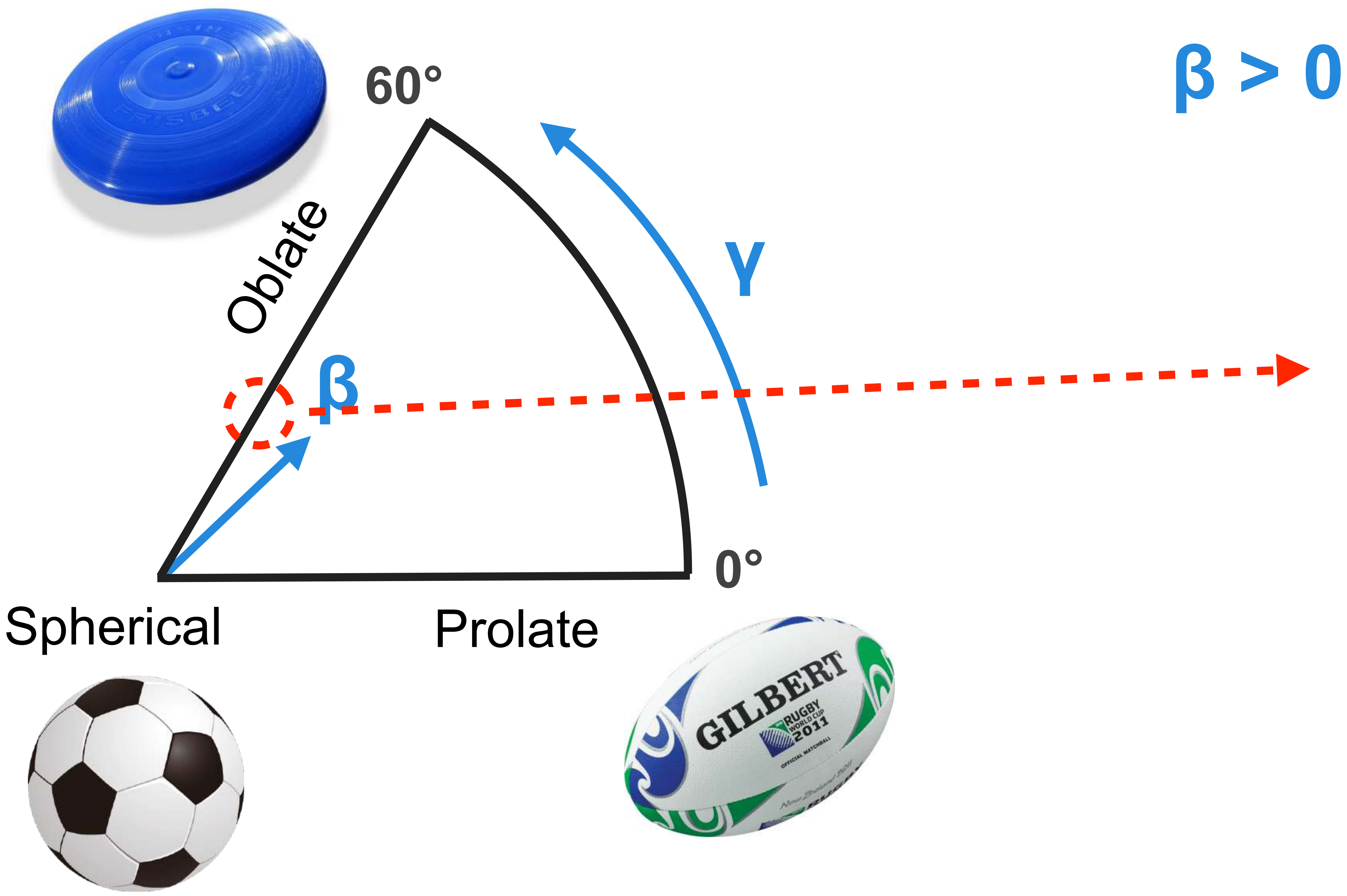

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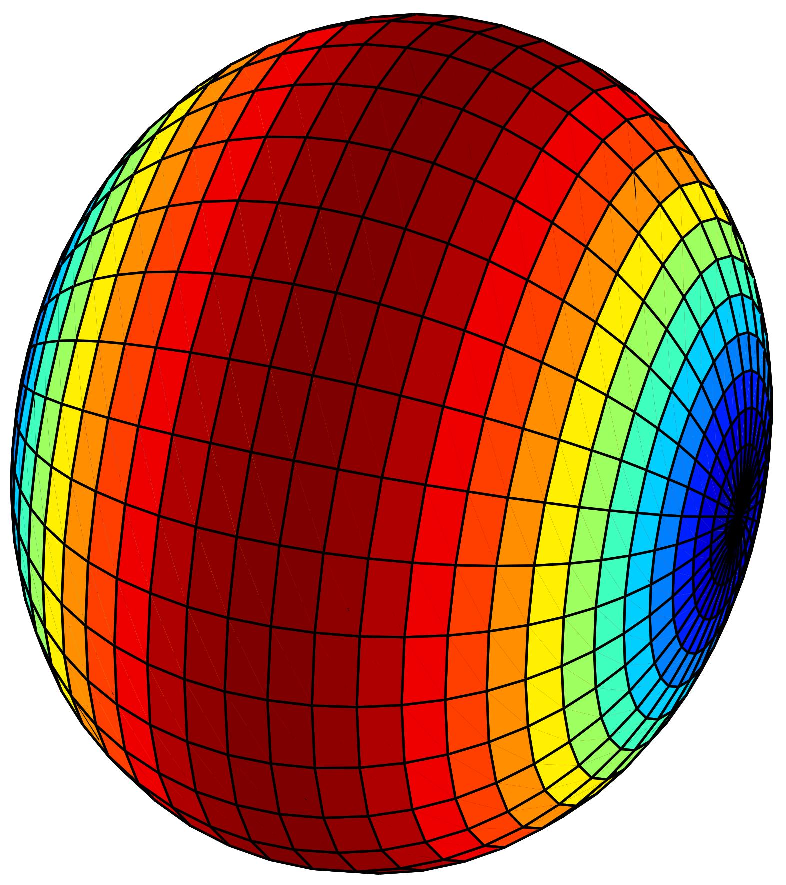

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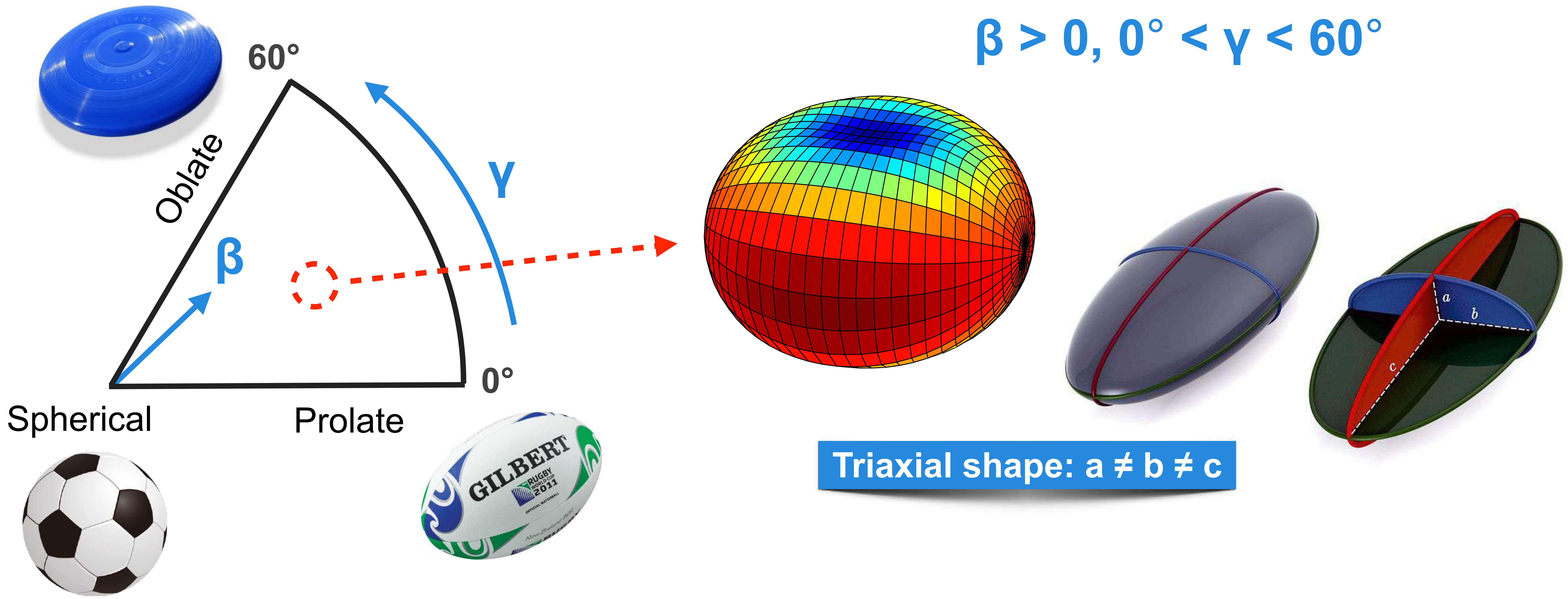
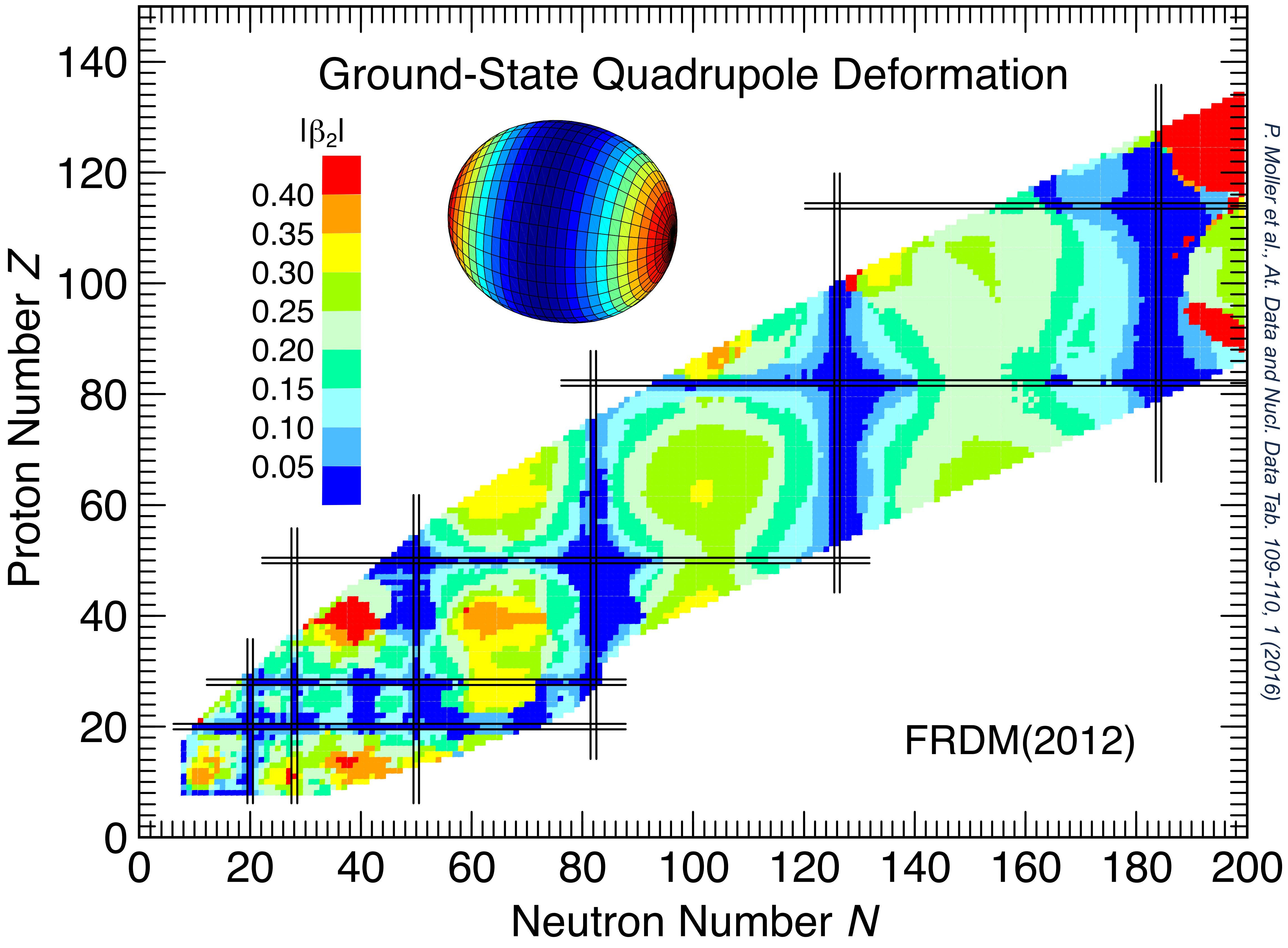
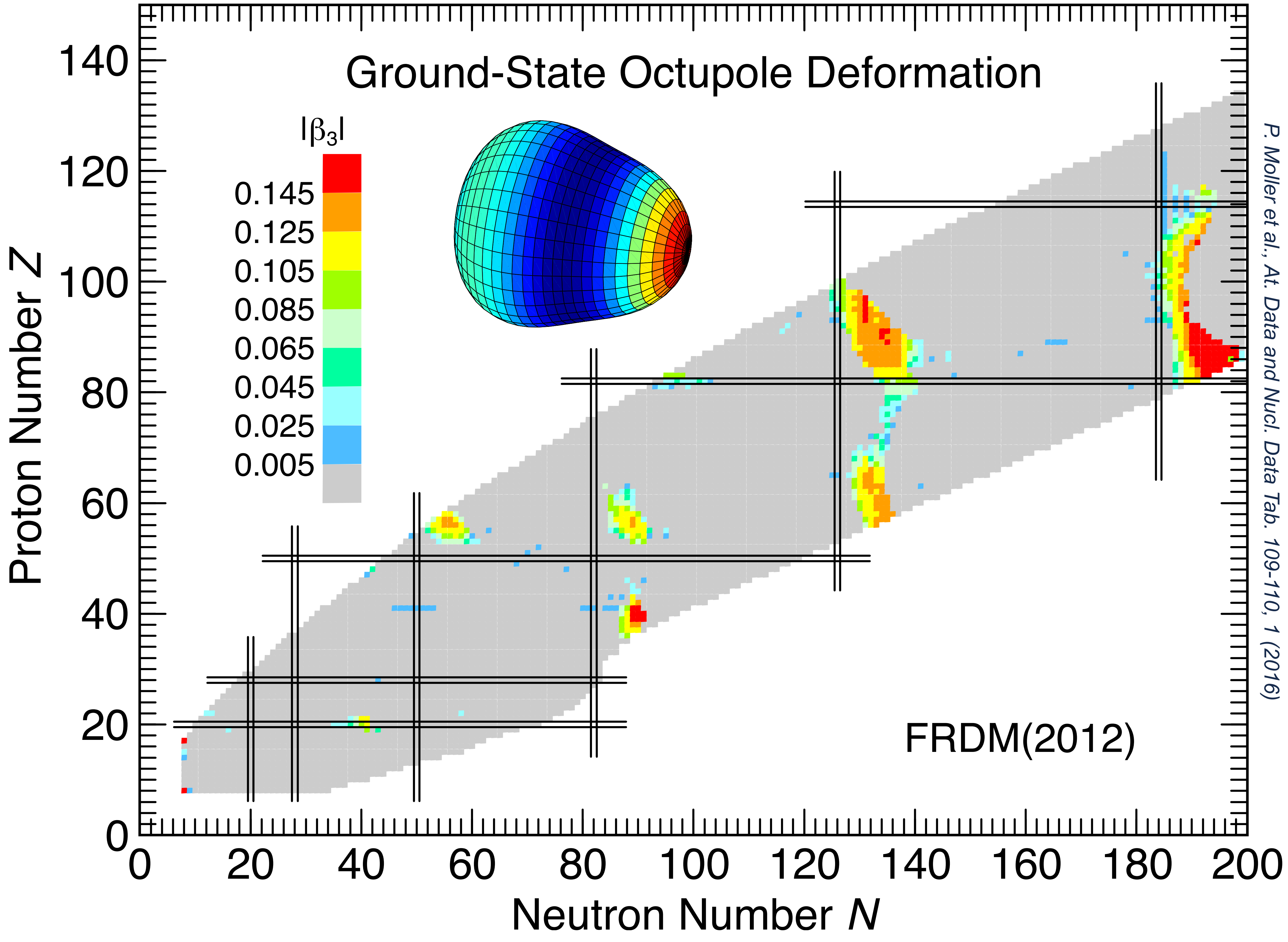


Nuclear Shape

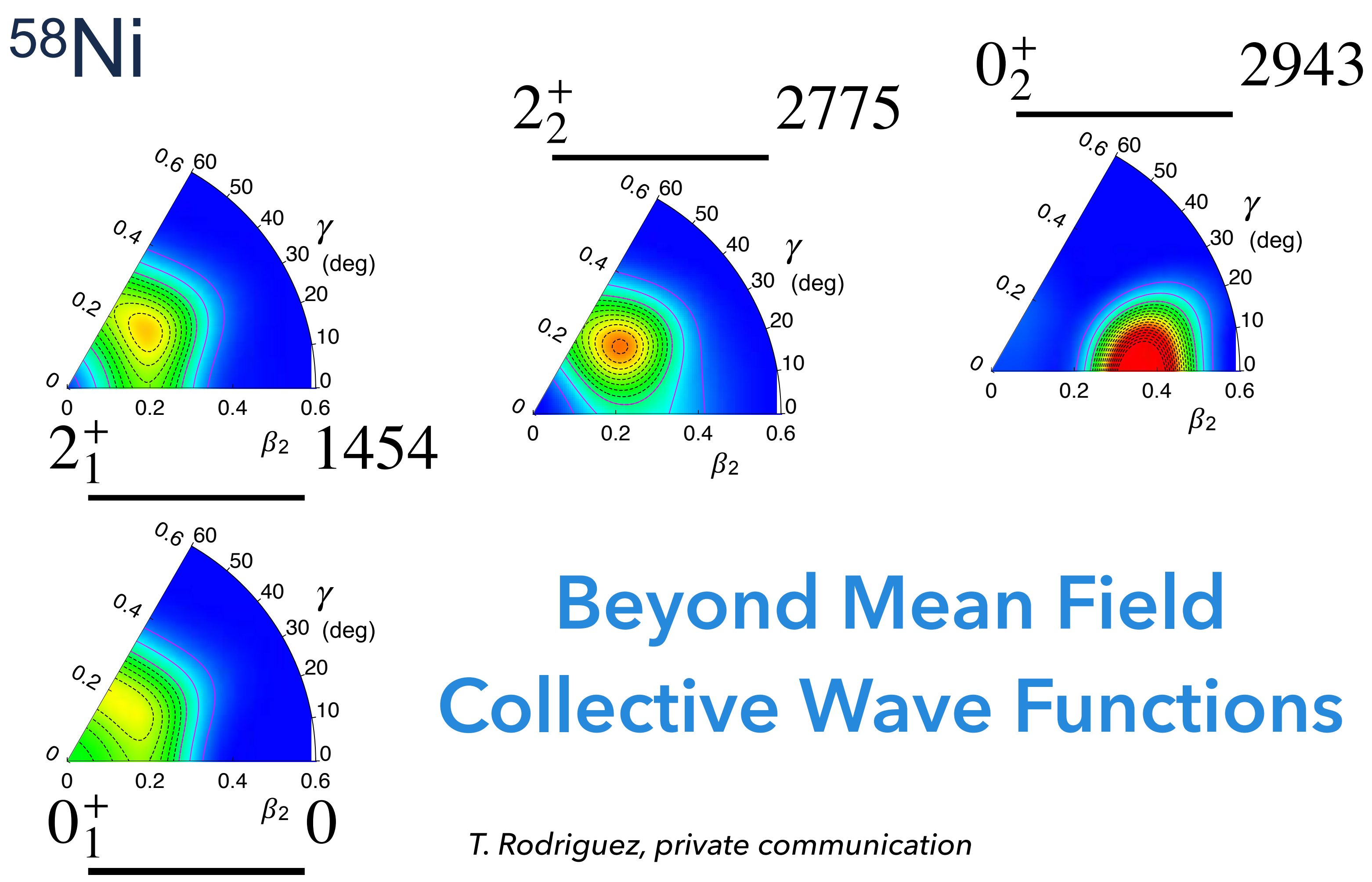
- Quadrupole shapes are predicted as the most frequent and important in atomic nuclei, followed by the octupole ones





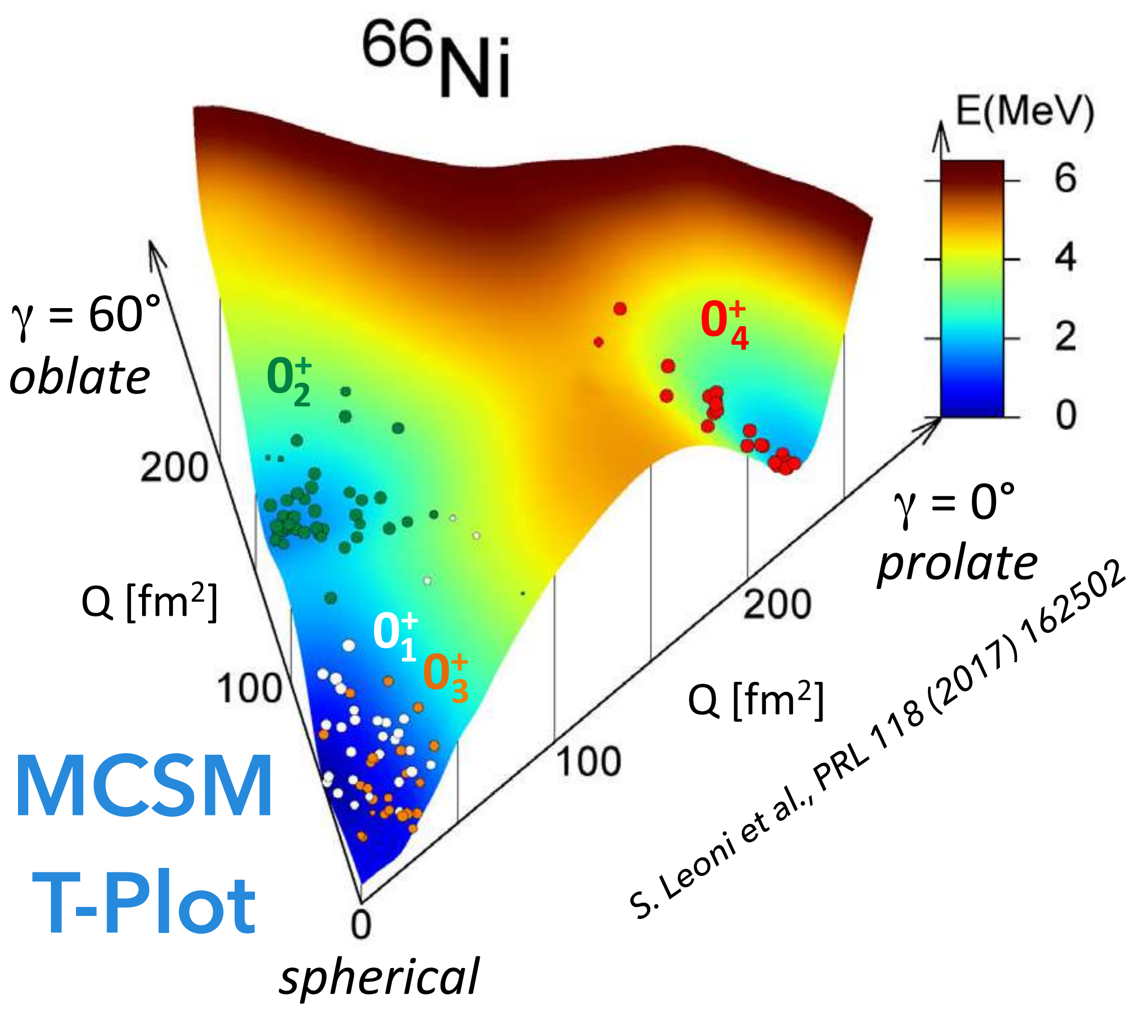
Theoretical Description of the Nuclear Shape

- The nuclear shape can be calculated with theoretical models for different isotopes in their ground and excited states, *e.g.*:
  - Beyond Mean Field — Collective Wave Functions: probability distribution (normalised to 1) to find a certain deformation in the (β, γ) plane
  - MCSM, LSSM — T-Plots: (β, γ) value for each wave-function component drawn with a circle (the circle's radius is proportional to the probability of the component), superimposed to the potential energy surface of the nucleus (state independent)

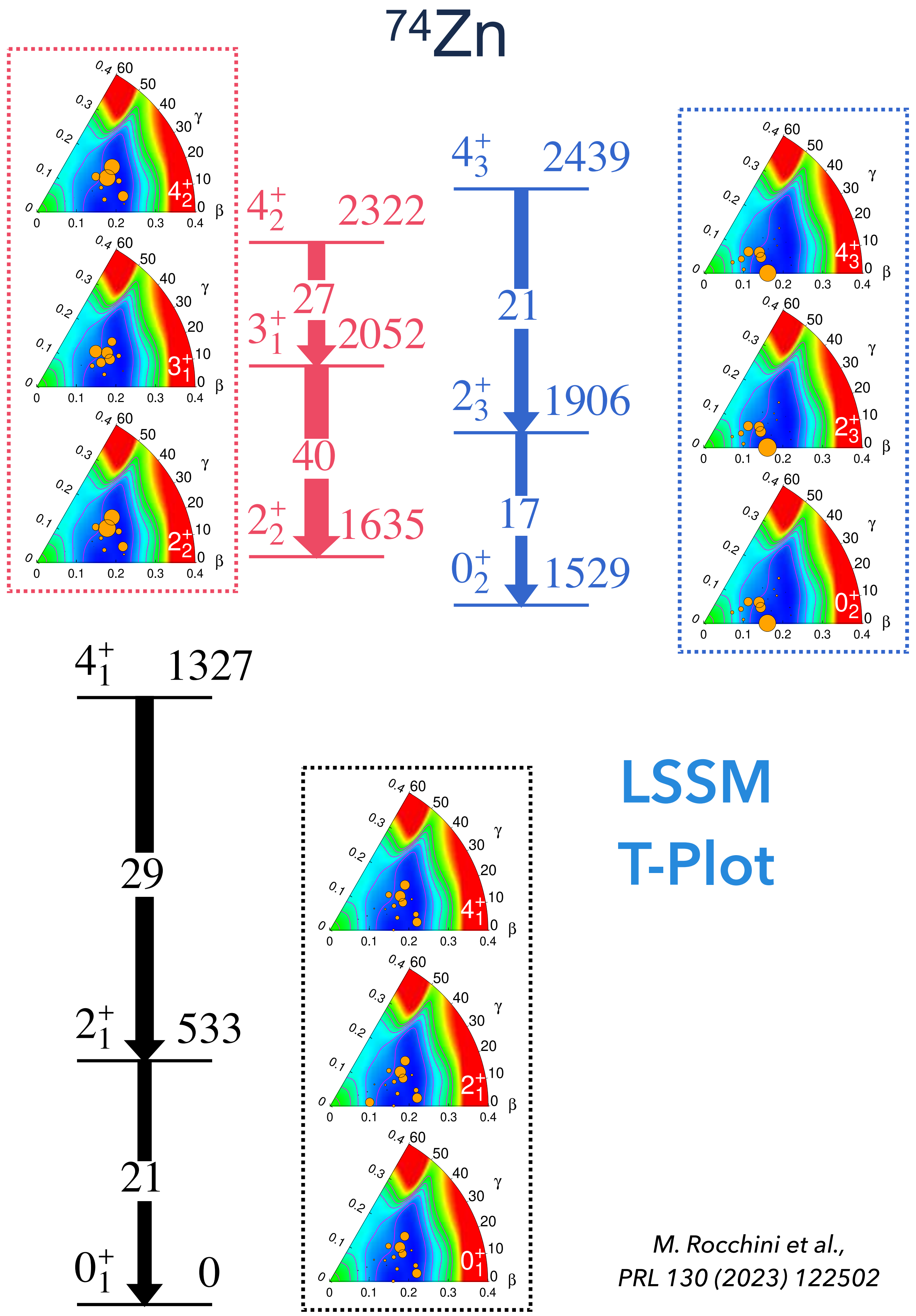


Beyond Mean Field Collective Wave Functions

T. Rodriguez, private communication



MCSM T-Plot

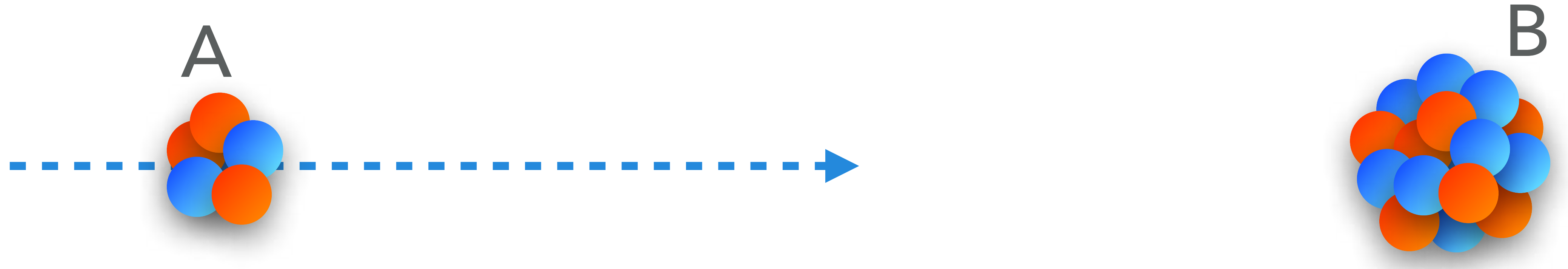


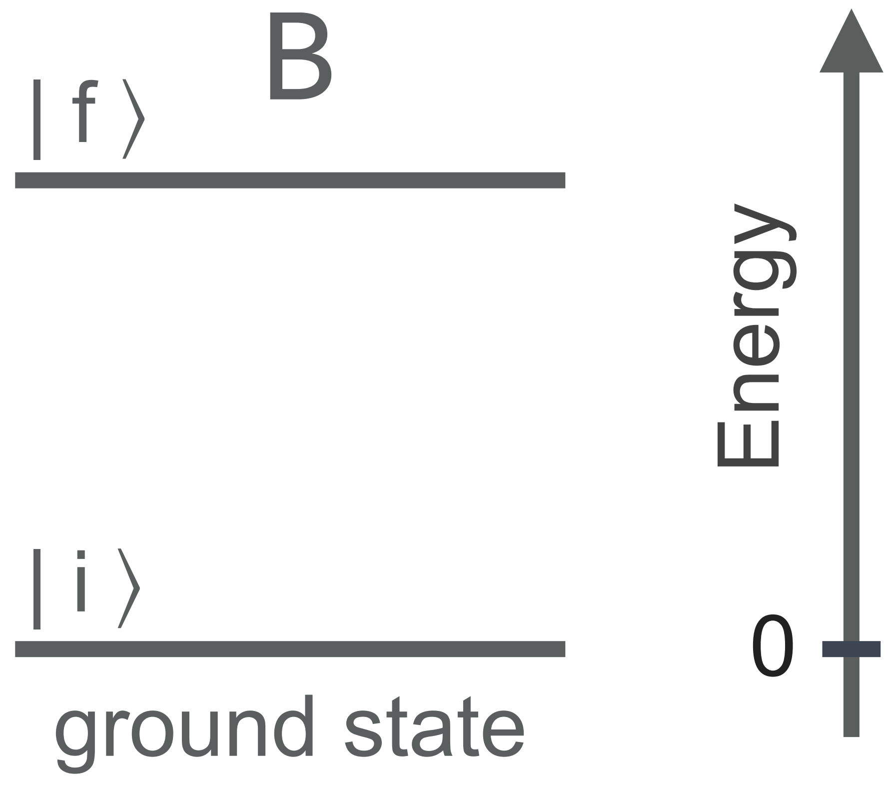
LSSM T-Plot

M. Rocchini et al., PRL 130 (2023) 122502

Low-Energy Coulomb Excitation (aka CoulEx)

- Inelastic scattering between two interacting nuclei, in the “purely” electromagnetic regime





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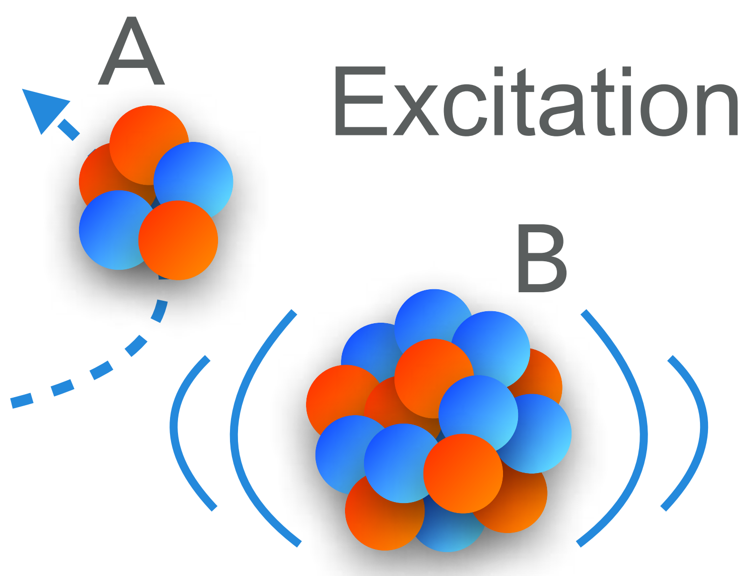
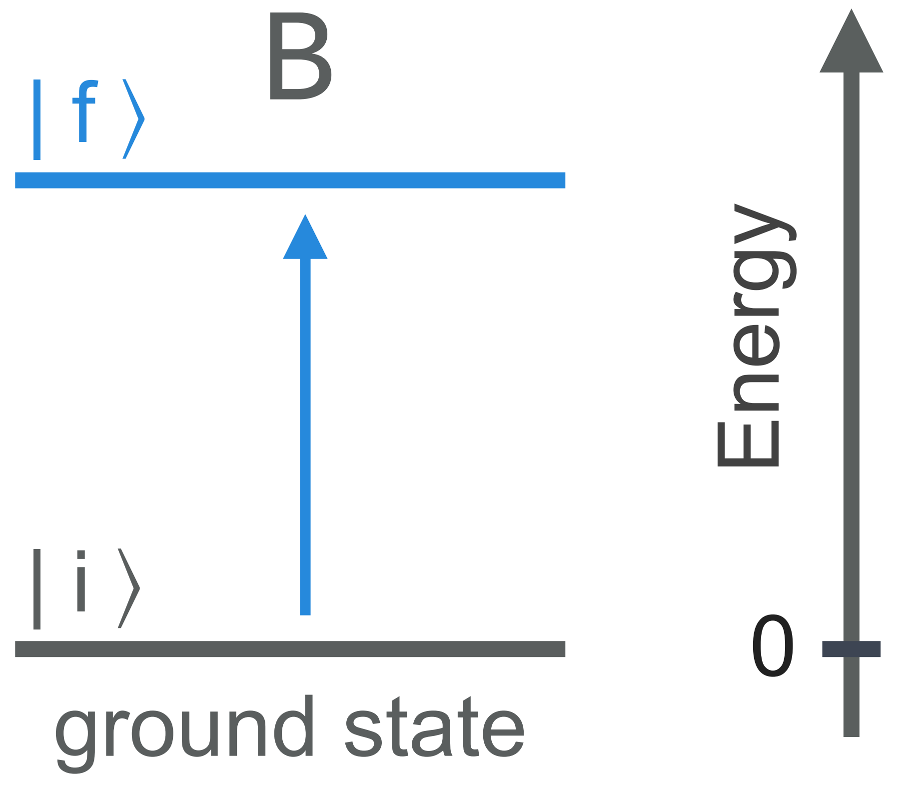


$$\frac{d\sigma_{clx}}{d\Omega} = \frac{d\sigma_{Ruth}}{d\Omega} \cdot P\,(i \longrightarrow f)$$



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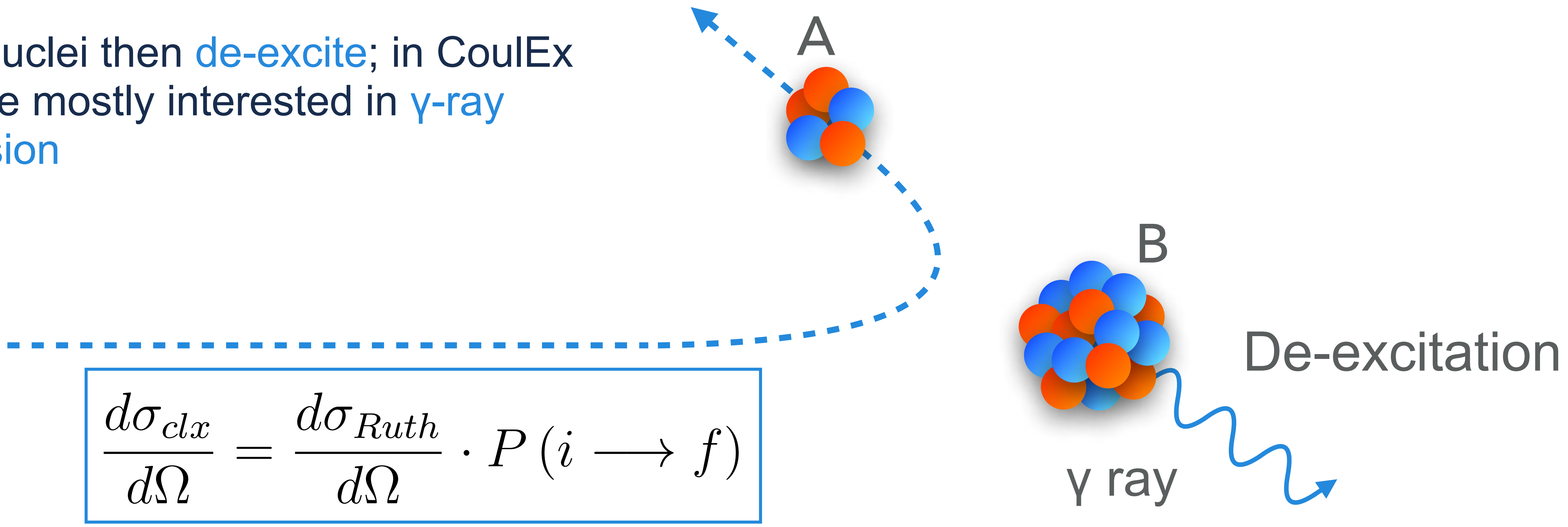

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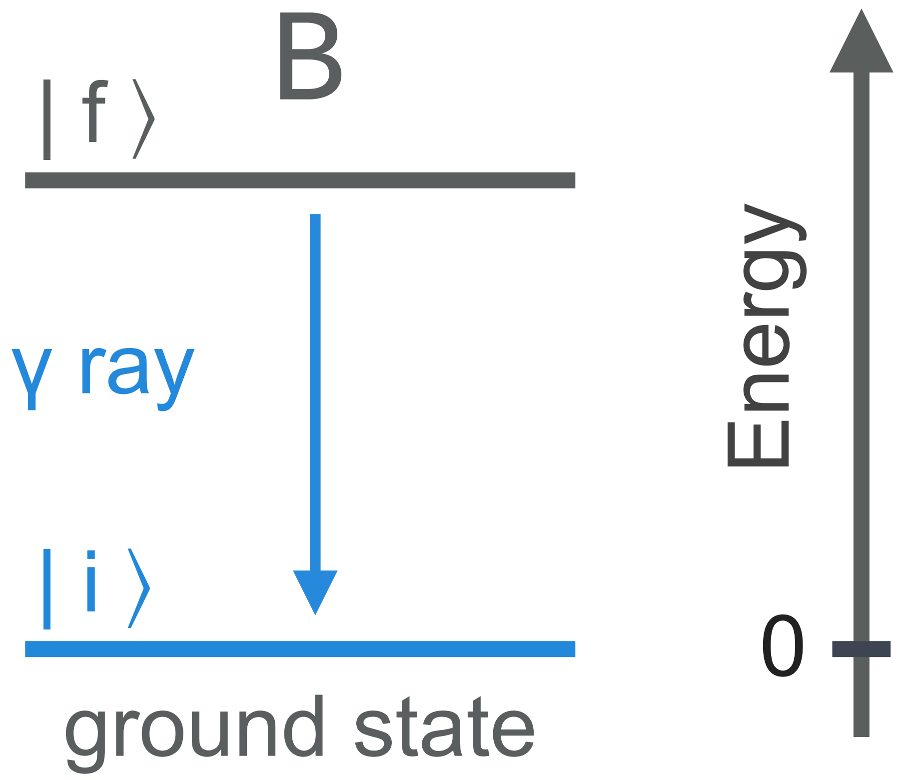

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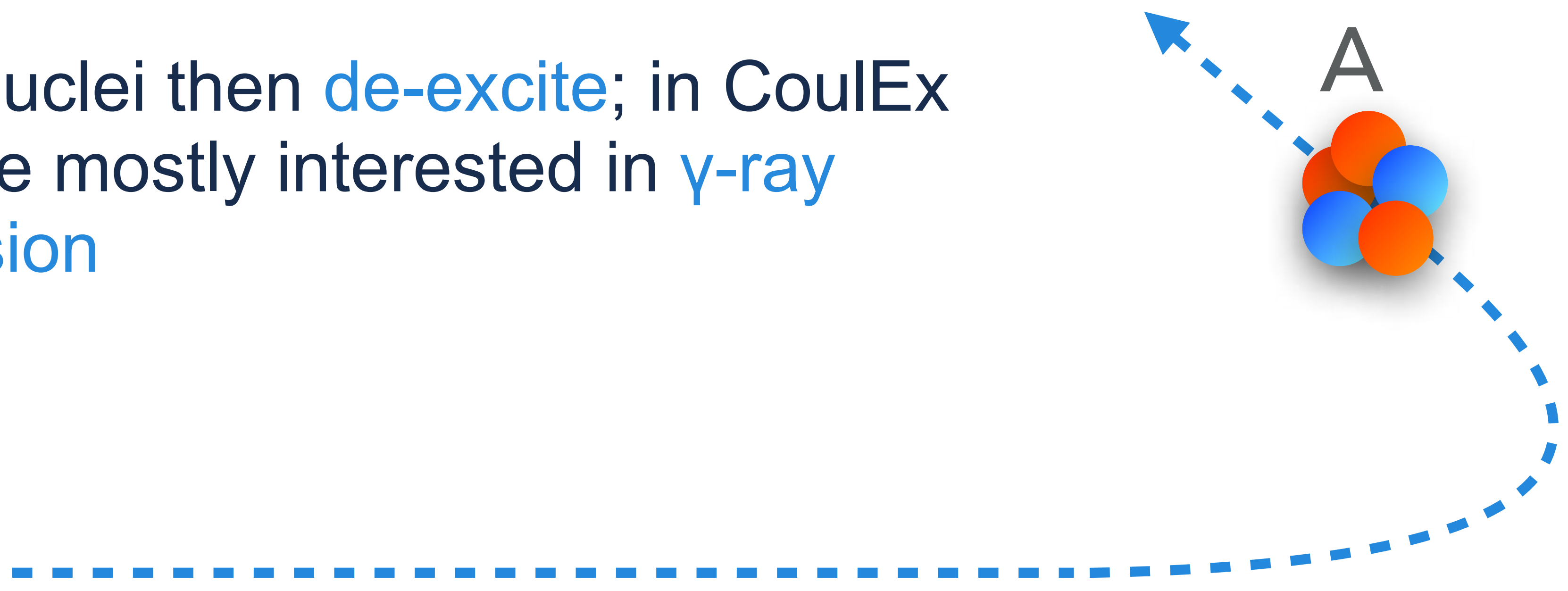
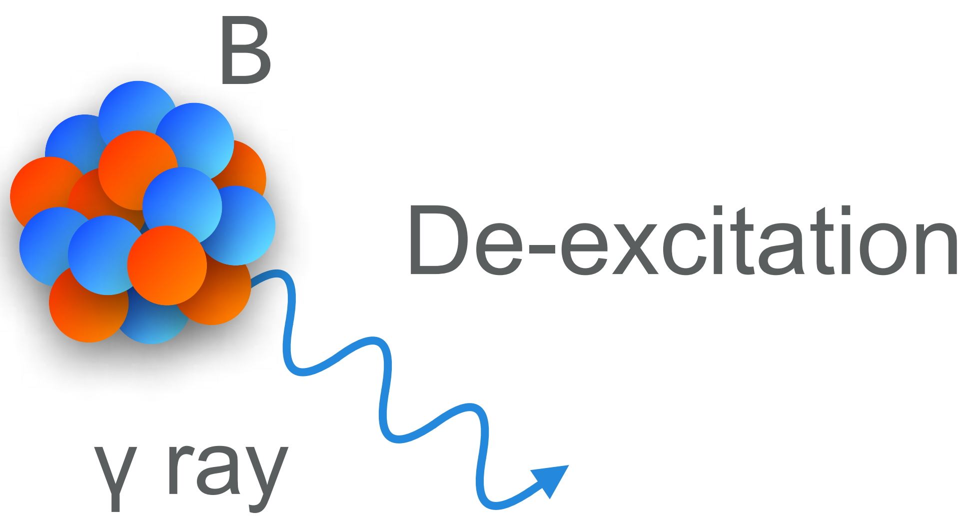
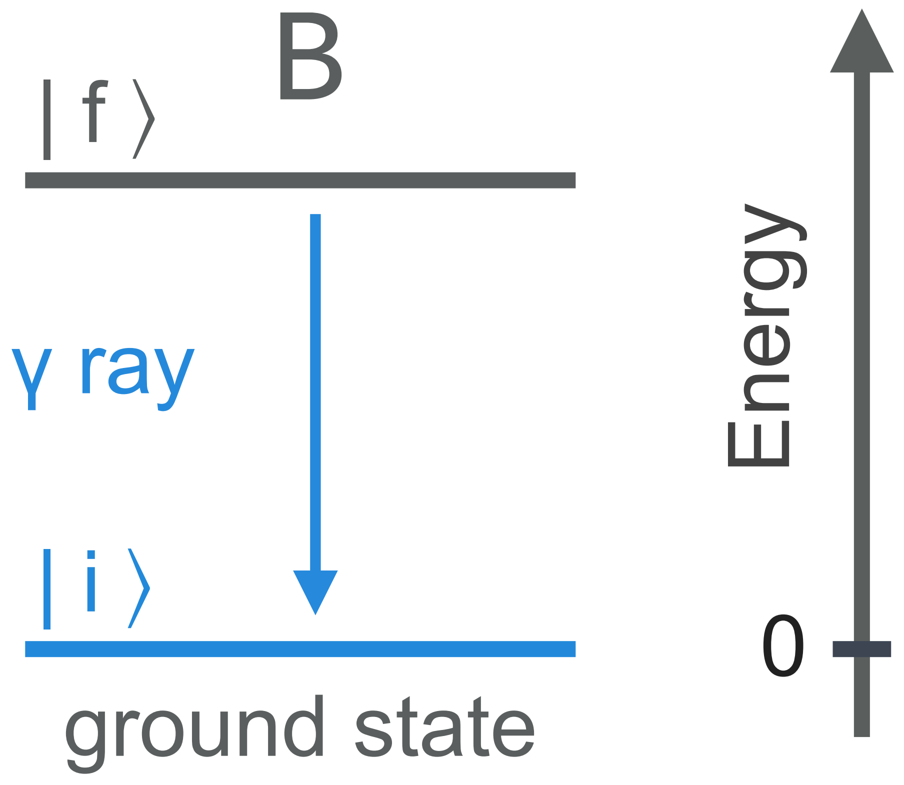


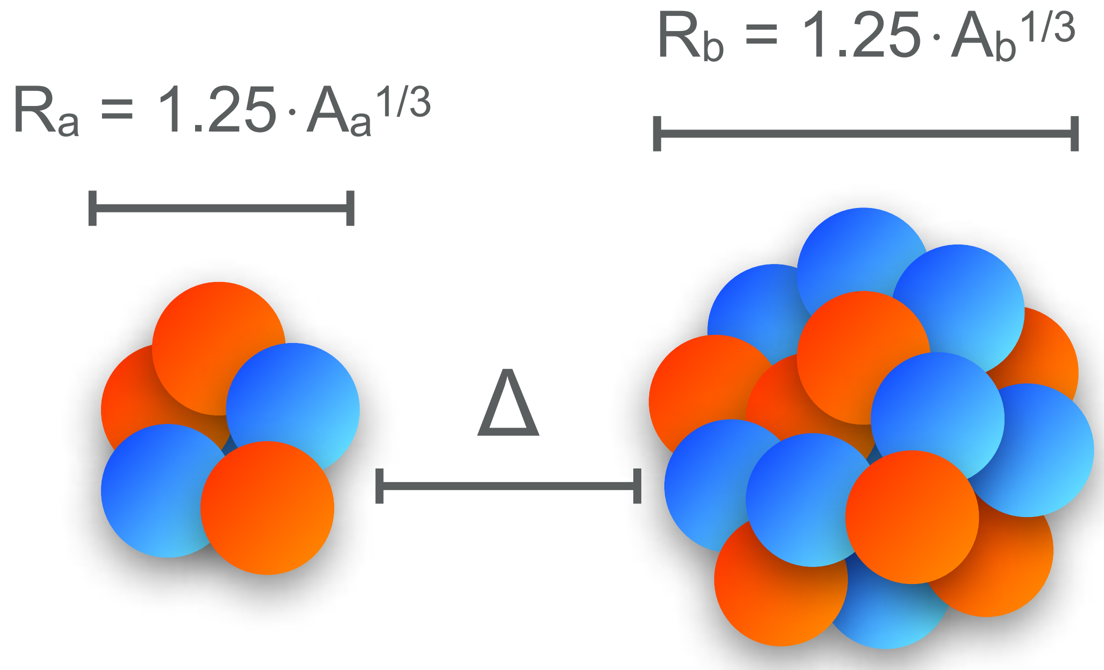




$$\frac{d\sigma_{clx}}{d\Omega} = \frac{d\sigma_{Ruth}}{d\Omega} \cdot P\,(i \longrightarrow f)$$

- Purely electromagnetic interaction if beam energy < safe energy (Cline's criterion):

$d_{min} > 1.25 \cdot (A_a^{1/3} + A_b^{1/3}) + \Delta$ [fm] (Δ = 5 fm) $\Longrightarrow$ $E_{safe} \sim 3$ - 5 MeV/A



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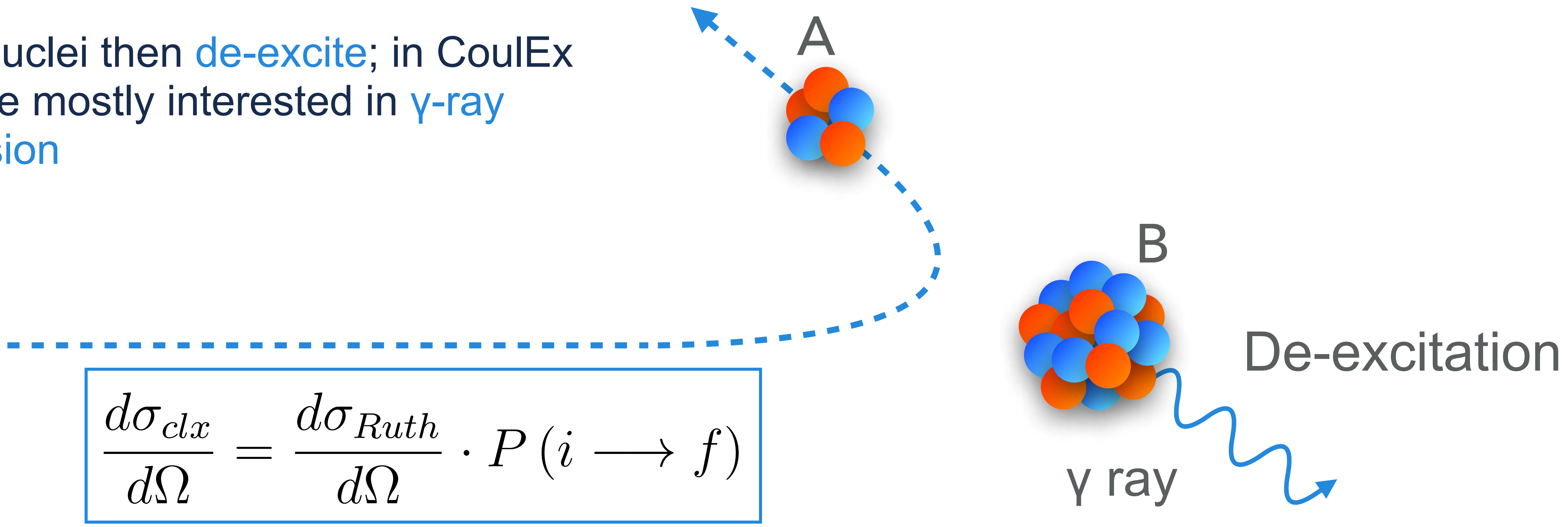

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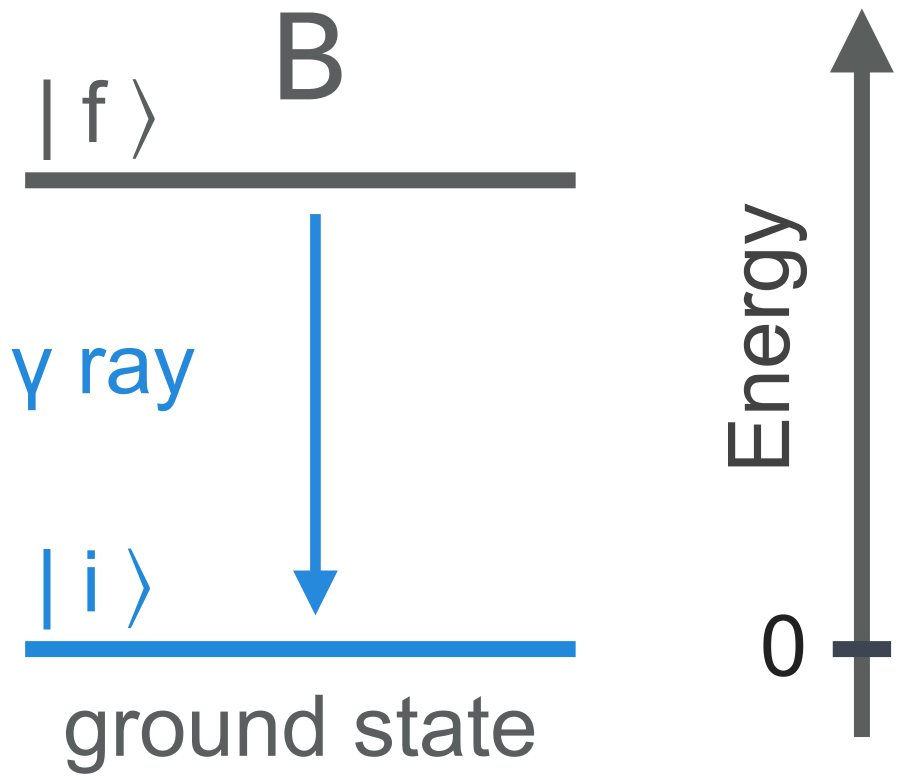


- Example: first 2^+ state in an even-even target nucleus

$\sim|\langle 2^+||E2||0^+\rangle|^2$ (for $B(E2)$), $\sim\langle 2^+||E2||2^+\rangle$ (for $Q_s(2^+)$)

$$P\left(0_1^+ \longrightarrow 2_1^+\right) = F\left(\theta, E_P\right) B(E2) \left[1 + 1.32 \frac{A_P}{Z_T} \frac{\Delta E}{\left(1 + \frac{A_P}{A_T}\right)} Q_s\left(2^+\right) K\left(\theta, E_P\right)\right]$$

Access to: transition probabilities, spectroscopic quadrupole moments

Observables in CoulEx

- Reduced transition probability, electric quadrupole case *B(E2)*:

$$B\left(E2; J \longrightarrow J_0\right) = \frac{1}{2J+1} \left|\langle J_0 \,||E2||\, J\rangle\right|^2$$

- Spectroscopic quadrupole moment Q_s:

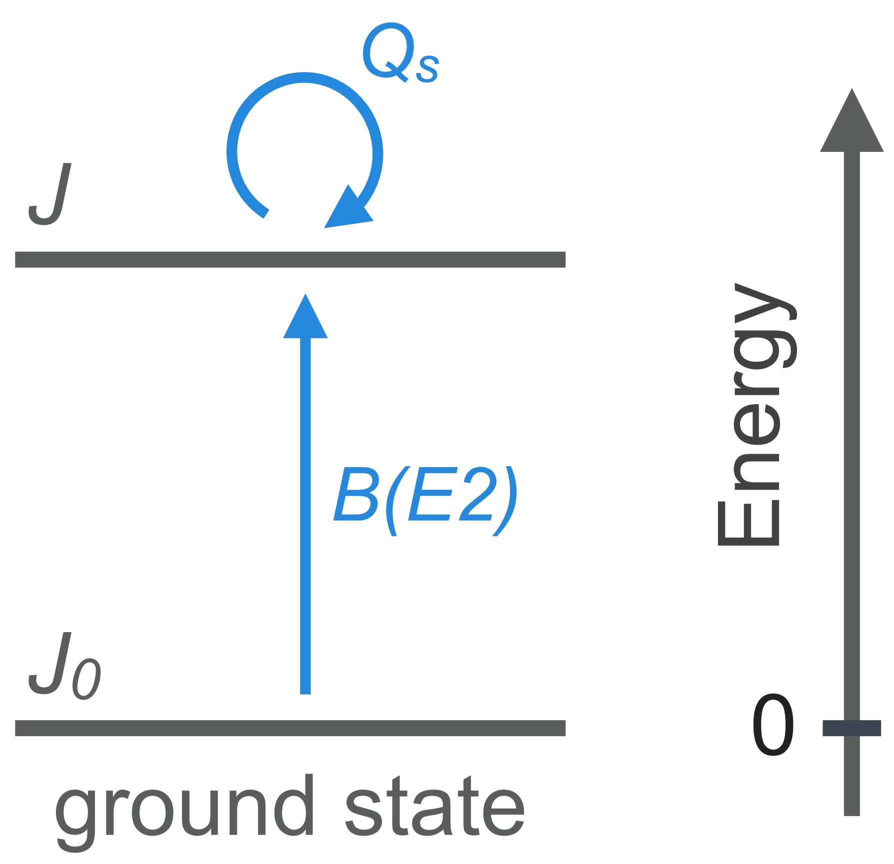
$$Q_s\left(J\right) = \sqrt{\frac{16\pi}{5}} \frac{\langle J J 20 | J J\rangle}{\sqrt{2J+1}} \langle J \,||E2||\, J\rangle$$

→ **Laboratory reference system**

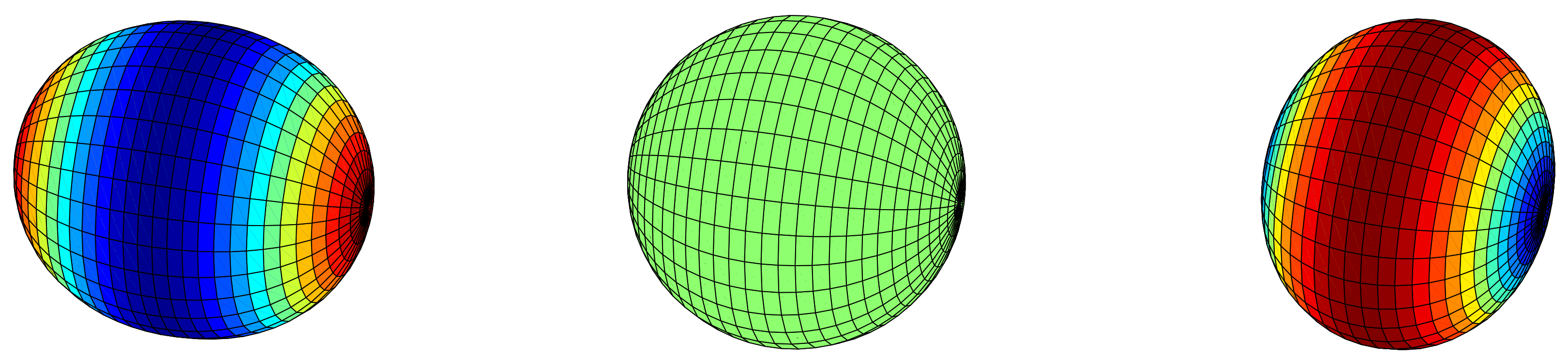
- Intrinsic quadrupole moment Q_0, using the rotational model:

$$Q_s\left(J\right) = \frac{3K^2 - J\left(J+1\right)}{\left(J+1\right)\left(2J+3\right)} Q_0\left(J\right)$$

→ **Intrinsic reference system**



Prolate Q > 0   Spherical Q = 0   Oblate Q < 0



Quadrupole Sum Rules & Rotational Invariants

- Unique feature of CoulEx ⟹ Possible to get relative signs of transitional matrix elements, together with spectroscopic quadrupole moments of short-living excited states with their sign
- Quadrupole Sum Rules ⟹ (β, γ) deformation parameters for g.s. and excited states in a model-independent way

K. Kumar, Phys. Rev. Lett. 28 (1972) 249 & D. Cline, Annu. Rev. Nucl. Part. Sci. 36 (1986) 683

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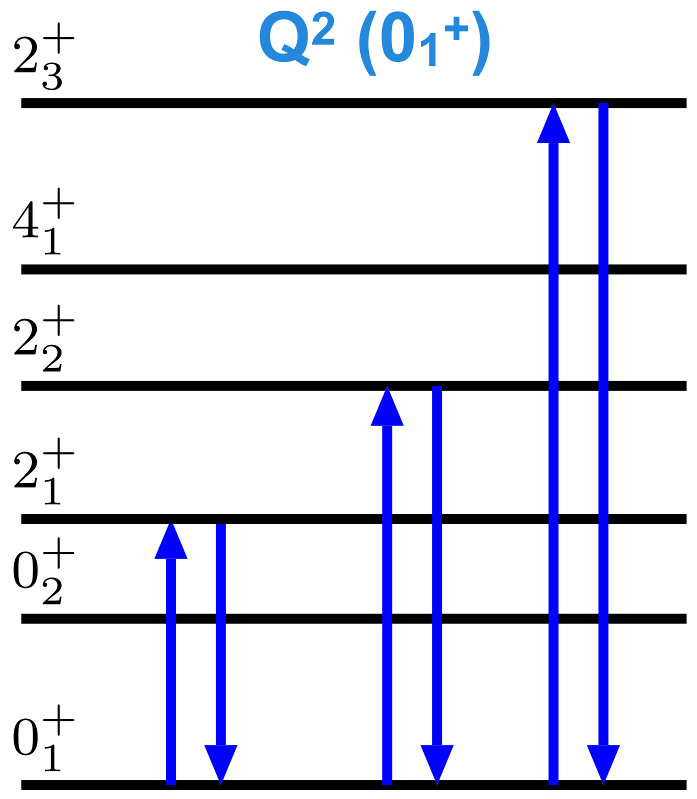
$$\langle i \left| Q^2 \right| i \rangle = \frac{\sqrt{5}}{\sqrt{2I_i+1}} \sum_t \langle i \,||E2||\, t\rangle \langle t \,||E2||\, i\rangle \begin{Bmatrix} 2 & 2 & 0 \\ I_i & I_i & I_t \end{Bmatrix}$$



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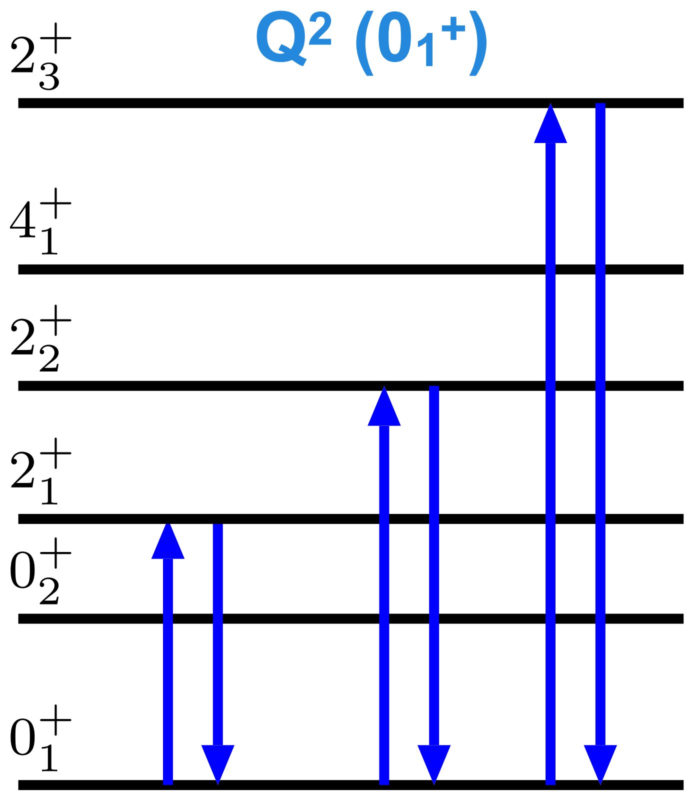
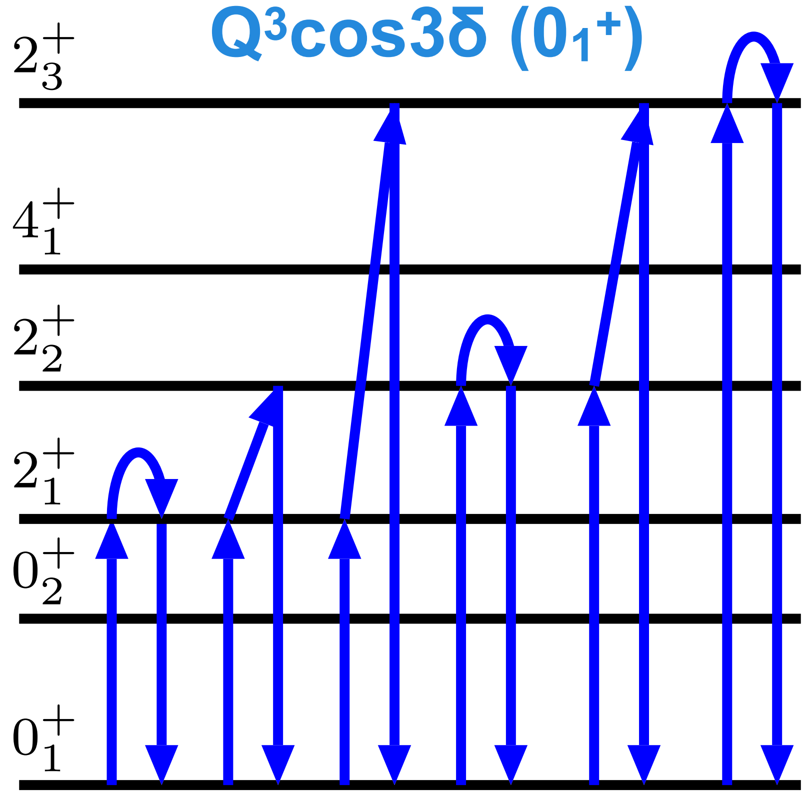
$$\langle i|Q^3\cos(3\delta)|i\rangle = -\frac{\sqrt{35}}{\sqrt{2}}\frac{1}{2I_i+1}\sum_{tu} \langle i||E2||t\rangle\langle t||E2||u\rangle\langle u||E2||i\rangle \begin{Bmatrix} 2 & 2 & 2 \\ I_i & I_t & I_u \end{Bmatrix}$$





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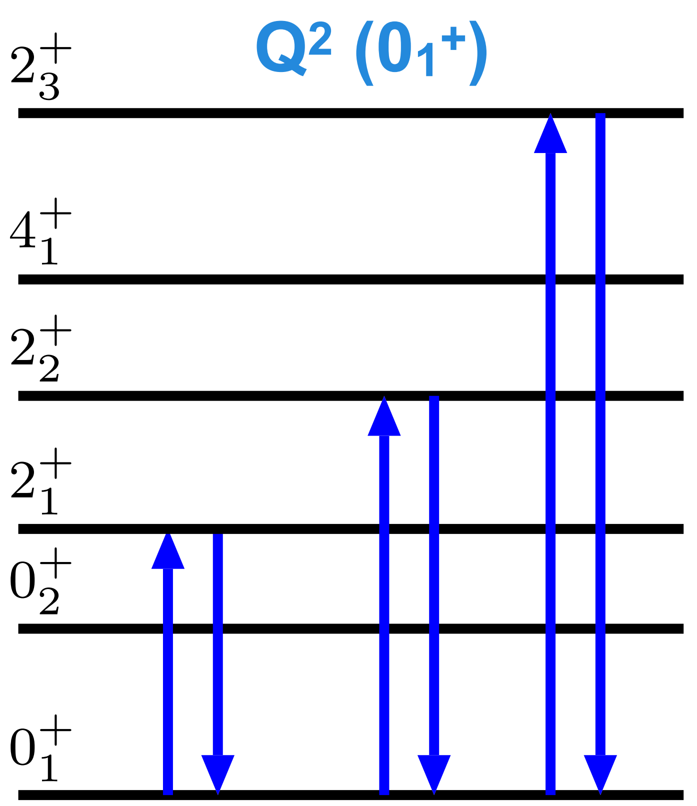
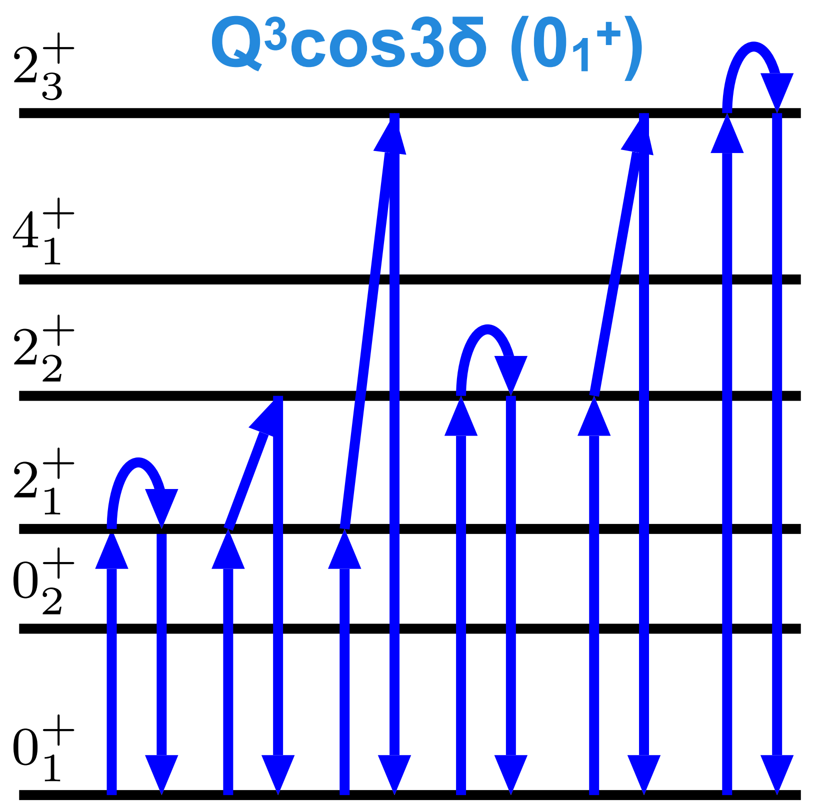
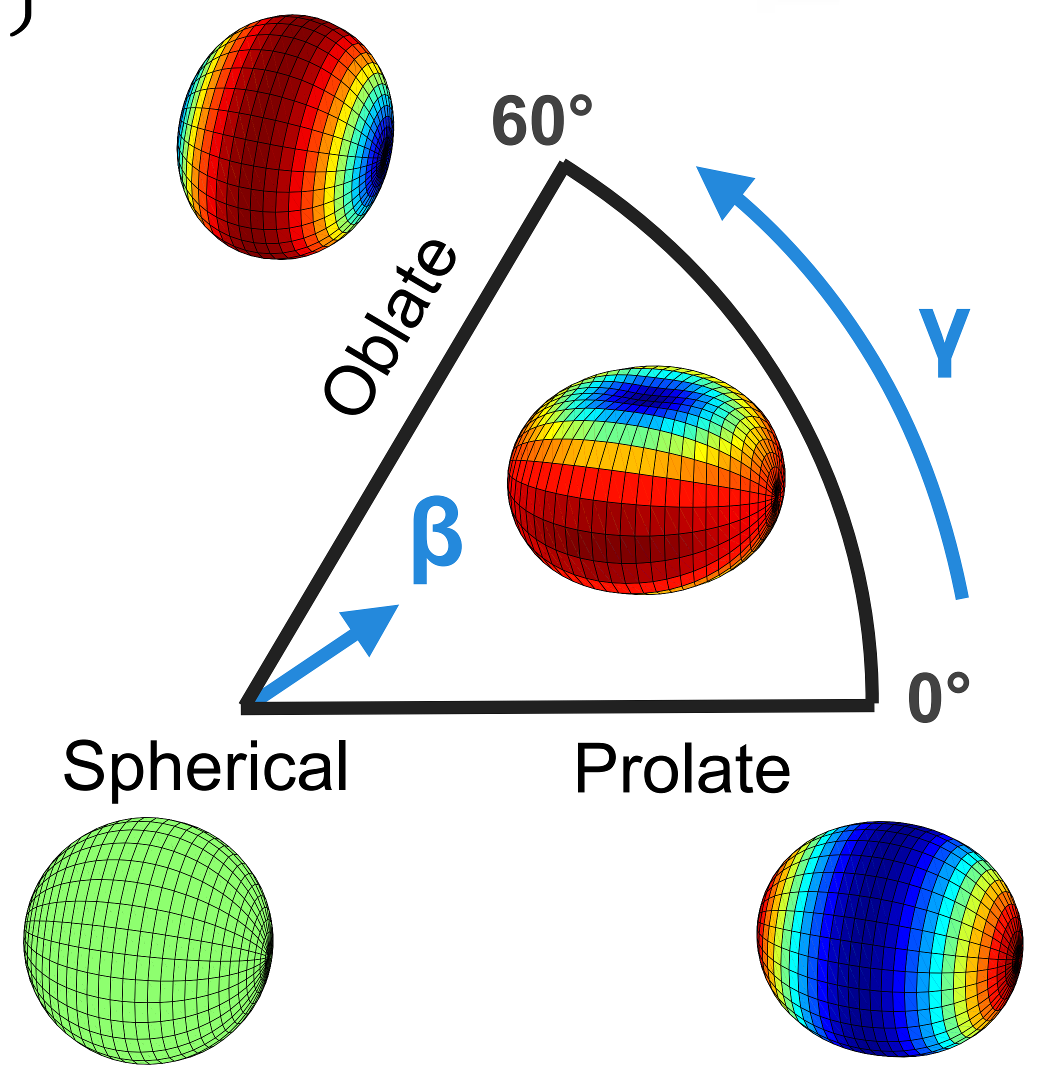
$$\langle i\,|Q^3\cos(3\delta)|\,i\rangle = -\frac{\sqrt{35}}{\sqrt{2}}\frac{1}{2I_i+1}\sum_{tu} \langle i\,||E2||\,t\rangle\,\langle t\,||E2||\,u\rangle\,\langle u\,||E2||\,i\rangle \begin{Bmatrix} 2 & 2 & 2 \\ I_i & I_t & I_u \end{Bmatrix}$$

(Q^2, δ) equivalent to (β, γ)







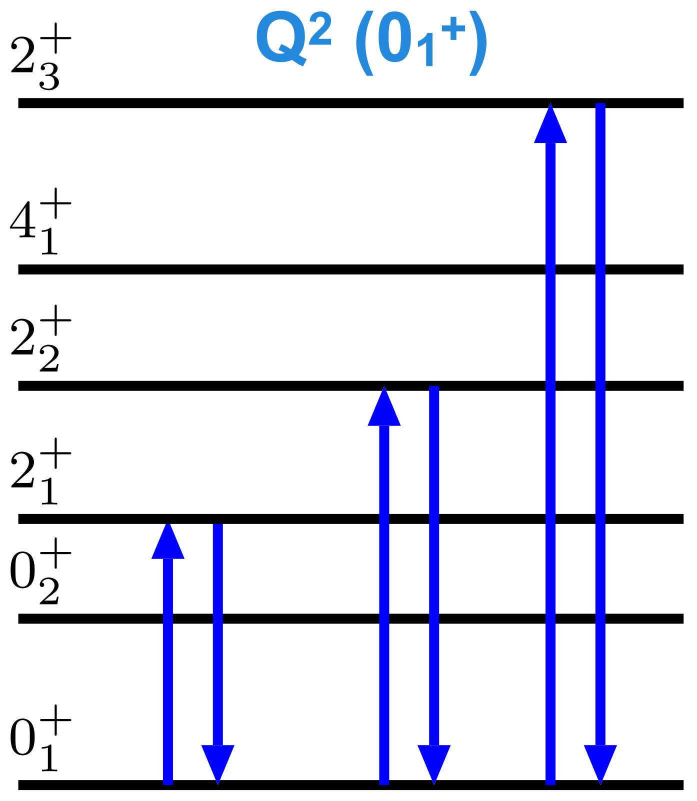
K. Kumar, Phys. Rev. Lett. 28 (1972) 249 & D. Cline, Annu. Rev. Nucl. Part. Sci. 36 (1986) 683

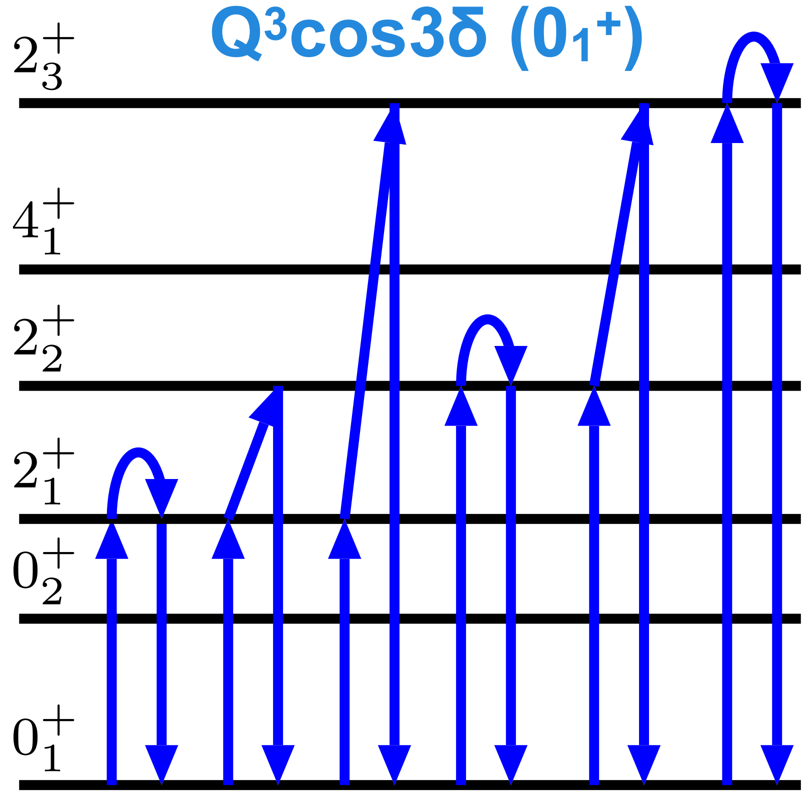
Quadrupole Sum Rules & Rotational Invariants

- Unique feature of CoulEx ⟹ Possible to get relative signs of transitional matrix elements, together with spectroscopic quadrupole moments of short-living excited states with their sign
- Quadrupole Sum Rules ⟹ (β, γ) deformation parameters for g.s. and excited states in a model-independent way

$$\langle i\,|Q^2|\,i\rangle = \frac{\sqrt{5}}{\sqrt{2I_i+1}} \sum_t \langle i\,||E2||\,t\rangle \langle t\,||E2||\,i\rangle \begin{Bmatrix} 2 & 2 & 0 \\ I_i & I_i & I_t \end{Bmatrix}$$

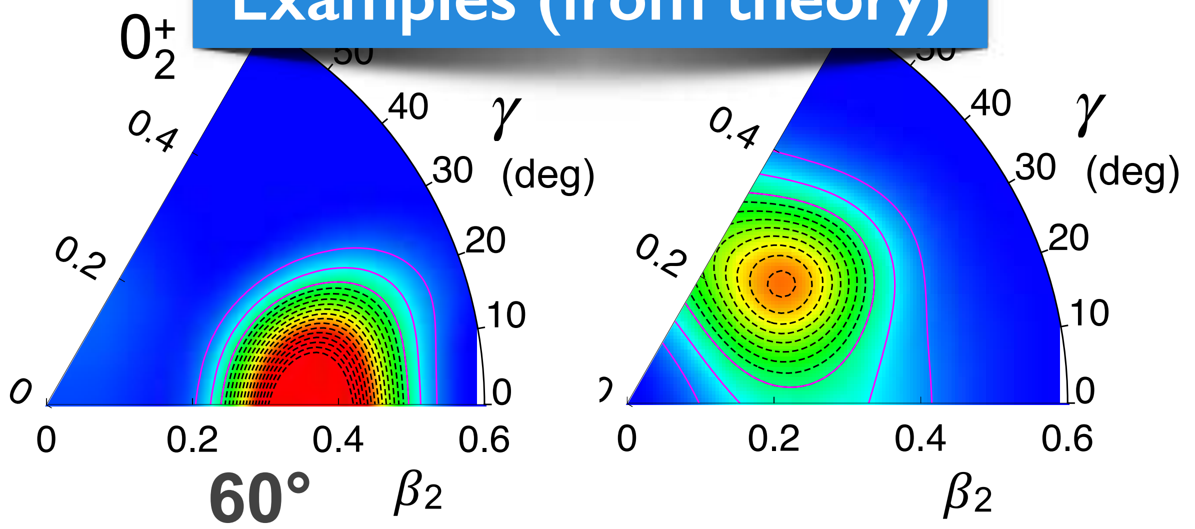
$$\langle i\,|Q^3\cos(3\delta)|\,i\rangle = -\frac{\sqrt{35}}{\sqrt{2}}\frac{1}{2I_i+1} \sum_{tu} \langle i\,||E2||\,t\rangle \langle t\,||E2||\,u\rangle \langle u\,||E2||\,i\rangle \begin{Bmatrix} 2 & 2 & 2 \\ I_i & I_t & I_u \end{Bmatrix}$$



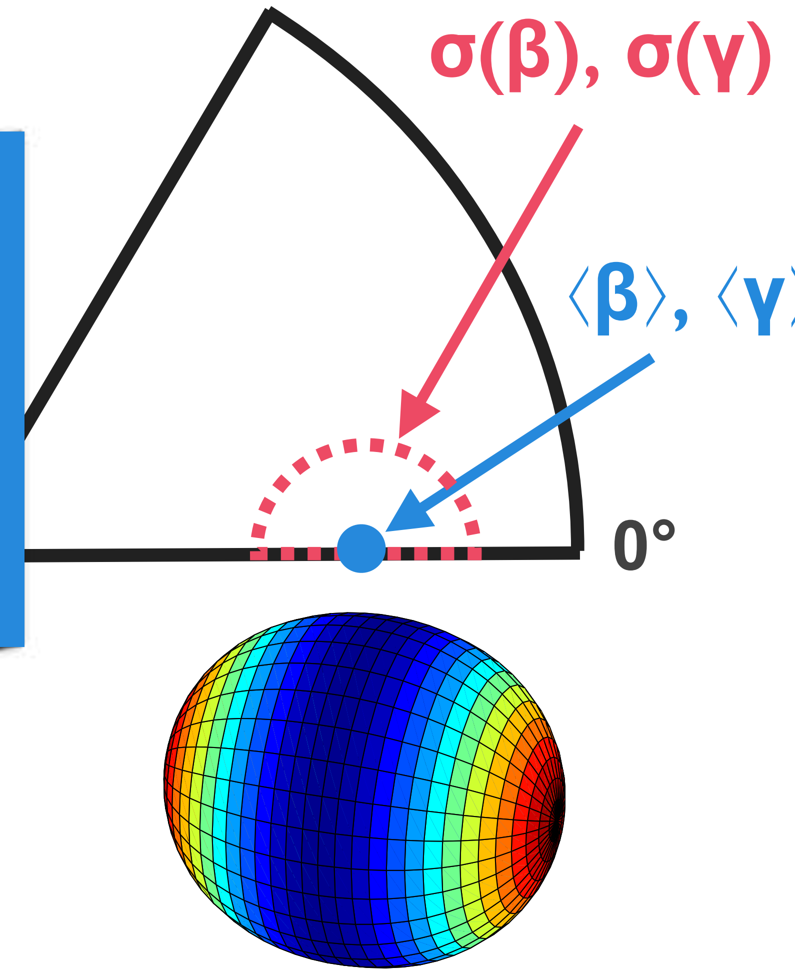


K. Kumar, Phys. Rev. Lett. 28 (1972) 249 & D. Cline, Annu. Rev. Nucl. Part. Sci. 36 (1986) 683

Examples (from theory)



Also possible to determine dispertions on beta and gamma (higher-order invariants)

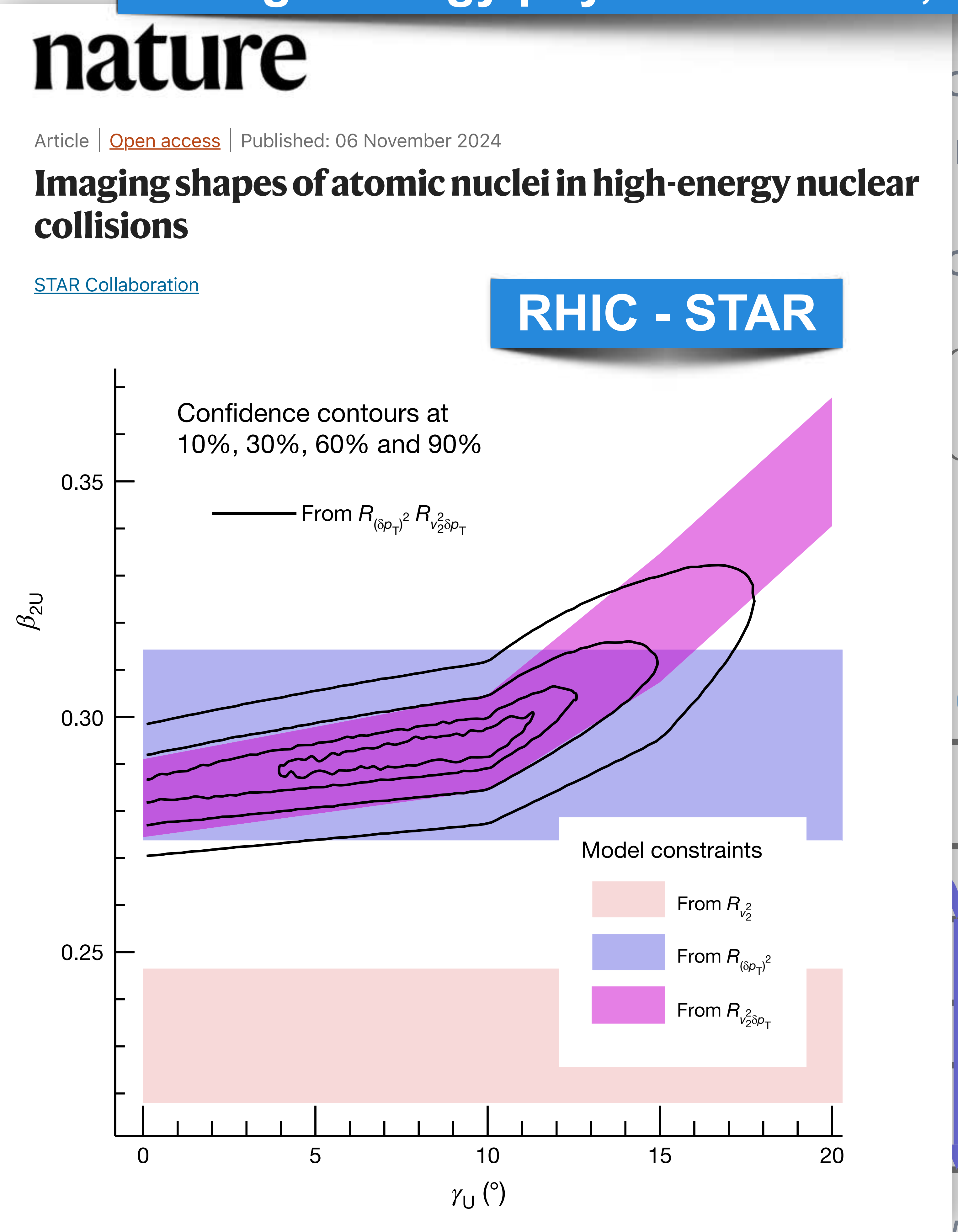


Quad… Invariants

Hot topic not only in low-energy nuclear physics ⇒ New studies at high-energy physics facilities, highly complementary to CoulEx

- Unique featu… of …ctroscopic quadrupole r…
- Quadrupole … or …nt way

$\langle i | Q^3$ c…



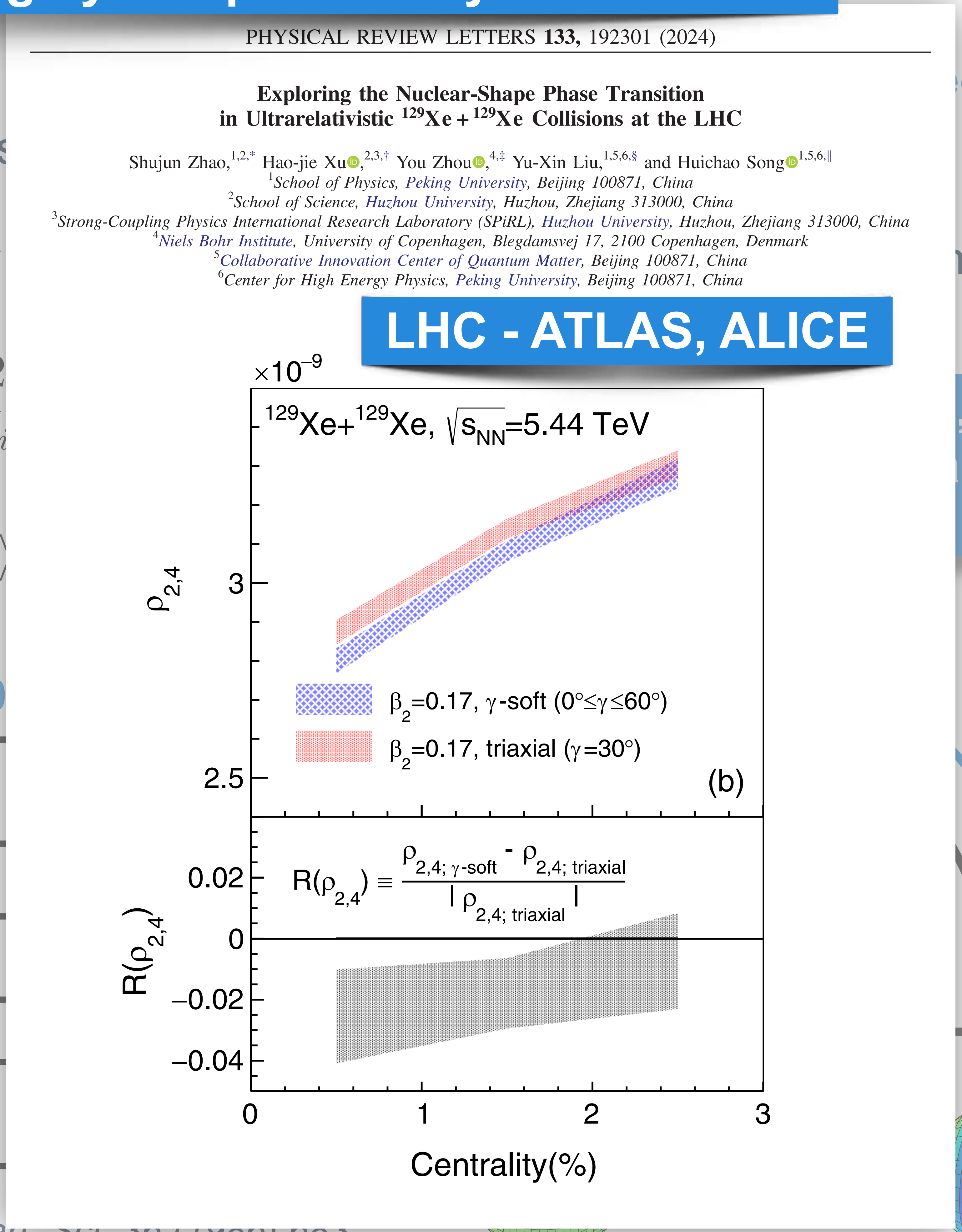
RHIC - STAR

nature

Article | Open access | Published: 06 November 2024

Imaging shapes of atomic nuclei in high-energy nuclear collisions

STAR Collaboration



LHC - ATLAS, ALICE

PHYSICAL REVIEW LETTERS 133, 192301 (2024)

Exploring the Nuclear-Shape Phase Transition in Ultrarelativistic ^{129}Xe + ^{129}Xe Collisions at the LHC

Shujun Zhao,[1,2,*] Hao-jie Xu,[2,3,†] You Zhou,[4,‡] Yu-Xin Liu,[1,5,6,§] and Huichao Song[1,5,6,‖]
[1]School of Physics, Peking University, Beijing 100871, China
[2]School of Science, Huzhou University, Huzhou, Zhejiang 313000, China
[3]Strong-Coupling Physics International Research Laboratory (SPiRL), Huzhou University, Huzhou, Zhejiang 313000, China
[4]Niels Bohr Institute, University of Copenhagen, Blegdamsvej 17, 2100 Copenhagen, Denmark
[5]Collaborative Innovation Center of Quantum Matter, Beijing 100871, China
[6]Center for High Energy Physics, Peking University, Beijing 100871, China

…, δ) …lent to … γ)

γ

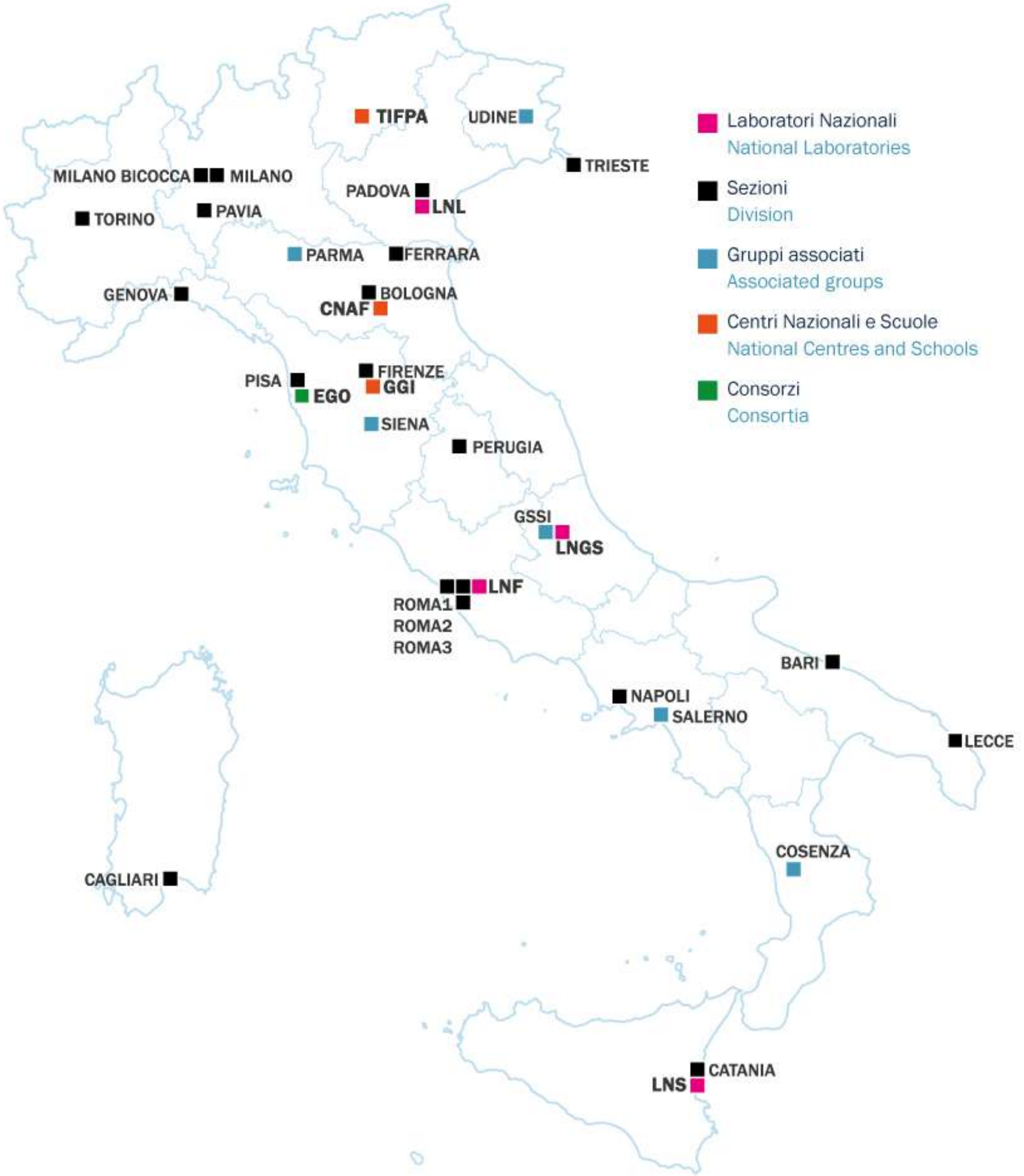
0°

K. Kumar, Phys. Rev. Lett. 28 (1972) 249 & D. Cline, Annu. Rev. Nucl. Part. Sci. 36 (1986) 683

INFN

Istituto Nazionale di Fisica Nucleare

- 20 Divisions
- 4 National Laboratories
- 6 Associated Groups
- 3 National Centres and Schools
- 1 Consortium (EGO, European Gravitational Observatory)
- 5 Lines of Research:
  - CSN1 — Particle Physics
  - CSN2 — Astroparticle Physics
  - CSN3 — Nuclear Physics
  - CSN4 — Theoretical Physics
  - CSN5 — Technological Physics



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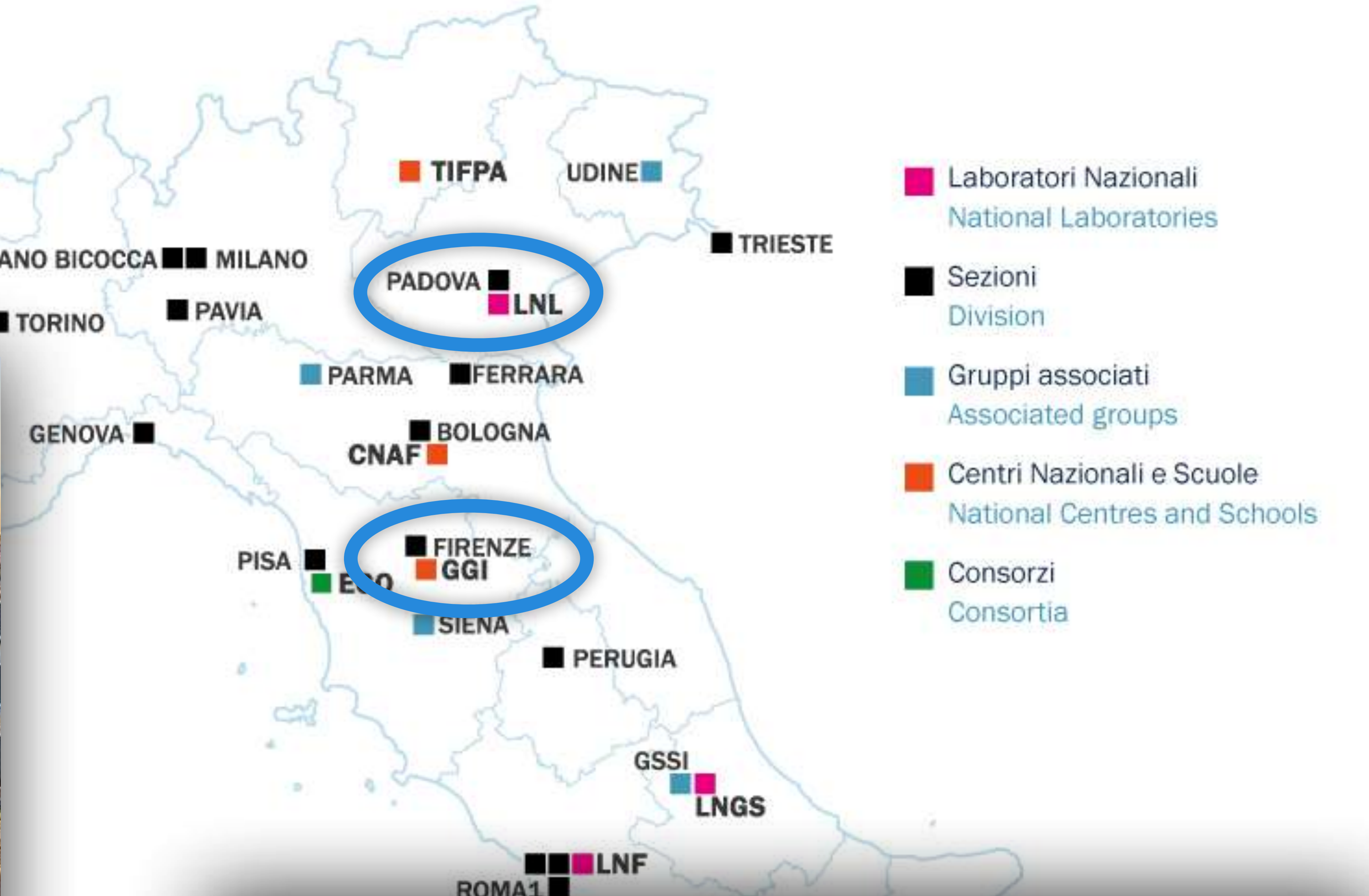

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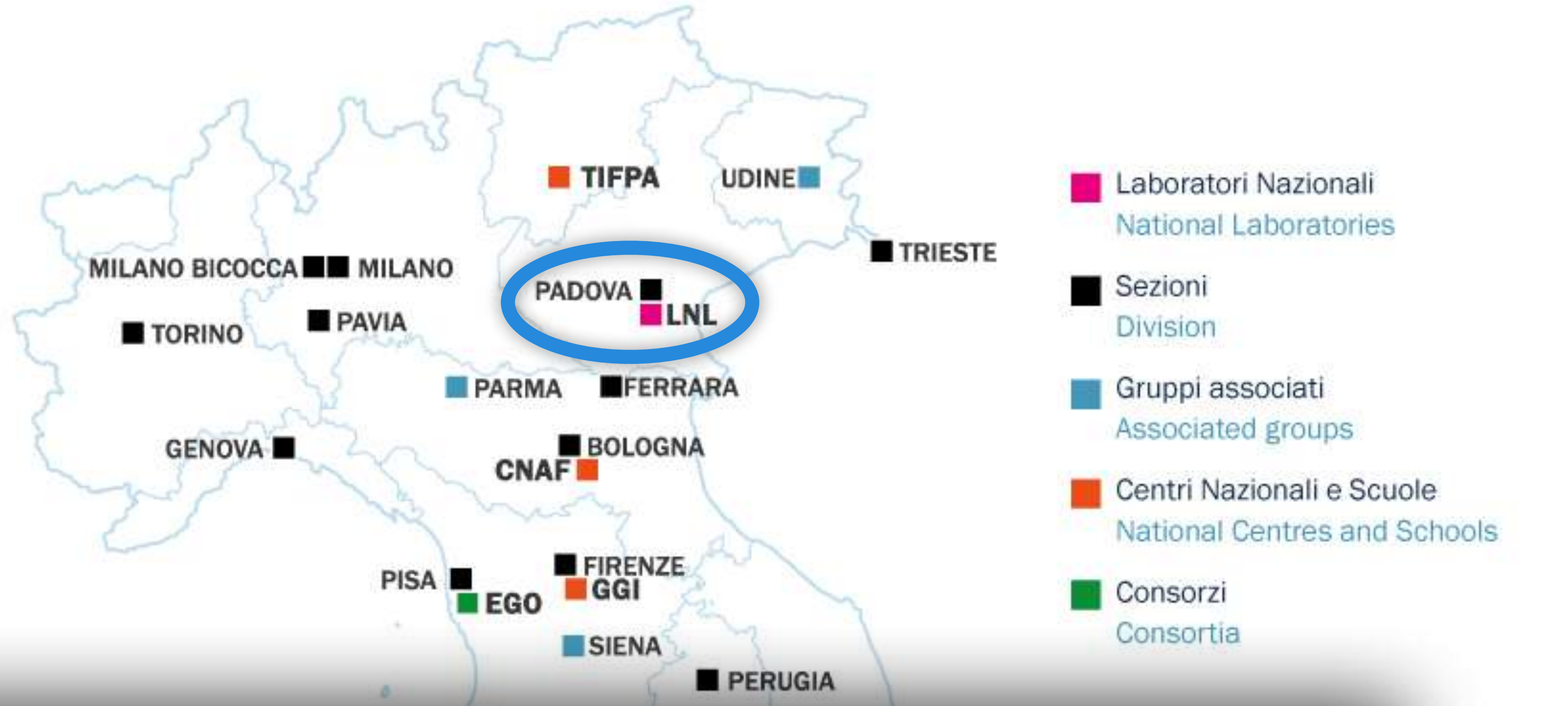
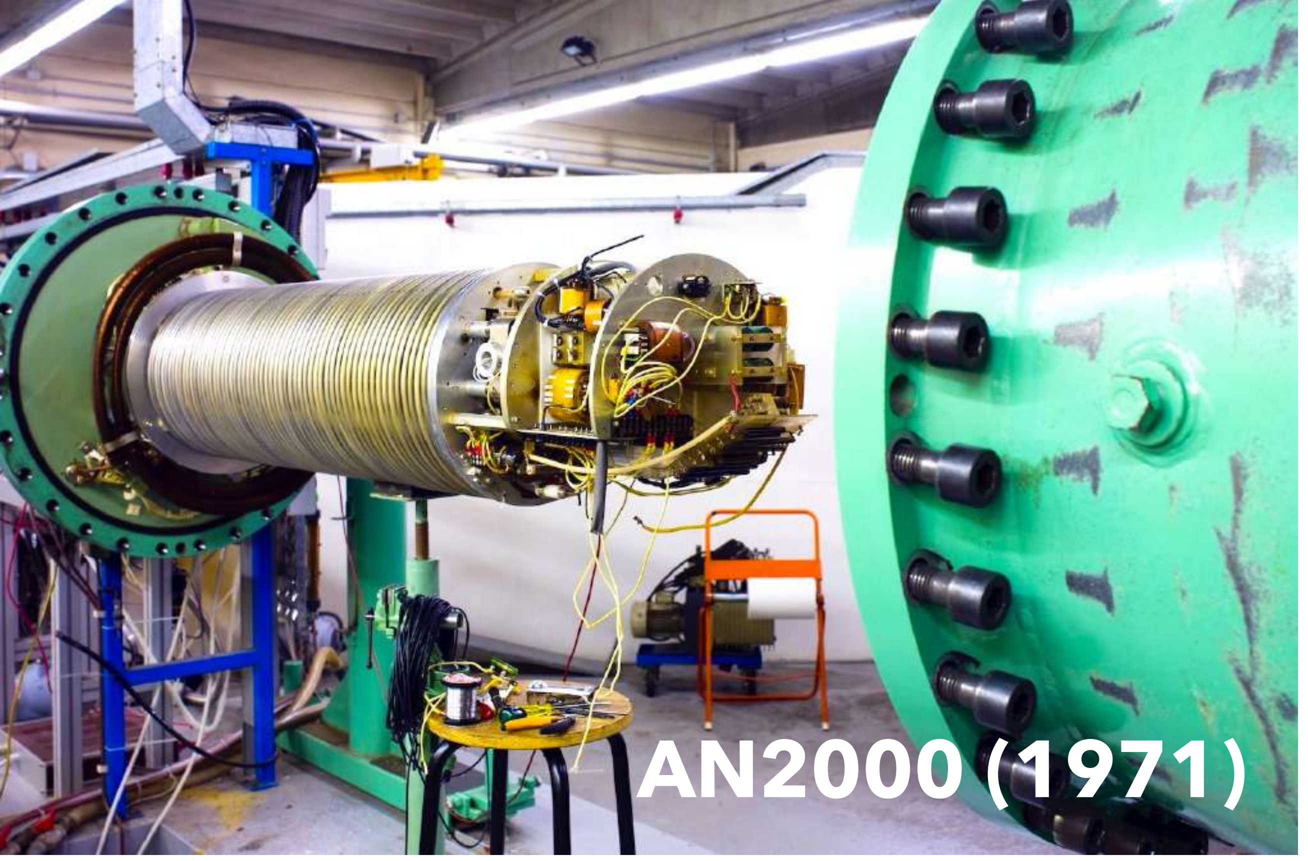


LNL Accelerators

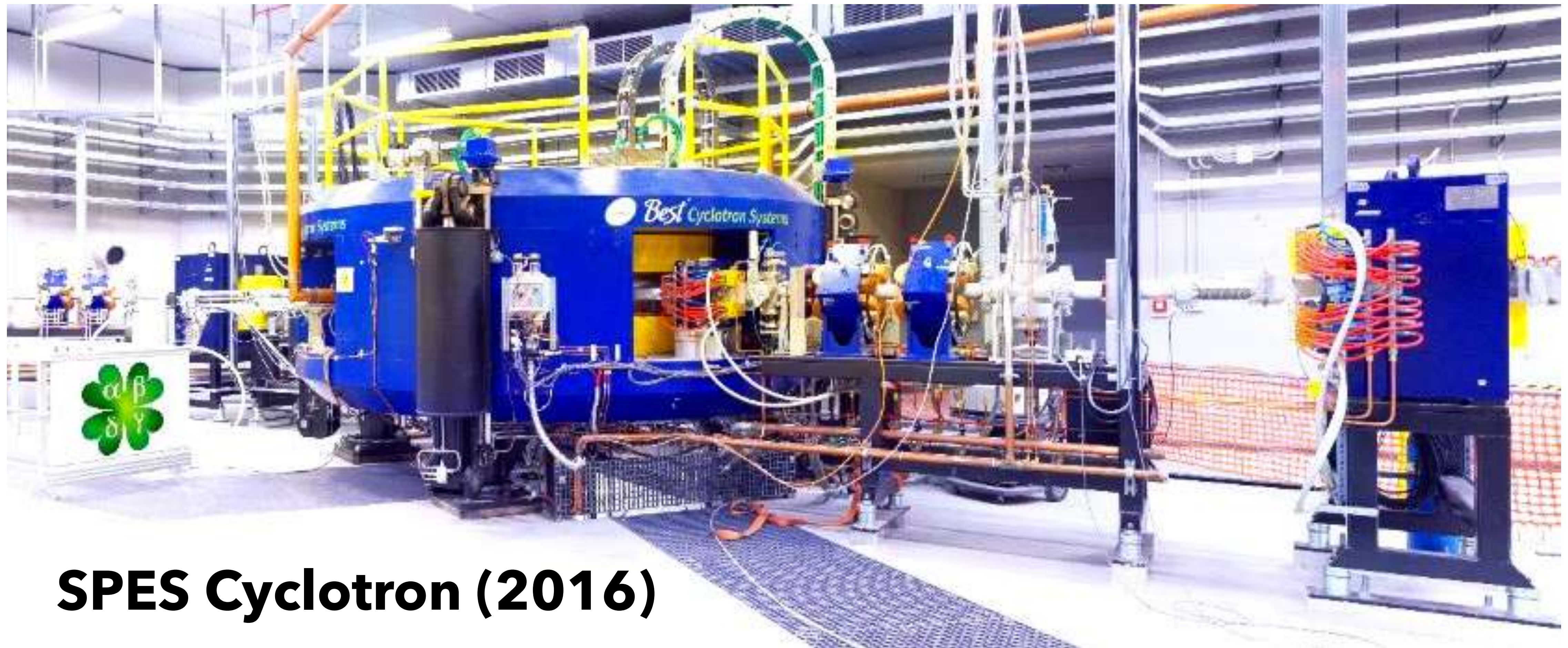

AN2000 (1971)


CN (1961)

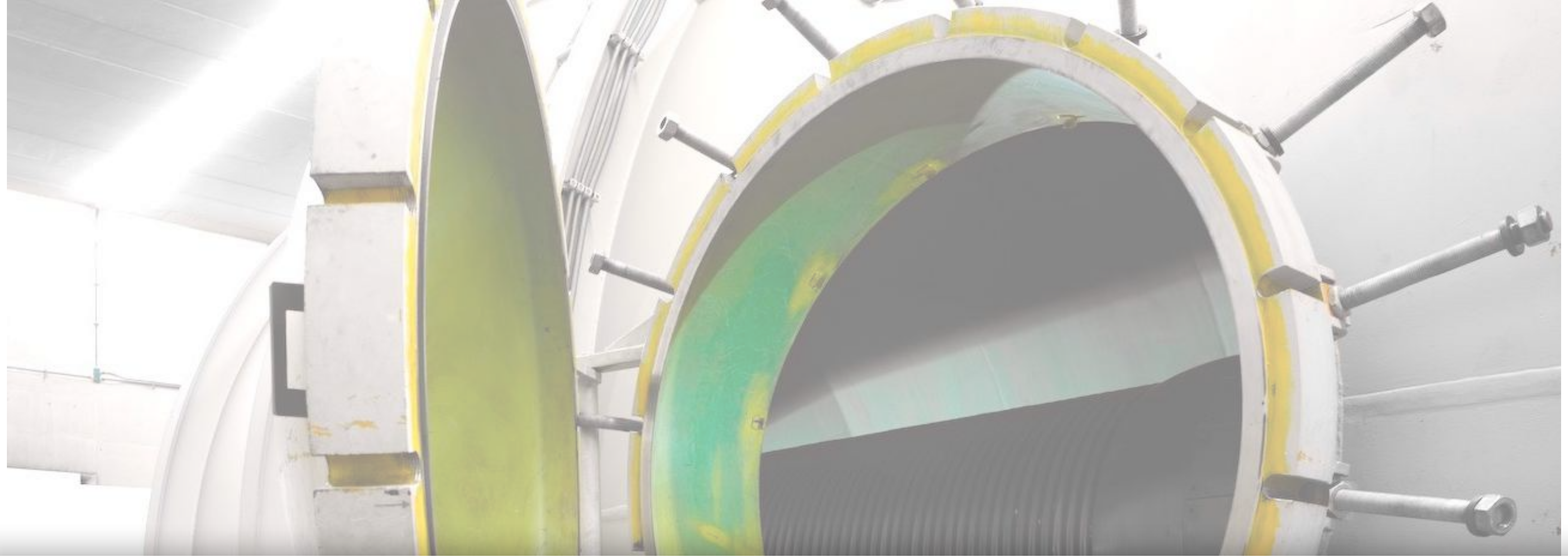

TANDEM (1981)


ALPI (1995)

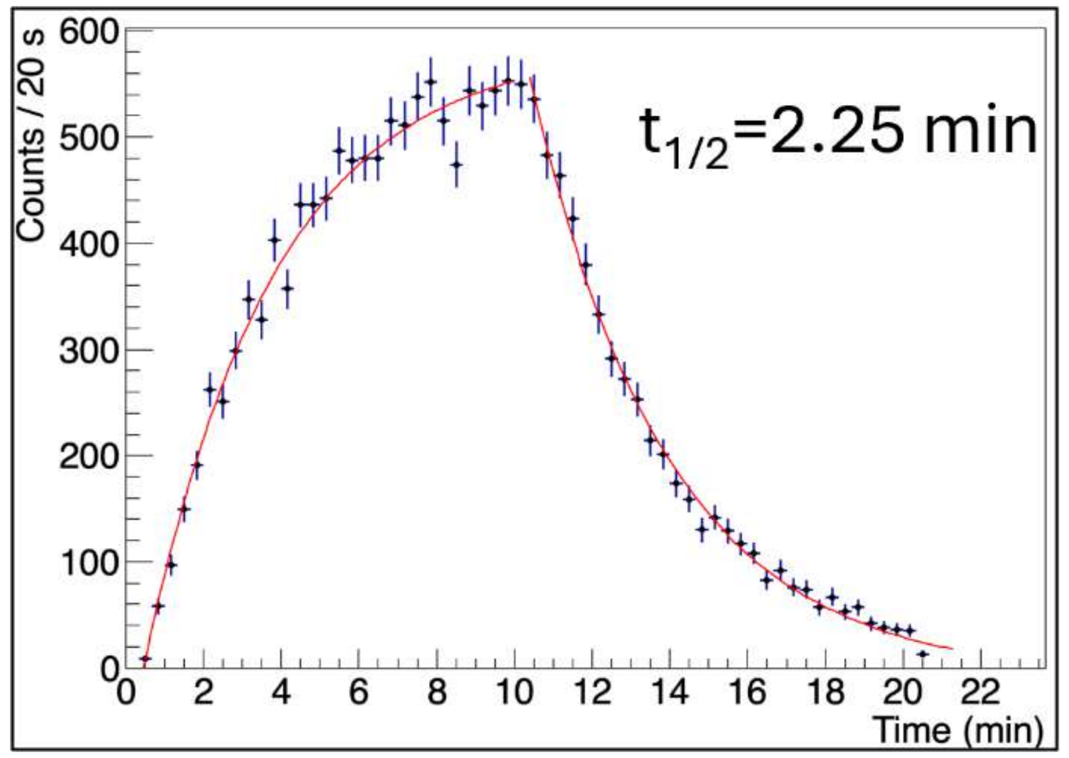

PIAVE (2005)



SPES Cyclotron (2016)



E. Fagotti, T. Marchi et al., 16th International Conference on Heavy Ion Accelerator Technology, East Lansing, MI, USA, 22-27 Jun 2025



First SPES RIB (2025)

- ^{28}Al RIB from SiC target, protons at 100 nA - 40 MeV
- HPGe with beta tag: ^{28}Si $2^+ \rightarrow 0^+$ observed

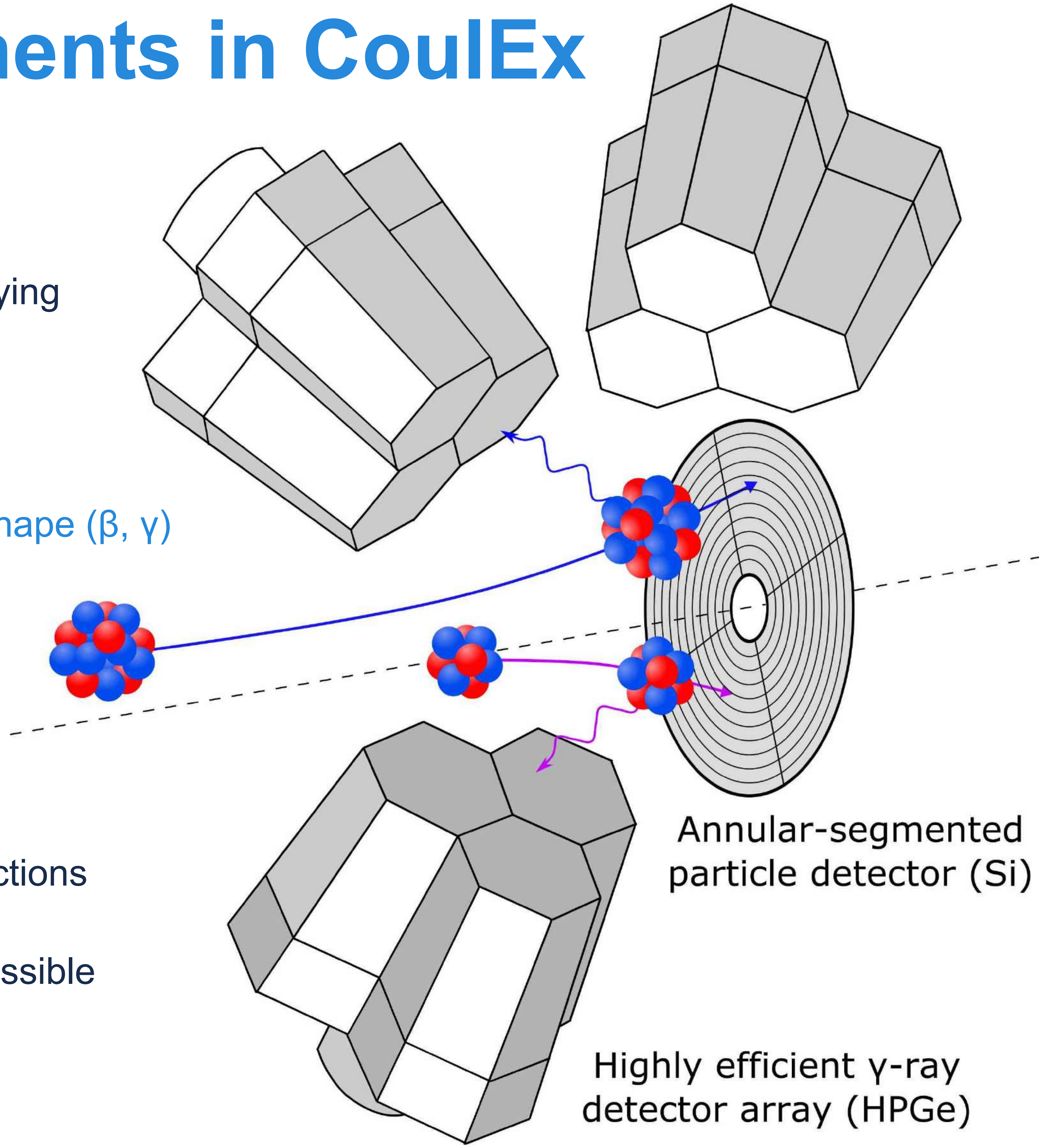
Experimental Requirements in CoulEx

What can we get?

- Reduced transition probabilities for transitions between low-lying states, mainly *E2* and *E3* multipolarities
- Spectroscopic quadrupole moments for excited states
- Rotational invariants (Q, δ) ⟹ Direct access to the nuclear shape (β, γ)

What do we need?

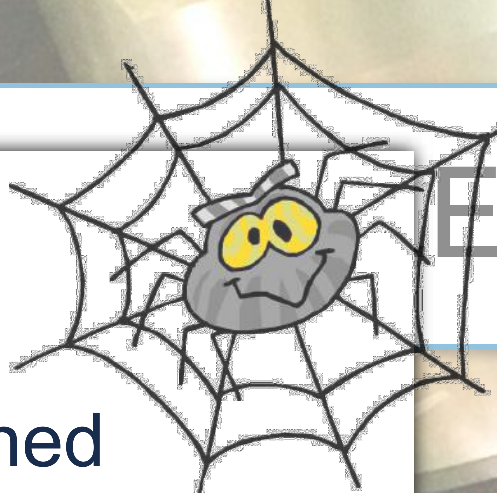
- Beam energies around 3 - 5 MeV/A
- Good technique for weak-intensity beams ⟹ Large cross sections
- Gamma-ray array and heavy ion detector with as good as possible efficiency, energy or time resolutions, and segmentations



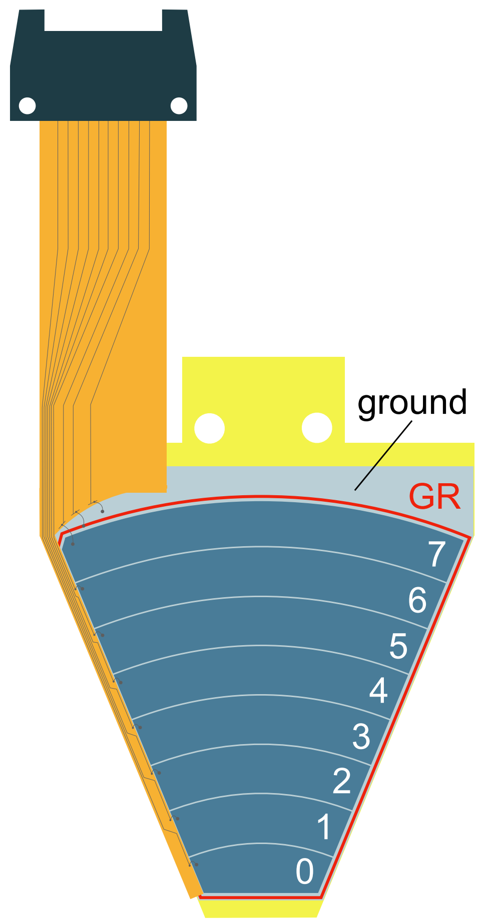
GALILEO with SPIDER (2016 - 2019)

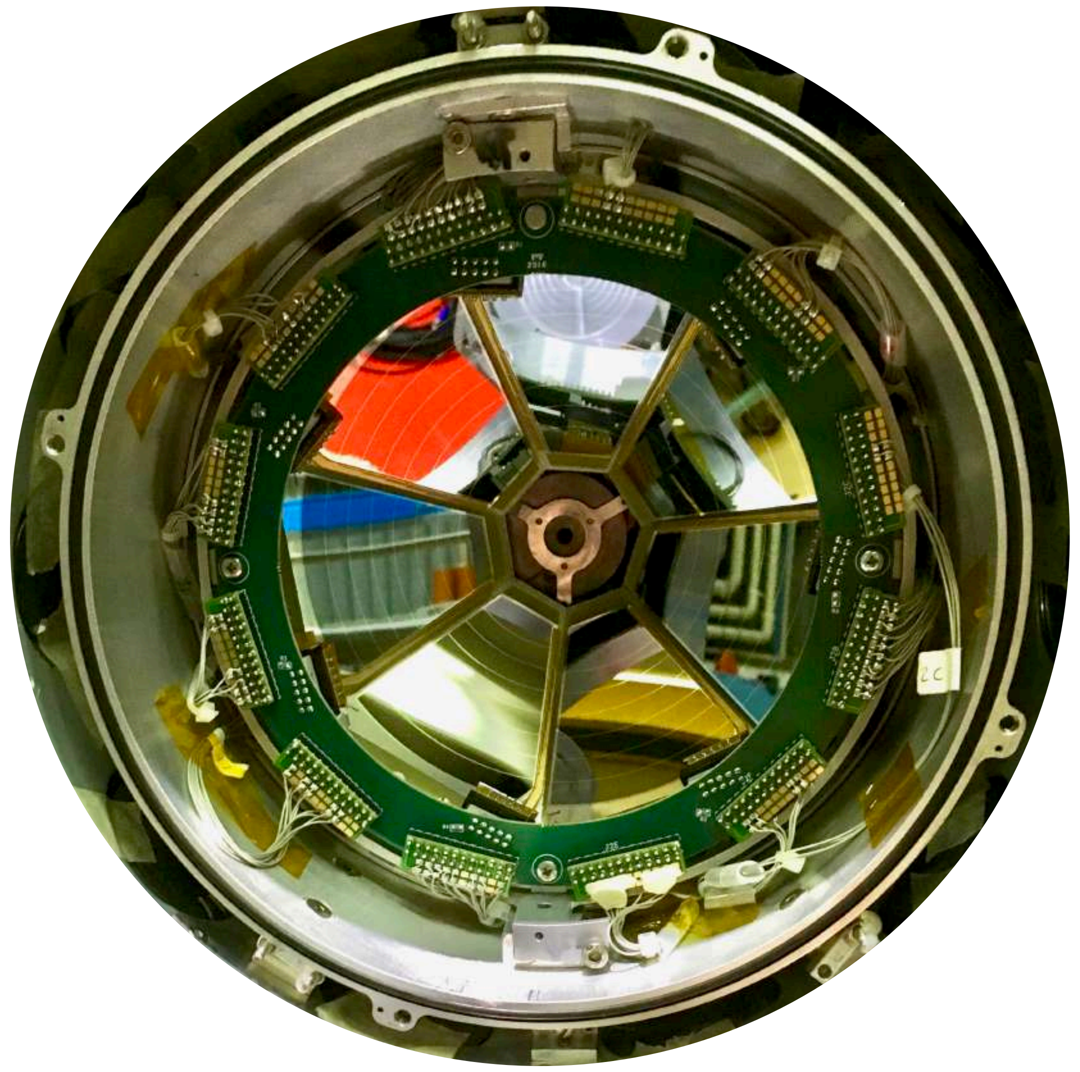
GALILEO with SPIDER (2016 - 2019)

SPIDER: the Silicon PIe DEtectoR

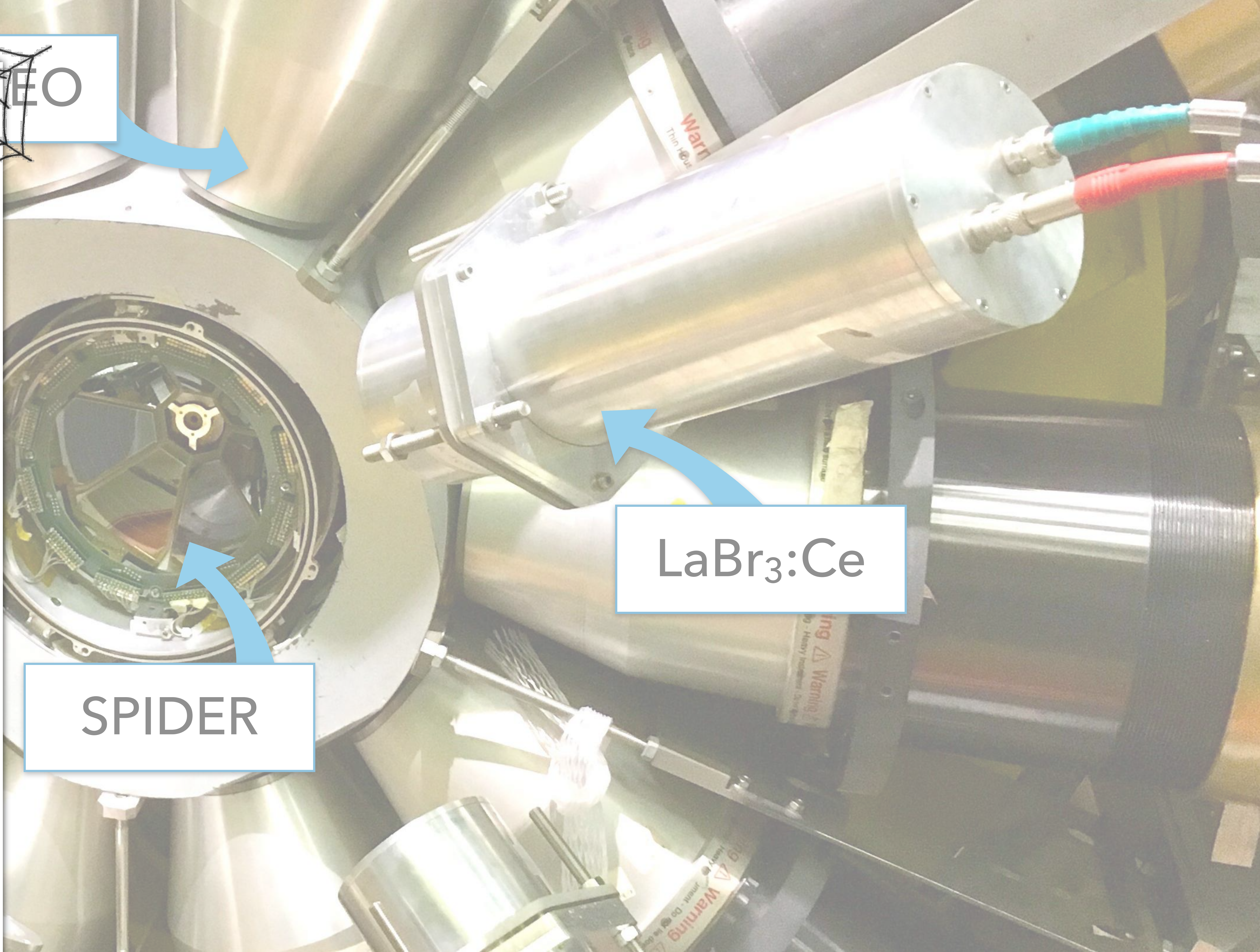


- Modular segmented silicon detector, designed for low-energy Coulomb-excitation measurements
- Independent sectors, 8 strips + guard ring
- Detector thickness ~ 300 µm, dead layers ~ 50 nm in the junction (front) side and ~ 350 nm in the ohmic (rear) side
- Cone configuration (7 sectors) at backward angles: 8.5 cm from the target ⟹ $\Delta\Theta = 37.4°$, $\Omega/4\pi = 17.3\%$





M. Rocchini, K. Hadyńska-Klęk, A. Nannini et al., Nuclear Inst. and Methods in Physics Research A 971 (2020) 164030



SPIDER (2016 - 2019)

GALILEO

GALILEO: the LNL Resident γ-Ray Spectrometer

- 25 HPGe Compton-suppressed detectors (GASP type)
- FWHM (@1332.5 keV) < 2.4 keV
- Efficiency (@1332.5 keV) = 2.1%
- Full digital electronics (takes advantage of the developments made for AGATA)
- Triggerless DAQ

A. Goasduff, D. Mengoni, F. Recchia, J.J. Valiente-Dobón et al., Nuclear Inst. and Methods in Physics Research A 1015 (2021) 165753

GALILE

AGATA with SPIDER (2022 - ongoing)

AGATA: the Advanced GAmma Tracking Array

- Latest generation γ-ray spectrometer composed of higly-segmented HPGe detectors
- Employs advanced PSA and γ-ray tracking methods to avoid Compton-suppressors and garantee high efficiency

J.J. Valiente-Dobón, R. Menegazzo, A. Goasduff et al., Nuclear Inst. and Methods in Physics Research A 1049 (2023) 168040

AGATA with SPID

AGATA: the Advanced GAmma Tracking Array

- Latest generation γ-ray
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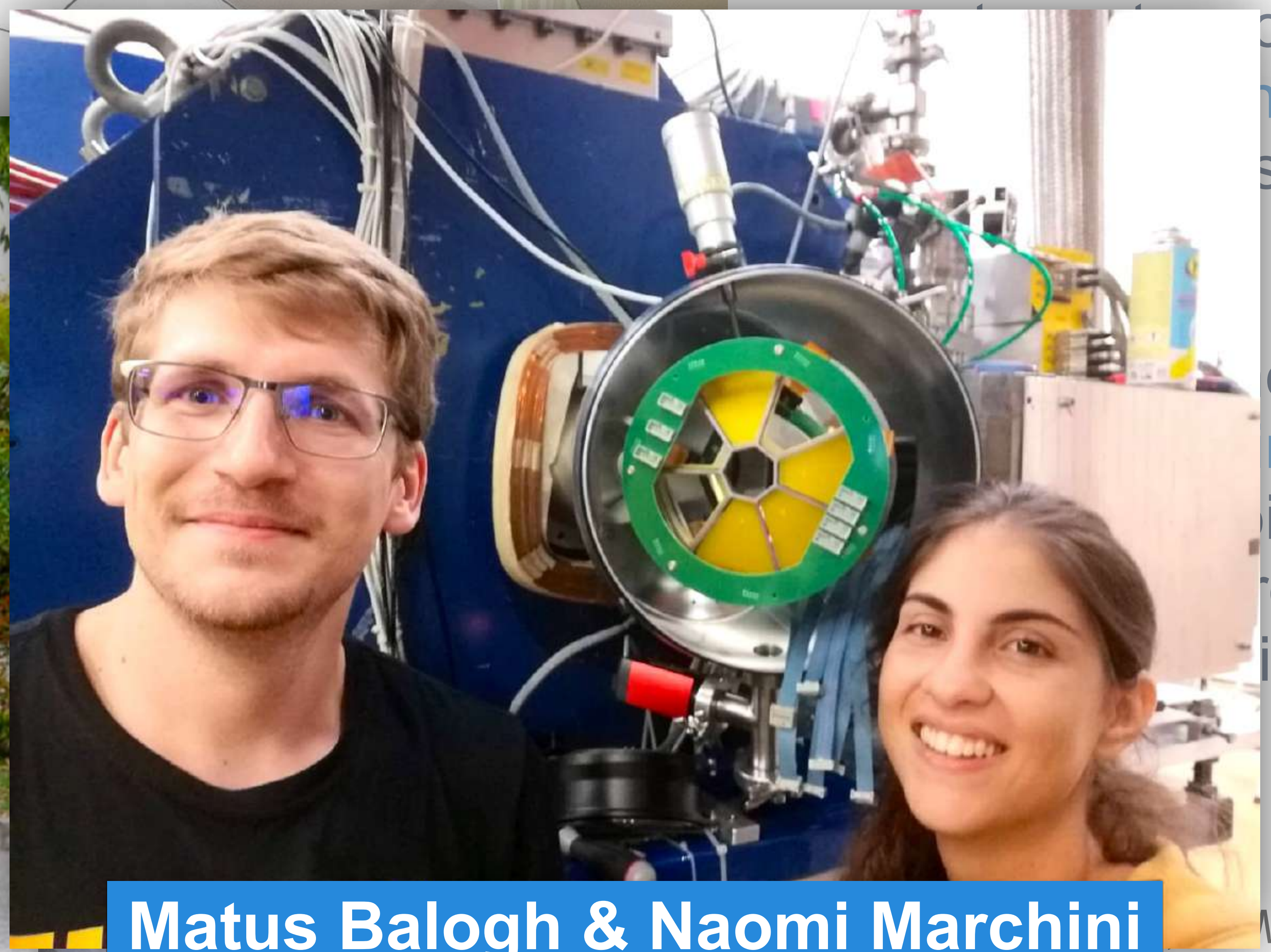


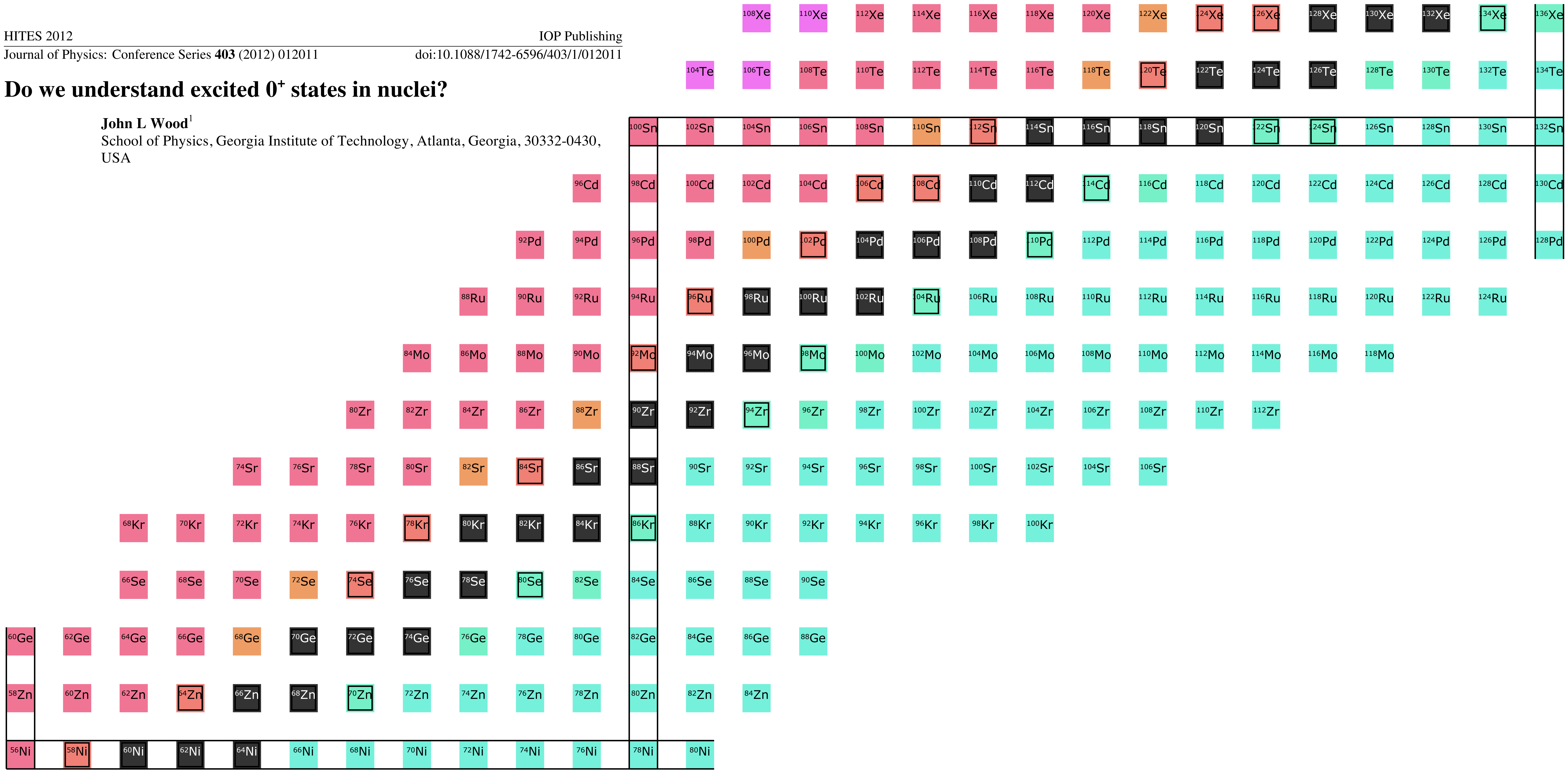
…Menegazzo, …Nuclear Inst. and … Research A 1049 (2023) 168040

Excited 0+ States in Mid-Mass Nuclei

HITES 2012 IOP Publishing
Journal of Physics: Conference Series **403** (2012) 012011 doi:10.1088/1742-6596/403/1/012011

Do we understand excited 0+ states in nuclei?

John L Wood[1]
School of Physics, Georgia Institute of Technology, Atlanta, Georgia, 30332-0430, USA



0^+ States with CoulEx at LNL and SPIDER

HITES 2012 IOP Publishing
Journal of Physics: Conference Series **403** (2012) 012011 doi:10.1088/1742-6596/403/1/012011

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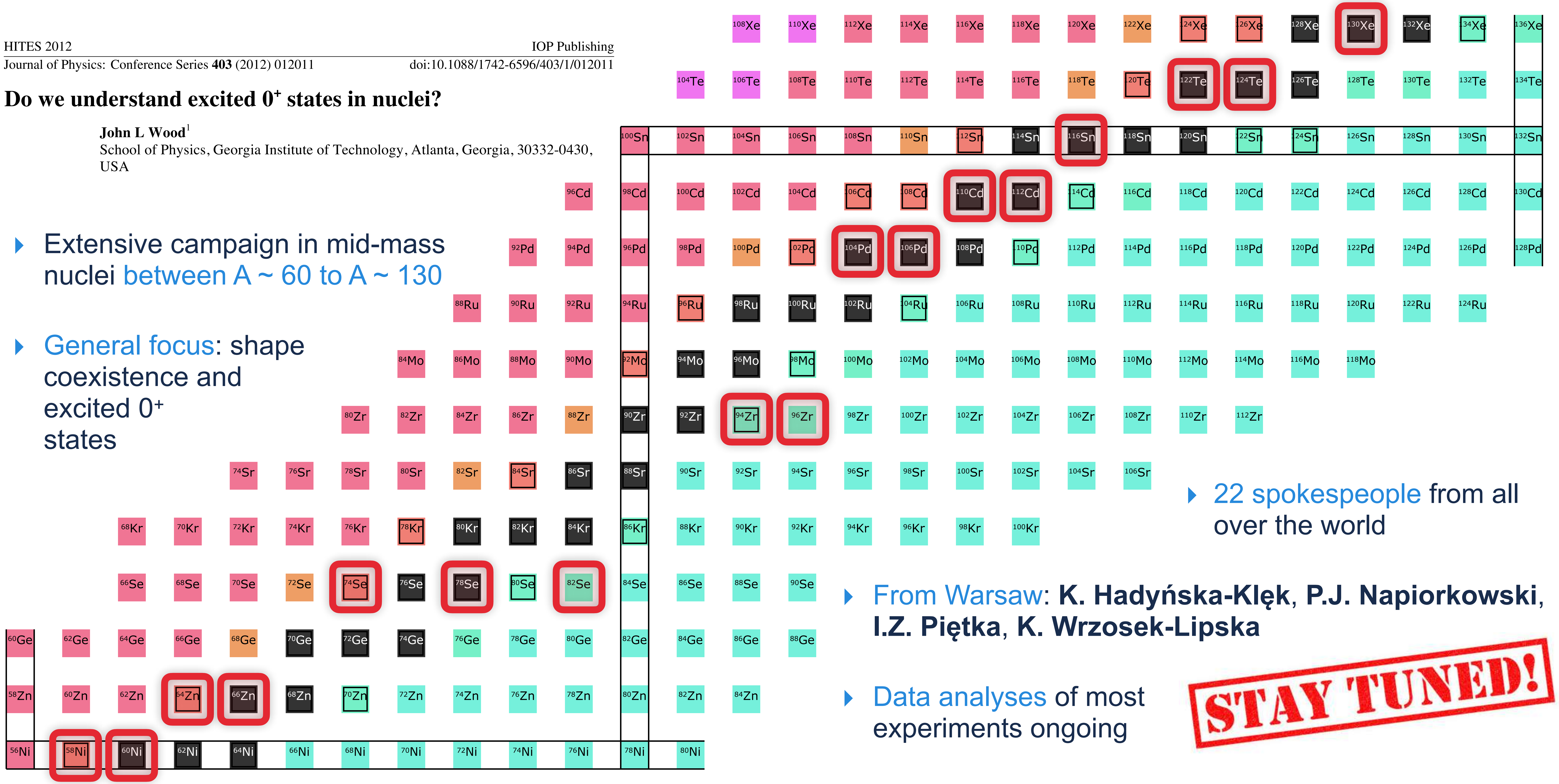
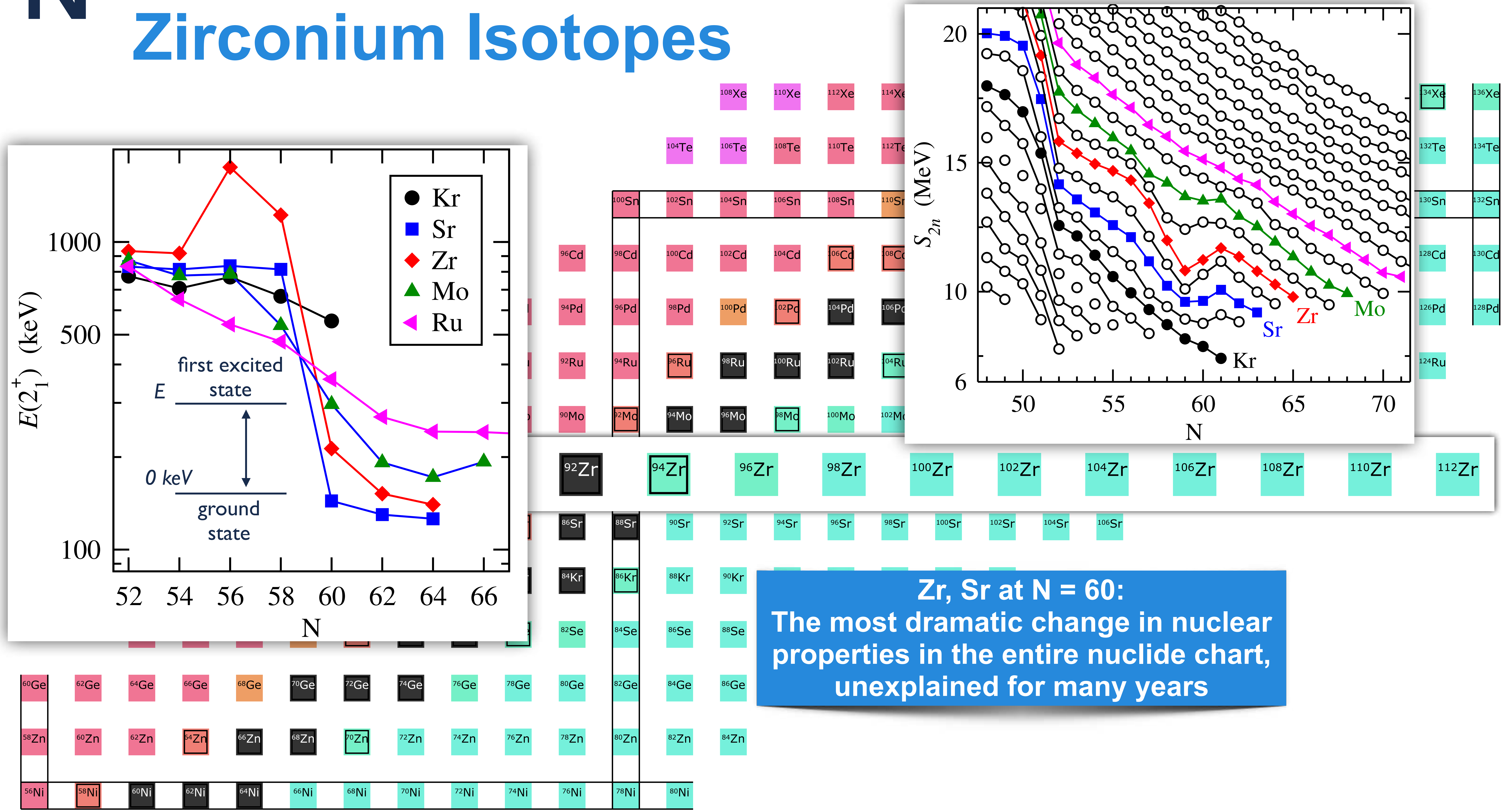


- Extensive campaign in mid-mass nuclei between A ~ 60 to A ~ 130
- General focus: shape coexistence and excited 0^+ states
- 22 spokespeople from all over the world
- From Warsaw: **K. Hadyńska-Klęk**, **P.J. Napiorkowski**, **I.Z. Piętka**, **K. Wrzosek-Lipska**
- Data analyses of most experiments ongoing

STAY TUNED!

Zirconium Isotopes



Zr, Sr at N = 60:
The most dramatic change in nuclear properties in the entire nuclide chart, unexplained for many years

Quantum Phase Transition

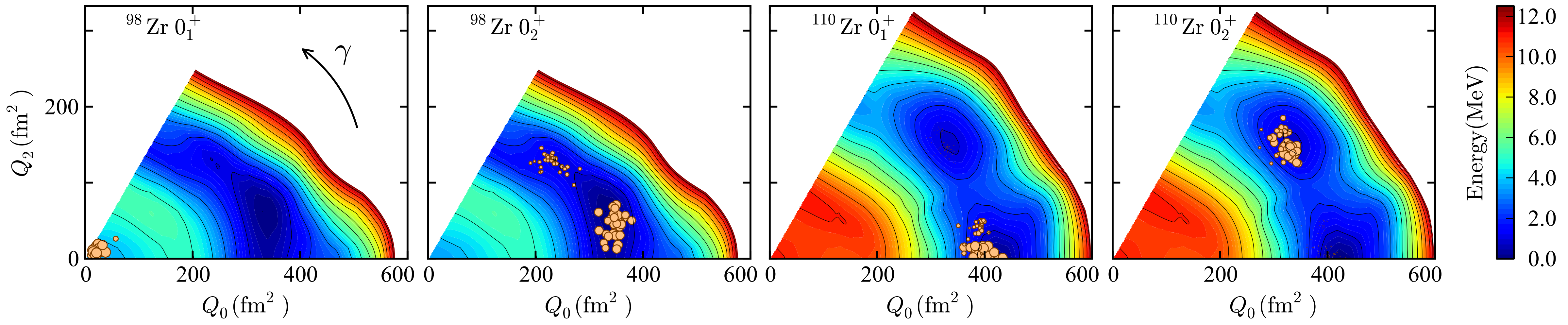
- Monte Carlo Shell Model (MCSM) ⟹ Excellent reproduction of experimental data, also around N = 60

PRL **117,** 172502 (2016) PHYSICAL REVIEW LETTERS week ending 21 OCTOBER 2016

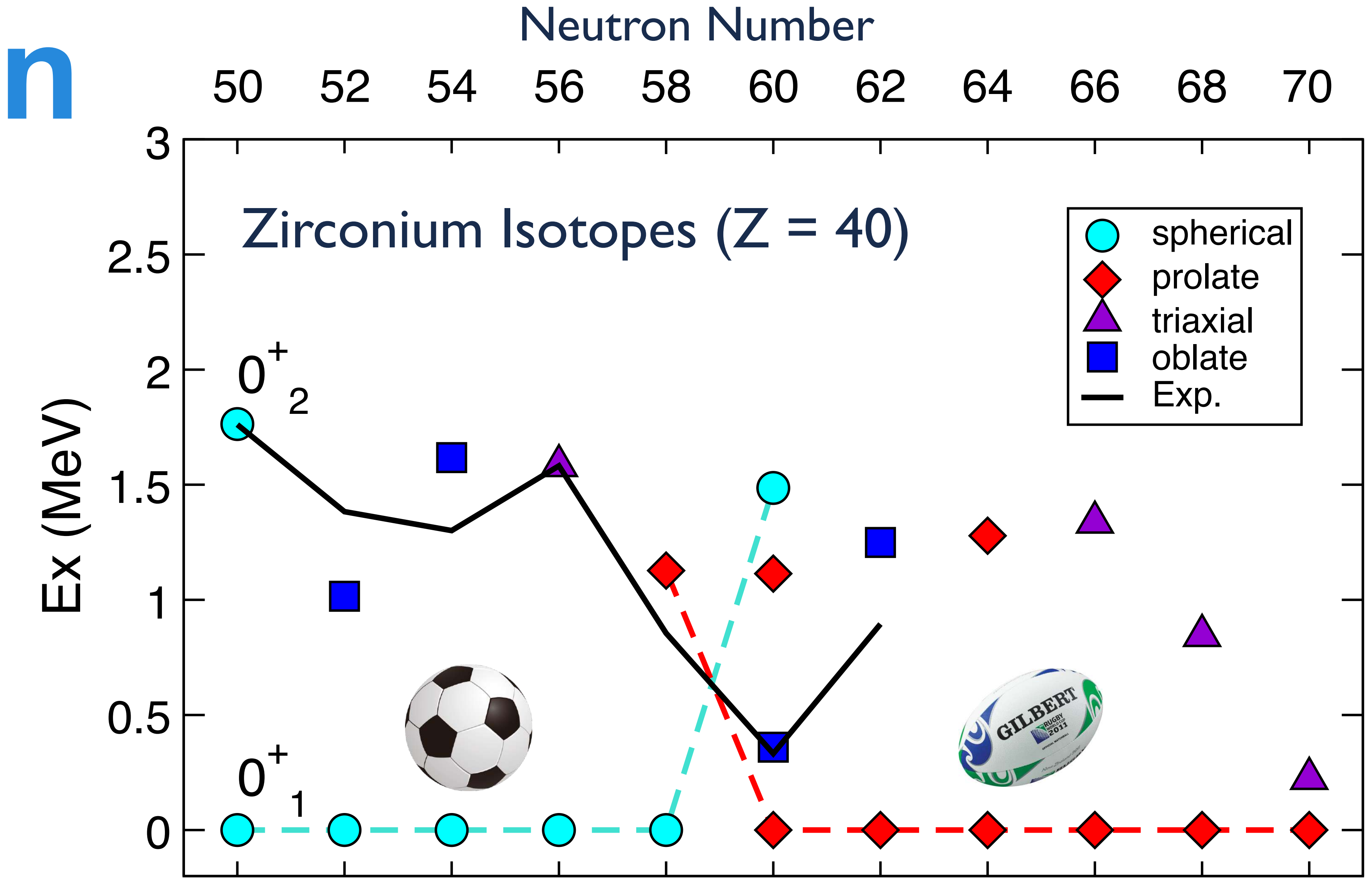
Quantum Phase Transition in the Shape of Zr isotopes

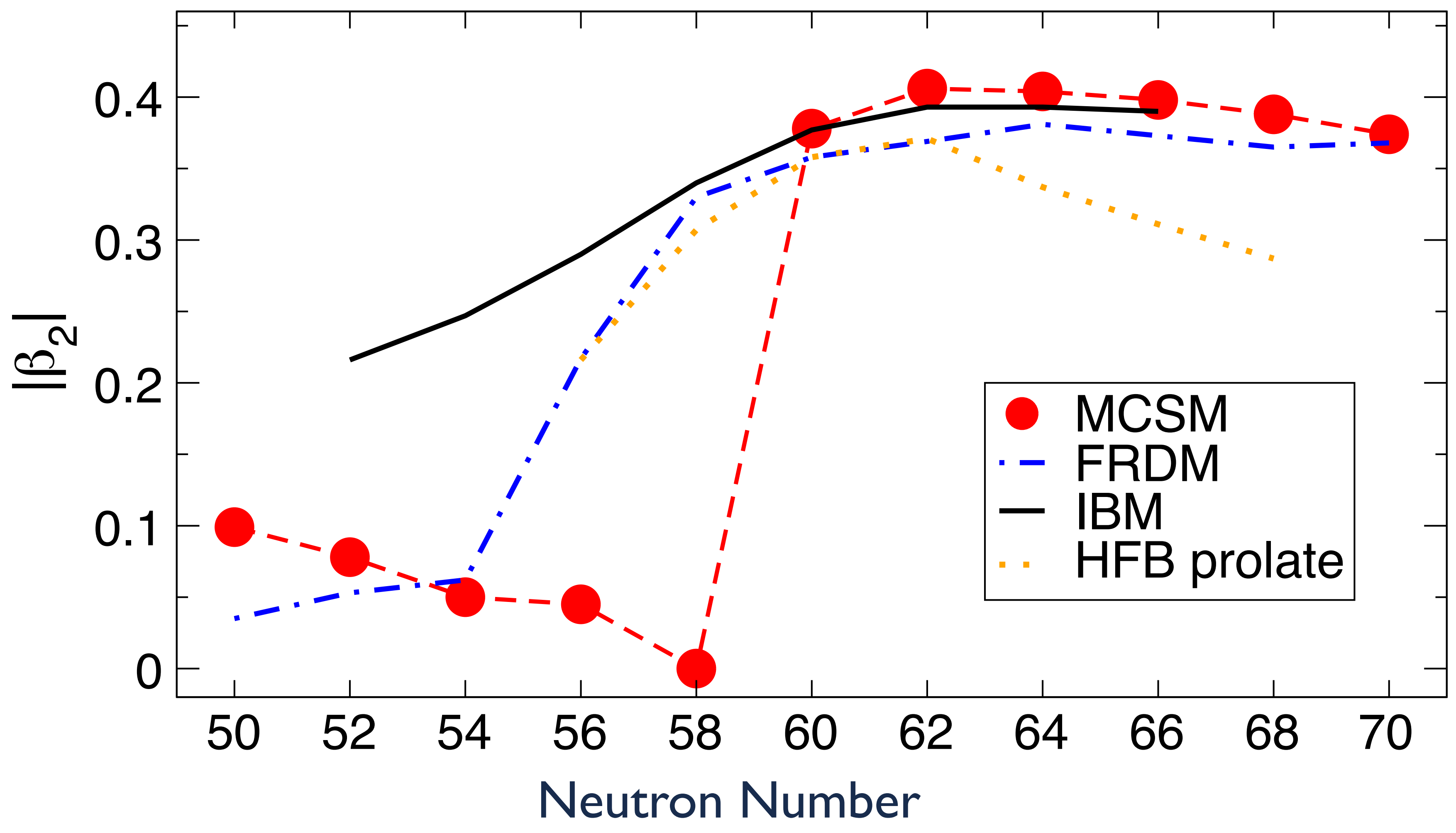
Tomoaki Togashi,[1] Yusuke Tsunoda,[1] Takaharu Otsuka,[1,2,3,4] and Noritaka Shimizu[1]

- Use of T-Plots for a direct calculation of the nuclear shape:



- Quantum Phase Transition (QPT):
  - Control Parameter ⟹ Neutron number
  - "Macroscopic" Quantity ⟹ Shape





- Self Organization: *"Process where some form of overall order arises from local interactions between parts of an initially disordered system"*
  - Phenomenon observed in Bose-Einstein condensates, superconductors, and even chemistry, biology, cybernetic, human sciences, road traffic
- Nuclear quantum self organization:

Positive Feedback

  - The shell structure can be modified by the occupation of individual nucleons
  - Increasing the number of neutrons induces deformation
  - Deformation promotes further excitations

Consequence: shape coexistence with weak mixing

Quantum Self Organisation

Intertwined Quantum Phase Transitions

- Interacting Boson Model with Configuration Mixing (IBM-CM) $\Longrightarrow$ Excellent reproduction of experimental data, also around N = 60

PHYSICAL REVIEW C **105**, 014305 (2022)

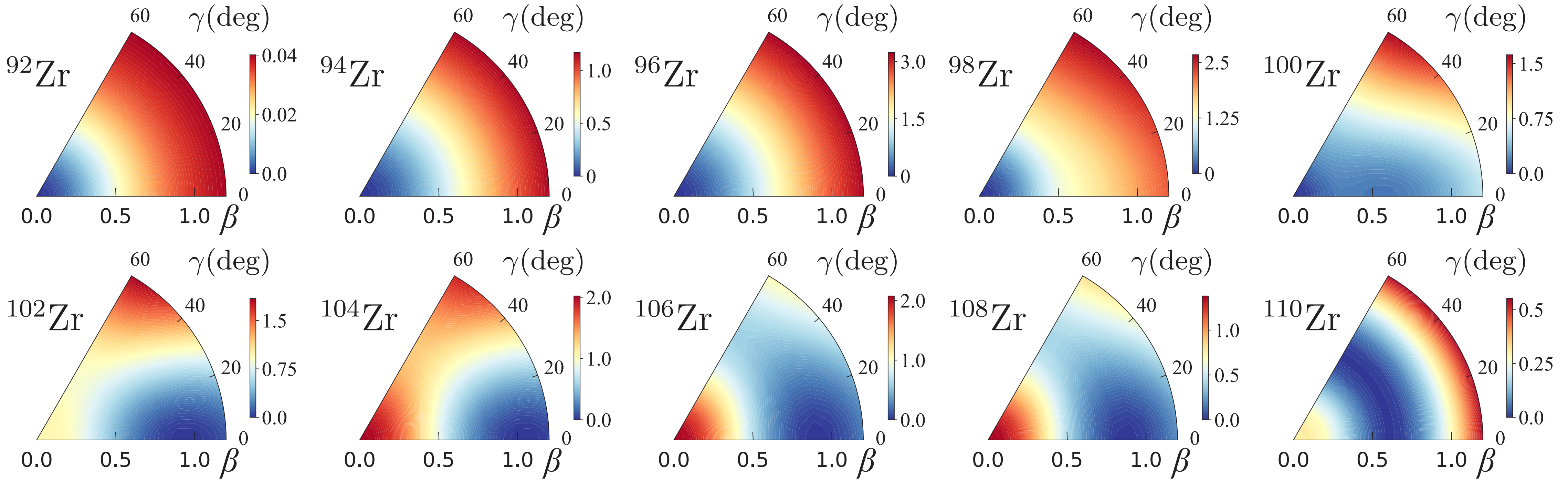
Zr isotopes as a region of intertwined quantum phase transitions

N. Gavrielov[1,2,*] A. Leviatan[1,†] and F. Iachello[2,‡]

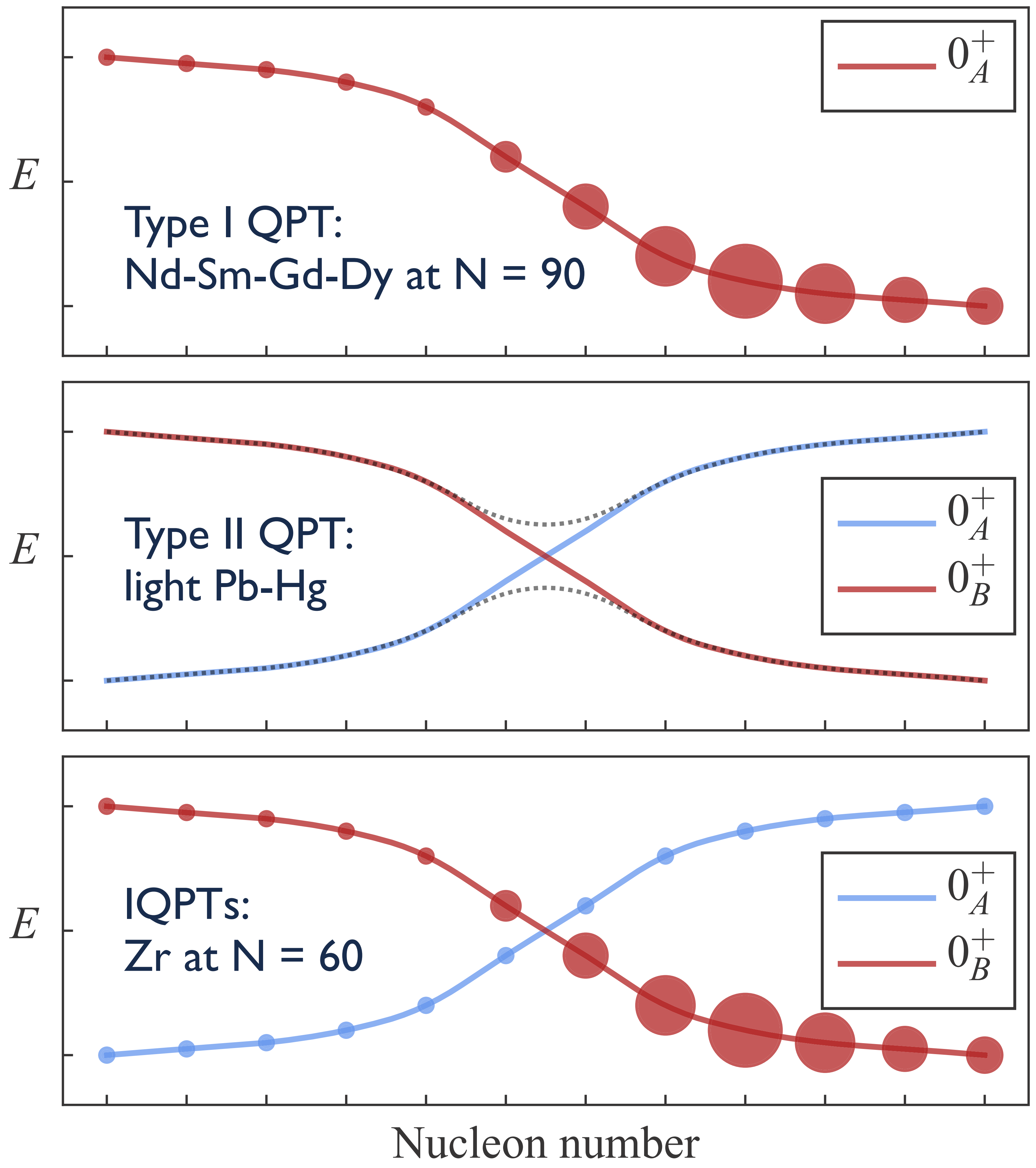
[1] *Racah Institute of Physics, The Hebrew University, Jerusalem 91904, Israel*

[2] *Center for Theoretical Physics, Sloane Physics Laboratory, Yale University, New Haven, Connecticut 06520-8120, USA*

- Two Intertwined Quantum Phase Transitions (IQPTs) exist in the Zr isotopic chain, appearing in the first two 0^+ states:
  - As in MCSM, the mixing between the coexisting 0^+ states is weak
  - At variance with MCSM, the ground state for N < 60 (0_2^+ state for N > 60) is always U(5)-like (*i.e.*, weakly deformed)

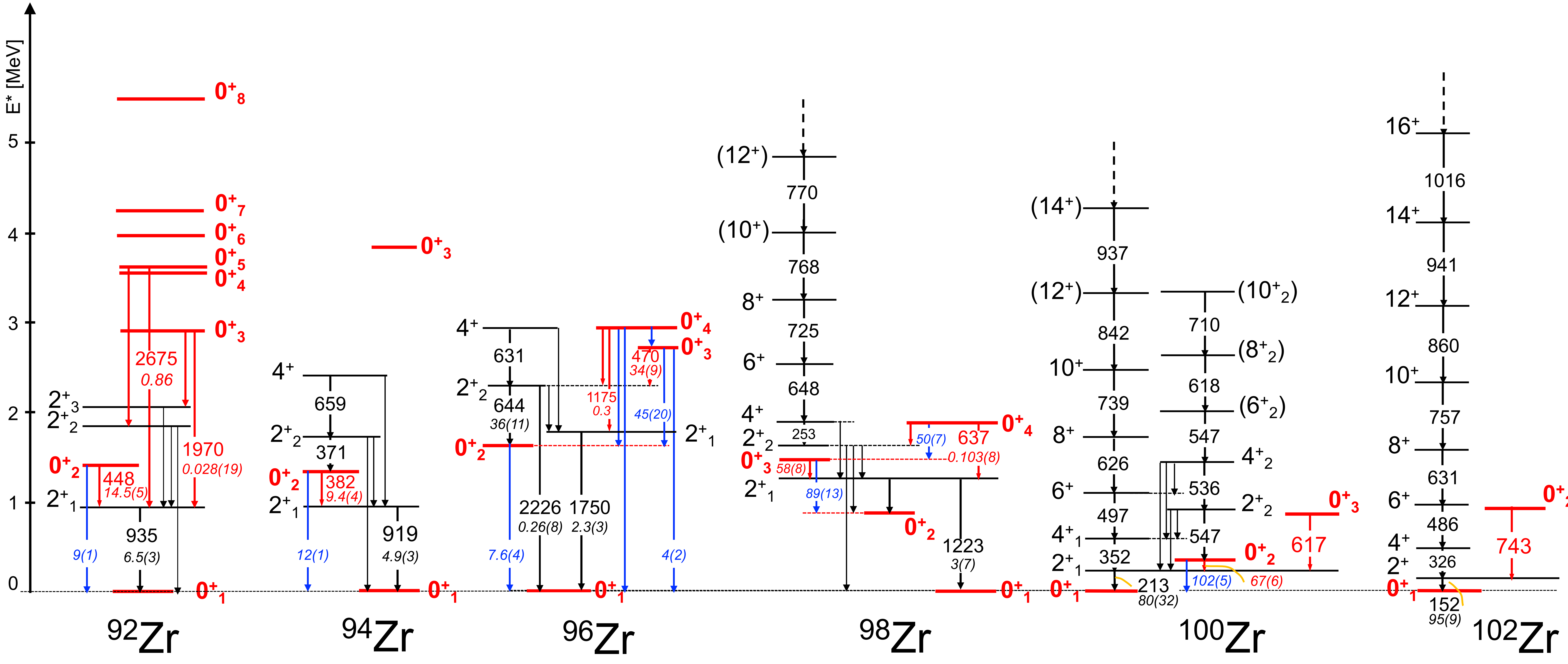


(Size of circles ∝ Deformation)



Experimental Knowledge on 0^+ States in Zr

- Excited 0^+ States in Even-Even Mid-Mass Nuclei
- Nuclear Shape
- Low-Energy Coulomb Excitation
- INFN and LNL
- SPIDER with GALILEO and AGATA
- The Zr Isotopic Chain and QPTs
- CoulEx Experiment on ^{94}Zr
- Experimental Shapes in ^{94}Zr



S. Leoni et al. Progress in Particle and Nuclear Physics 139 (2024) 104119

Excited 0+ States in Even-Even Mid-Mass Nuclei

Nuclear Shape

Low-Energy Coulomb Excitation

INFN and LNL

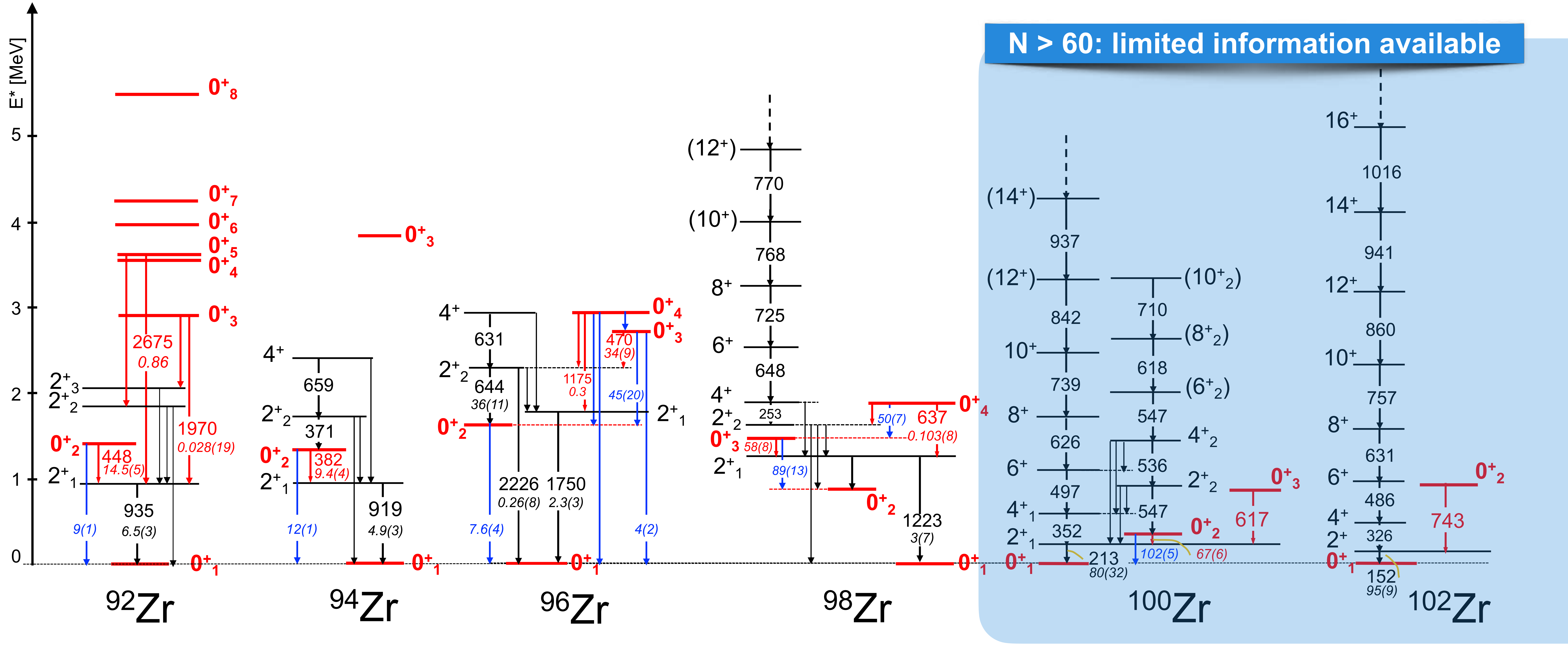
SPIDER with GALILEO and AGATA

The Zr Isotopic Chain and QPTs

CoulEx Experiment on 94Zr

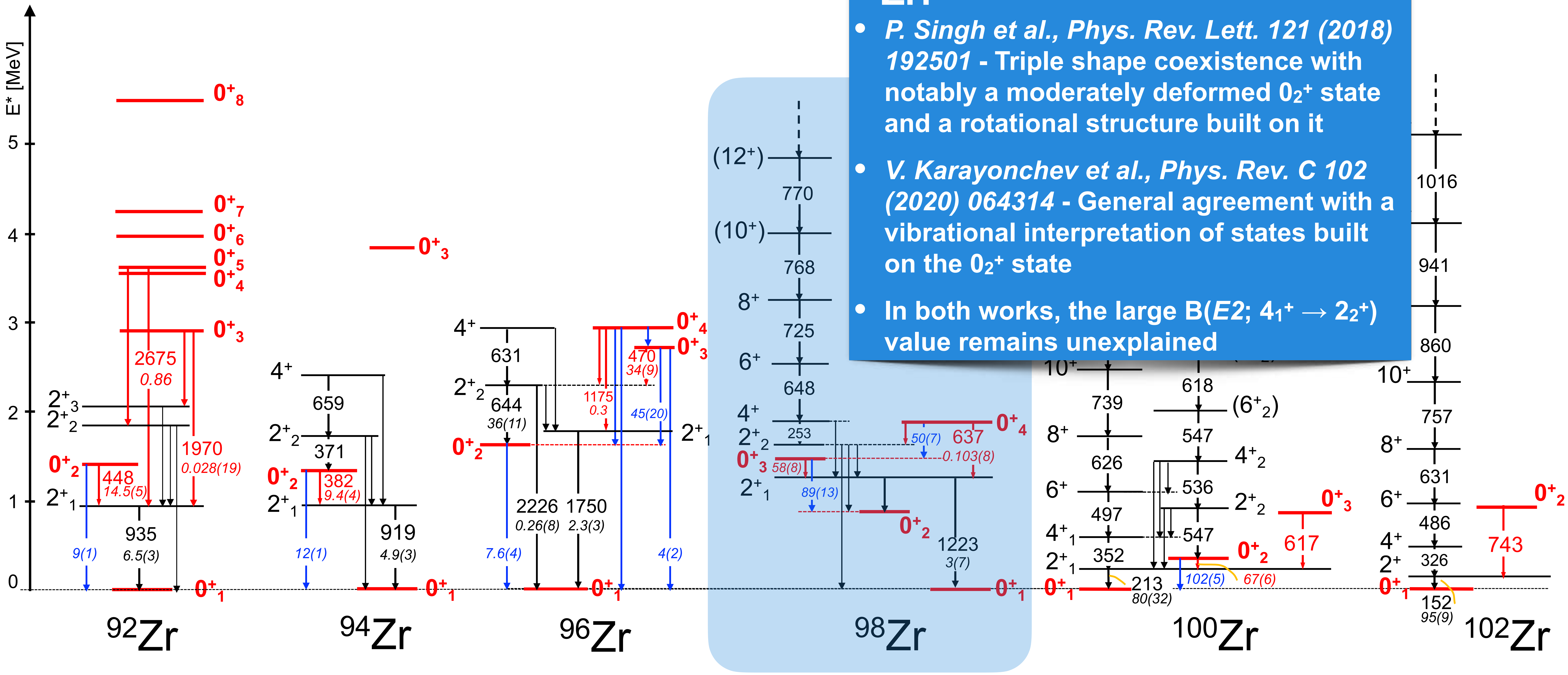
Experimental Shapes in 94Zr

Experimental Knowledge on 0+ States in Zr



S. Leoni et al. Progress in Particle and Nuclear Physics 139 (2024) 104119

Experimental Knowledge on 0^+ States in Zr



S. Leoni et al. Progress in Particle and Nuclear Physics 139 (2024) 104119

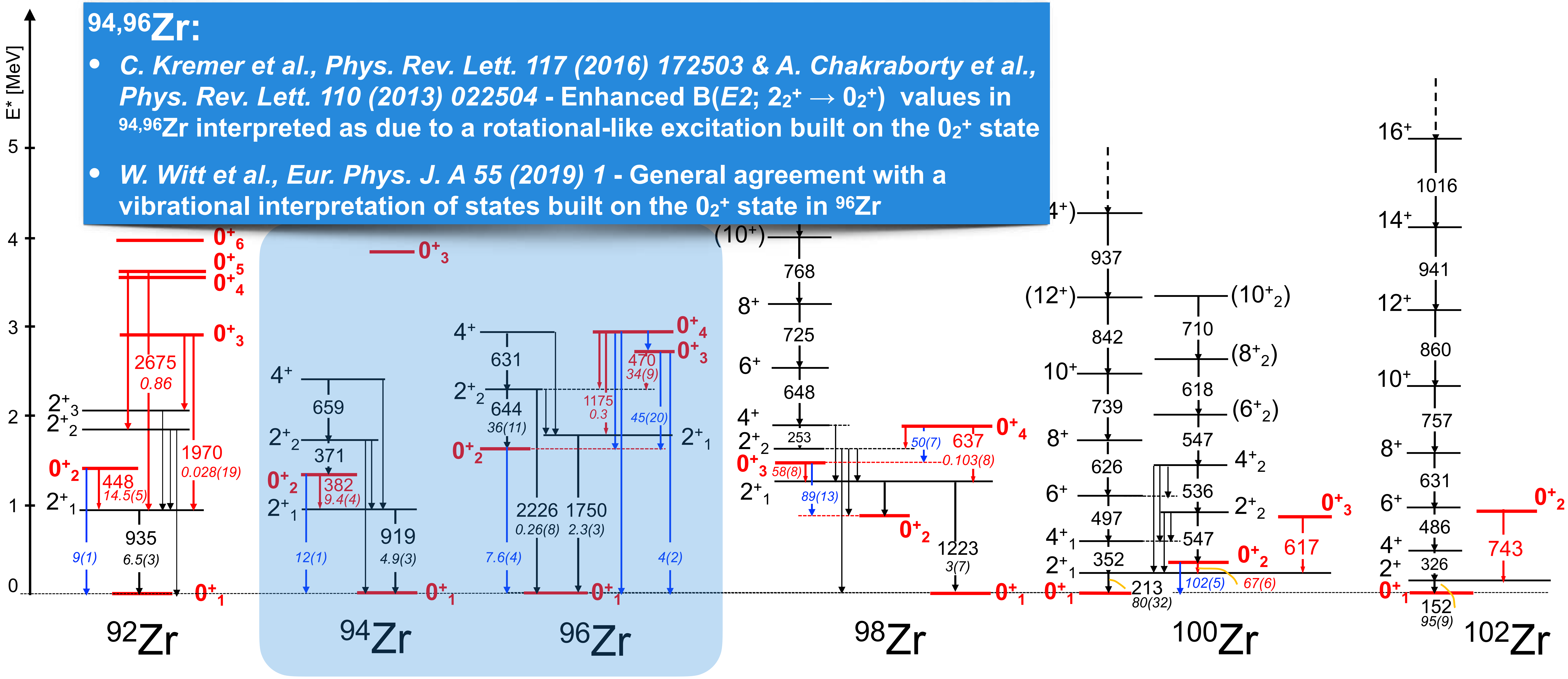
^{98}Zr:

- *P. Singh et al., Phys. Rev. Lett. 121 (2018) 192501* - **Triple shape coexistence with notably a moderately deformed 0_2^+ state and a rotational structure built on it**
- *V. Karayonchev et al., Phys. Rev. C 102 (2020) 064314* - **General agreement with a vibrational interpretation of states built on the 0_2^+ state**
- **In both works, the large $B(E2; 4_1^+ \rightarrow 2_2^+)$ value remains unexplained**

Experimental Knowledge on 0⁺ States in Zr

^{94,96}Zr:

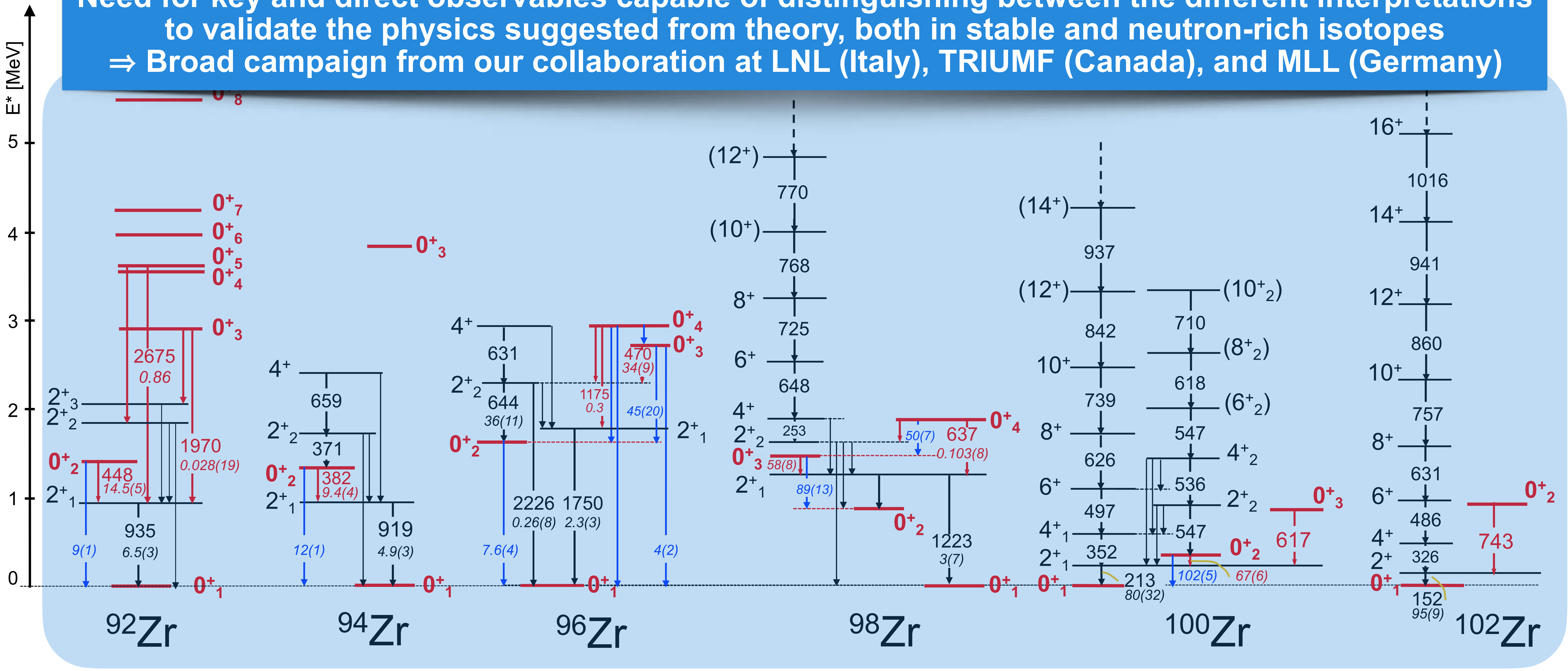
- *C. Kremer et al., Phys. Rev. Lett. 117 (2016) 172503 & A. Chakraborty et al., Phys. Rev. Lett. 110 (2013) 022504* - Enhanced $B(E2; 2_2^+ \rightarrow 0_2^+)$ values in ^{94,96}Zr interpreted as due to a rotational-like excitation built on the 0_2^+ state
- *W. Witt et al., Eur. Phys. J. A 55 (2019) 1* - General agreement with a vibrational interpretation of states built on the 0_2^+ state in ^{96}Zr



S. Leoni et al. Progress in Particle and Nuclear Physics 139 (2024) 104119

Experimental Knowledge on 0⁺ States in Zr

Need for key and direct observables capable of distinguishing between the different interpretations to validate the physics suggested from theory, both in stable and neutron-rich isotopes
⇒ Broad campaign from our collaboration at LNL (Italy), TRIUMF (Canada), and MLL (Germany)

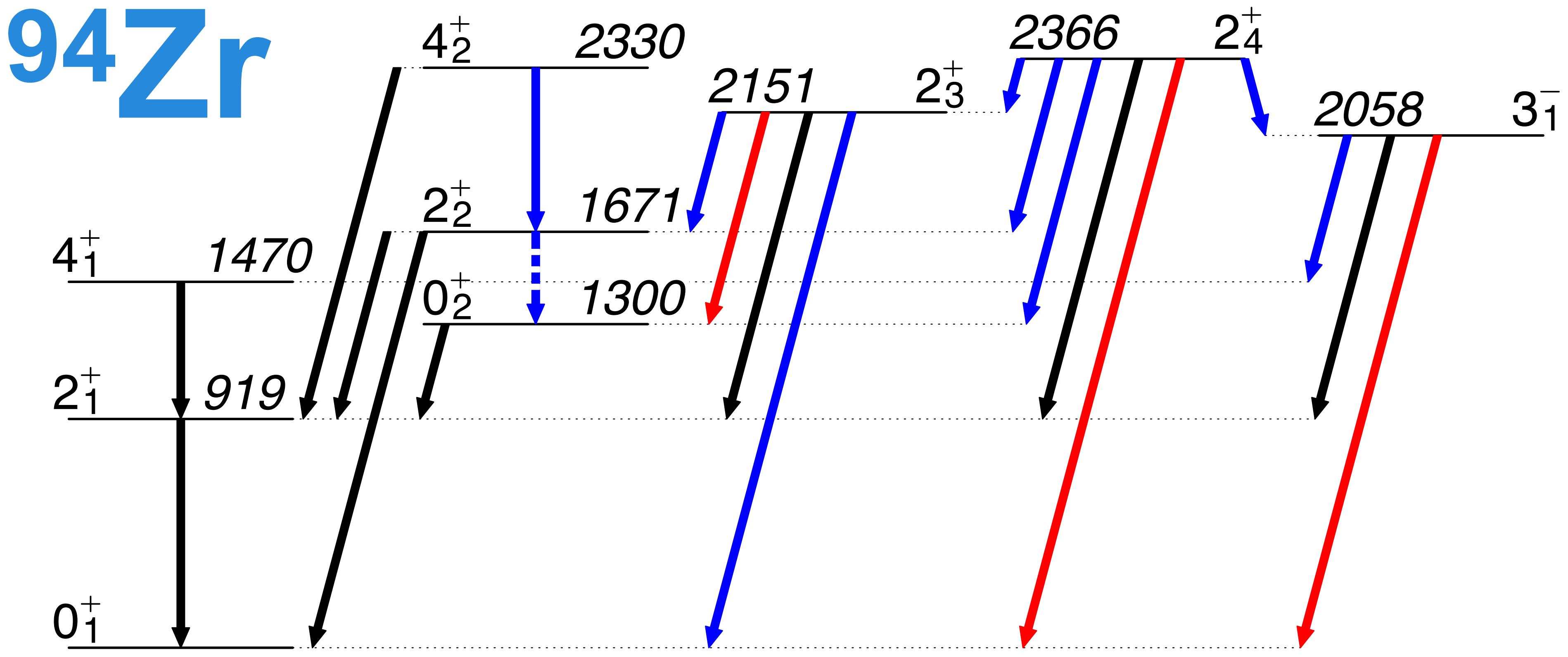


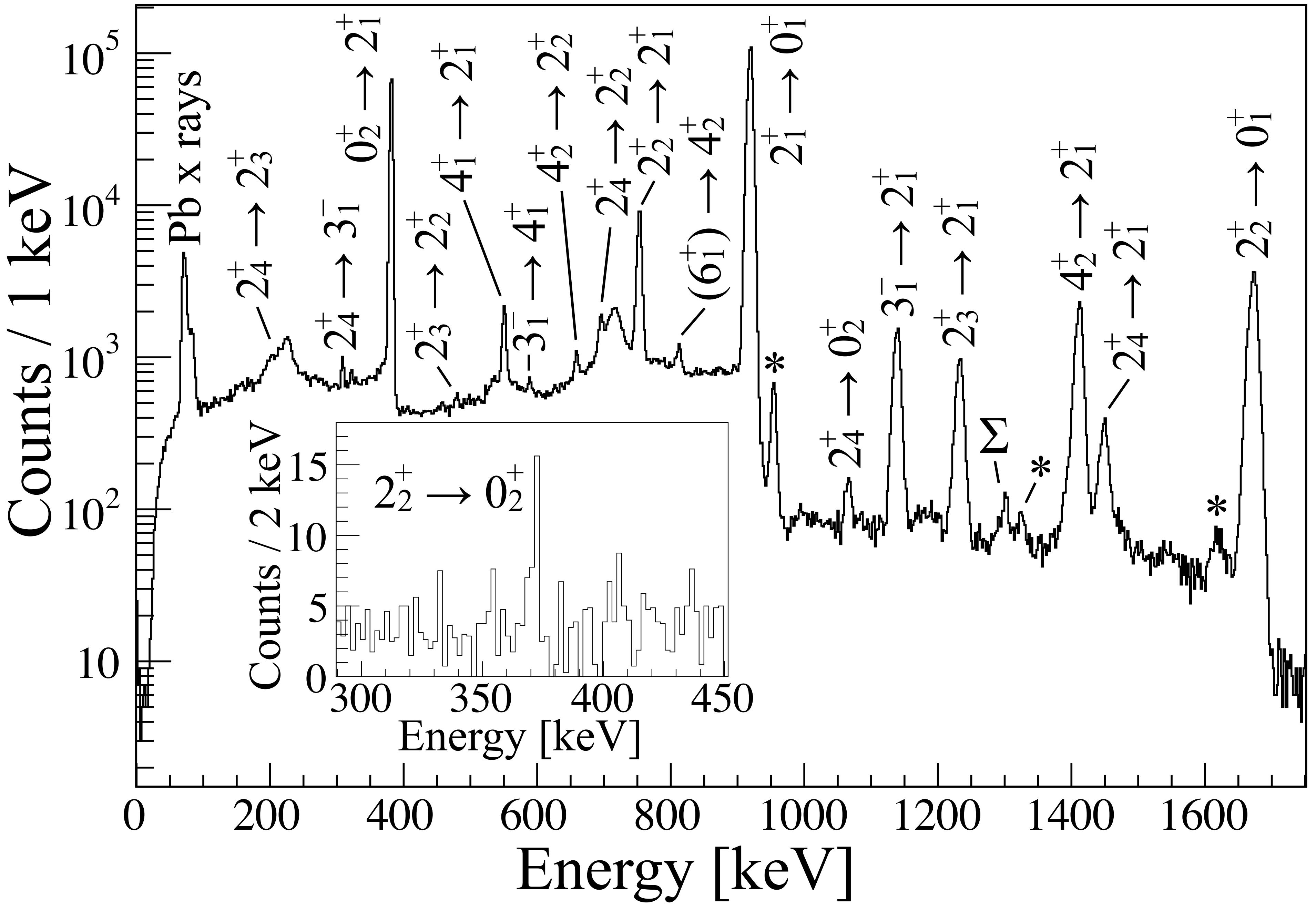
S. Leoni et al. Progress in Particle and Nuclear Physics 139 (2024) 104119

Coulomb Excitation of ^{94}Zr

- Excited 0+ States in Even-Even Mid-Mass Nuclei
- Nuclear Shape
- Low-Energy Coulomb Excitation
- INFN and LNL
- SPIDER with GALILEO and AGATA
- The Zr Isotopic Chain and QPTs
- CoulEx Experiment on ^{94}Zr
- Experimental Shapes in ^{94}Zr

- Experiment performed with GALILEO and SPIDER:
  - ^{94}Zr beam @370 MeV (0.1 pnA) on a self-supporting ^{208}Pb target with 1 mg/cm^2 thickness
  - GALILEO with 25 HPGe Compton-suppressed detectors
  - SPIDER at backward angles (126° - 162°)
  - 6 $LaBr_3$:Ce detectors to increase the efficiency in γ-γ coinc.
  - *Spokespersons: D.T. Doherty (University of Surrey, UK), M. Rocchini, M. Zielinska (CEA Saclay, France)*
  - *Analysis by: N. Marchini (INFN Firenze), M. Rocchini*
- 18 gamma-ray transitions observed, 8 states populated
- Known spectroscopic data added in the analysis:
  - 13 branching ratios, 10 lifetimes, 7 mixing ratios

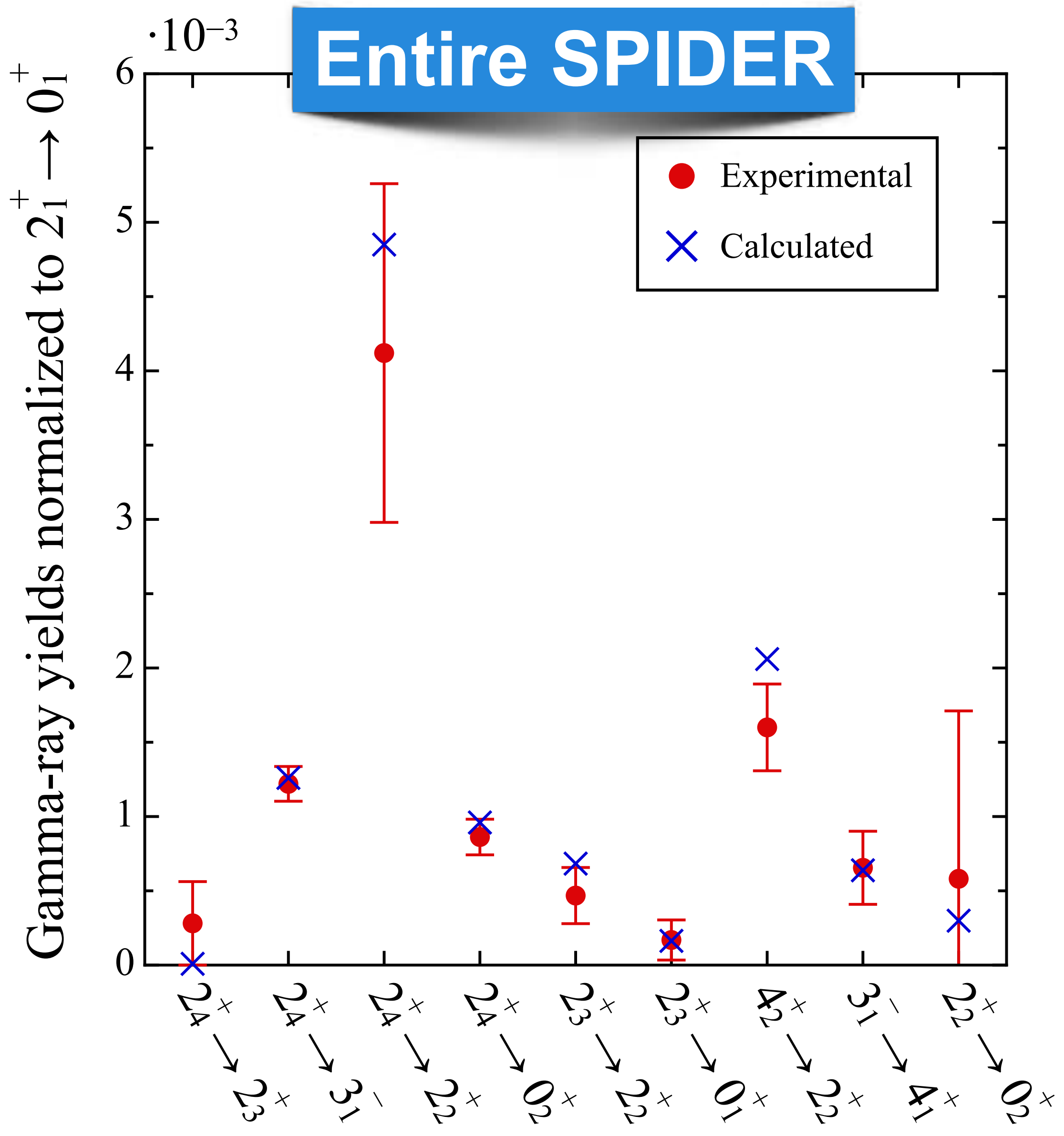


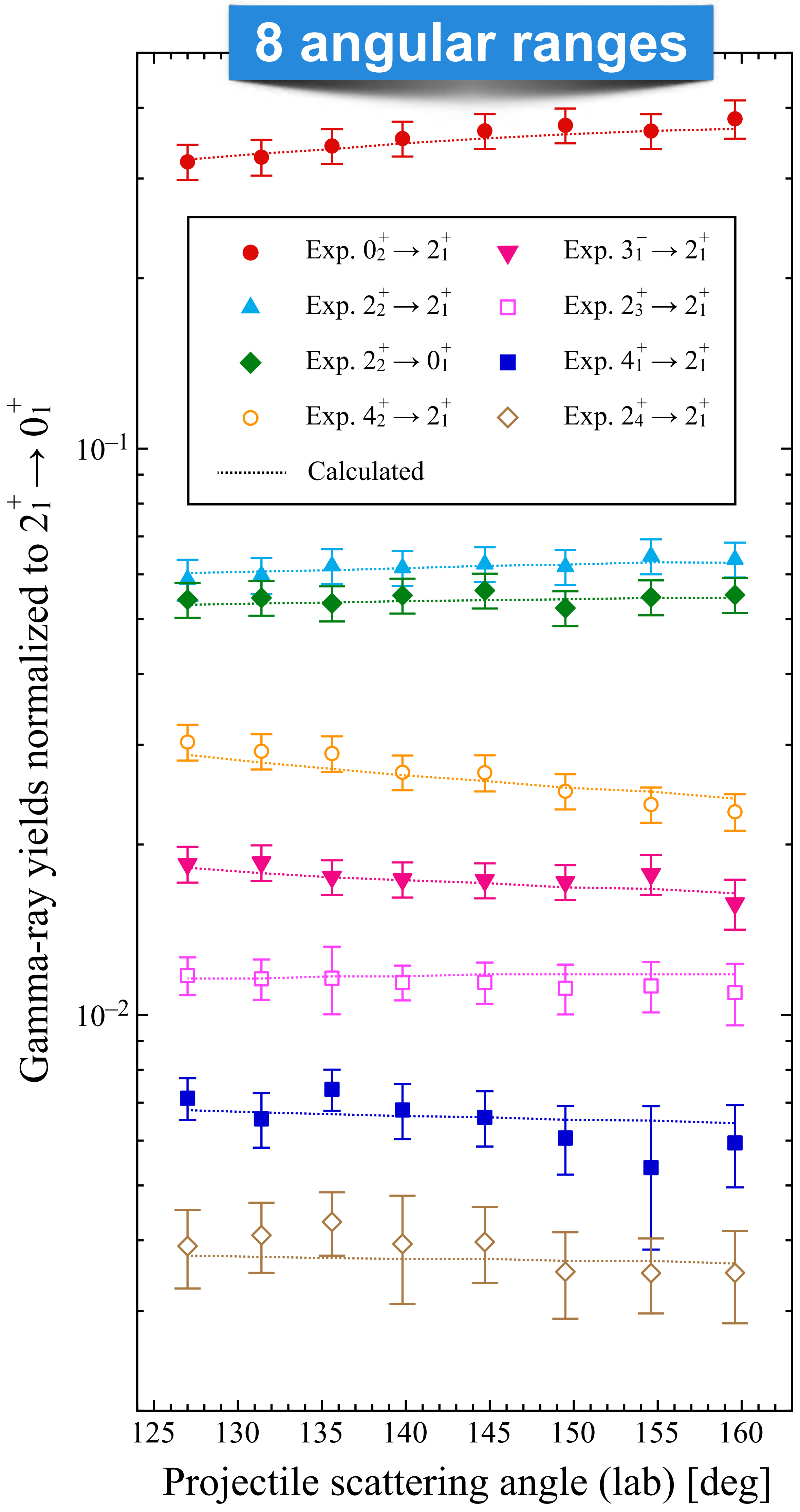


N. Marchini, M. Rocchini, M. Zielinska et al. submitted to Phys. Lett. B

Results

- 8 transitions analysed by dividing the statistics into 8 angular ranges, 9 transitions considering the entire SPIDER
- GOSIA analysis: excellent reproduction of experimental yields





17 reduced transition probabilities extracted + new IBM-CM calculations

$J_i \to J_f$	$\langle J_f \| E2 \| J_i \rangle$ [eb]	$B(E2; J_i \to J_f)$ [W.u.]	
	Exp.	Exp.	IBM-CM
$2_1^+ \to 0_1^+$	$+0.250(7)^{ab}$	$4.8(2)^{b}$	2.7
$0_2^+ \to 2_1^+$	$+0.155(4)^{a}$	9.5(5)	9.3
$4_1^+ \to 2_1^+$	$+0.141(4)^{a}$	0.87(5)	—[c]
$2_2^+ \to 0_2^+$	$+0.484(12)^{a}$	18.5(9)	20.2
$2_2^+ \to 2_1^+$	+0.03(3)	< 0.3	1.49
$2_2^+ \to 0_1^+$	+0.221(6)	3.9(2)	0.82
$4_2^+ \to 2_2^+$	$+0.96(3)^{a}$	40(3)	26.6
$4_2^+ \to 2_1^+$	+0.607(16)	16.1(9)	2.1
$2_3^+ \to 2_2^+$	$+0.249^{+0.019}_{-0.040}$	$4.9^{+0.7}_{-1.6}$	17.3
$2_3^+ \to 0_2^+$	$< 0.13^{d}$	< 1.3	0.07
$2_3^+ \to 2_1^+$	$+0.290^{+0.014\,a}_{-0.012}$	$6.6^{+0.6}_{-0.5}$	1.2
$2_3^+ \to 0_1^+$	$-0.0182^{+0.0016}_{-0.0020}$	$0.026^{+0.005}_{-0.006}$	0.001
$2_4^+ \to 2_3^+$	$-0.02^{+0.06}_{-0.03}$	< 0.2	2.44
$2_4^+ \to 2_2^+$	$\pm 0.073^{+0.030}_{-0.018}$	$0.4^{+0.3}_{-0.2}$	0.1
$2_4^+ \to 0_2^+$	$\pm 0.177^{+0.010}_{-0.005}$	$2.47^{+0.30}_{-0.14}$	0.06
$2_4^+ \to 2_1^+$	$+0.092^{+0.006\,a}_{-0.004}$	$0.67^{+0.09}_{-0.06}$	0.001
$2_4^+ \to 0_1^+$	$-0.001^{+0.003}_{-0.006}$	$< 4 \cdot 10^{-3}$	$3 \cdot 10^{-4}$

[a] Sign imposed in the analysis (see text for details).
[b] Determined from literature data.
[c] Outside IBM-CM model space (see Ref. [24] for details).
[d] Positive sign determined.

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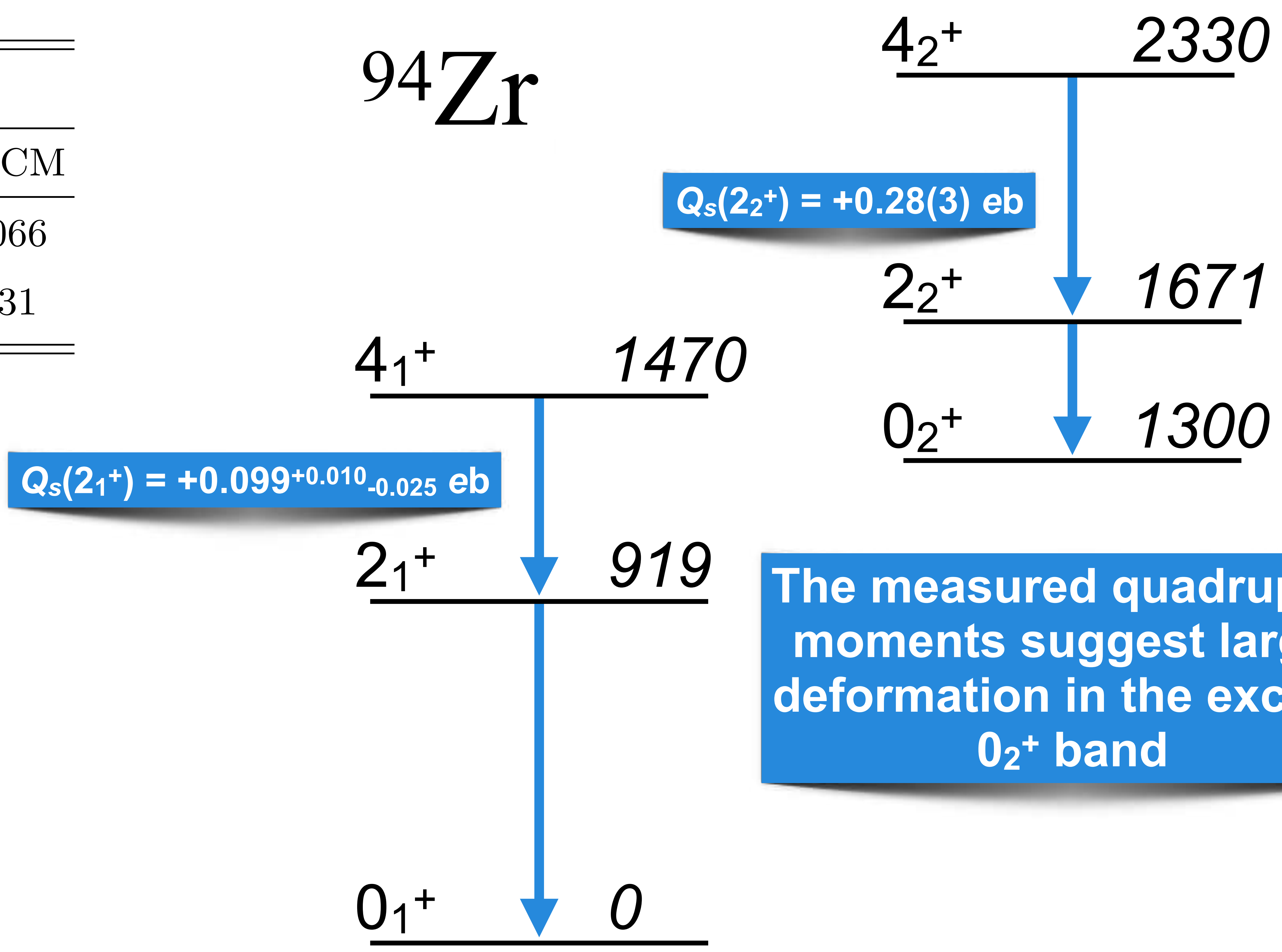
Shapes of the ^{94}Zr $0_{1,2}^+$ States

- First determination of spectroscopic quadrupole moments for the $2_{1,2}^+$ states in the Zr isotopic chain:

J_i	$\langle J_i \Vert E2 \Vert J_i \rangle$ [eb]	$Q_s(J_i)$ [eb]	
	Exp.	Exp.	IBM-CM
2_1^+	$+0.131^{+0.013}_{-0.030}$	$+0.099^{+0.010}_{-0.025}$	$+0.066$
2_2^+	$+0.37(4)$	$+0.28(3)$	$+0.31$



The measured quadrupole moments suggest larger deformation in the excited 0_2^+ band

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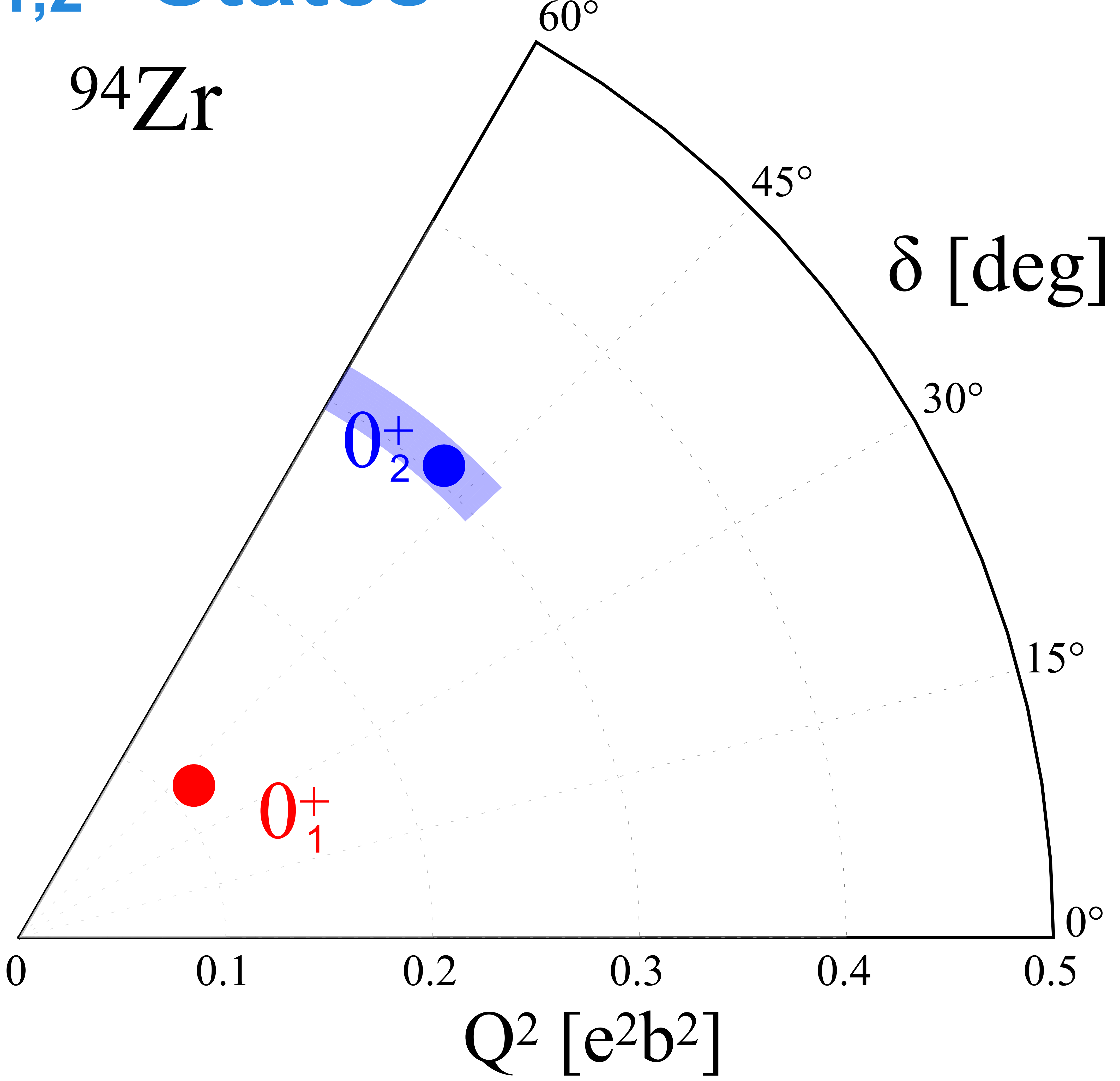
- First experimental application of quadrupole sum rules to the $0^+_{1,2}$ states in the Zr isotopic chain, and one of the few determinations of $\sigma(Q^2) = (\langle Q^4\rangle - (\langle Q^2\rangle)^2)^{1/2}$ in the entire nuclide chart:

J_i	$\langle Q^2\rangle$ [e^2b^2]		$\sigma(Q^2)$ [e^2b^2]		$\langle\cos(3\delta)\rangle$	
	Exp.	Th.	Exp.	Th.	Exp.	Th.
0^+_1	0.112(4)				−0.37(7)	
0^+_2	0.305(12)				−0.8(2)	



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Generic Ellipsoid

$$R(\theta,\phi) = R_0 \left[1 + \beta \sqrt{\frac{5}{16\pi}} \left(\cos\gamma (3\cos^2\theta - 1) + \sqrt{3}\sin\gamma \sin^2\theta \cos 2\phi \right) \right]$$

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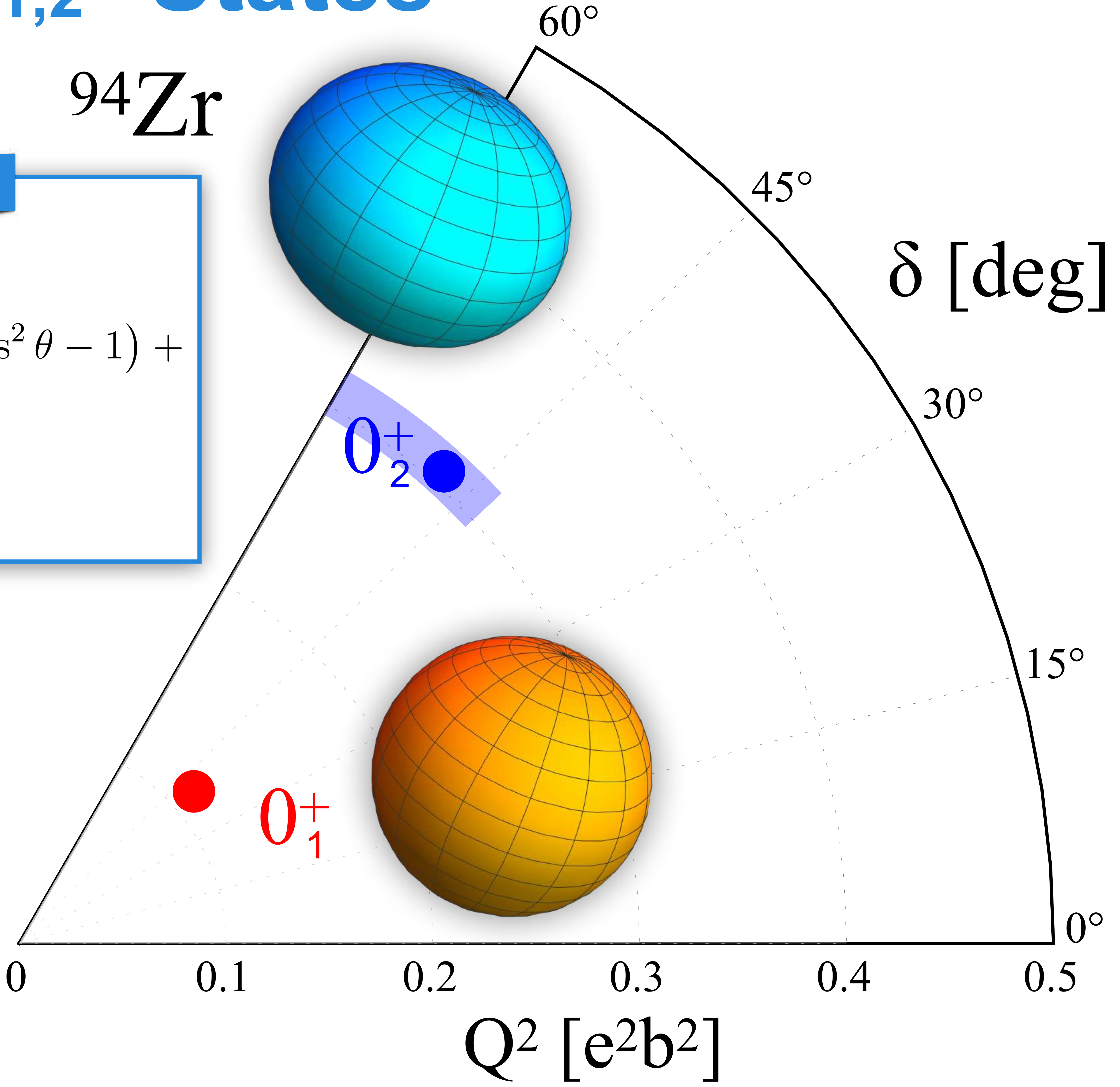

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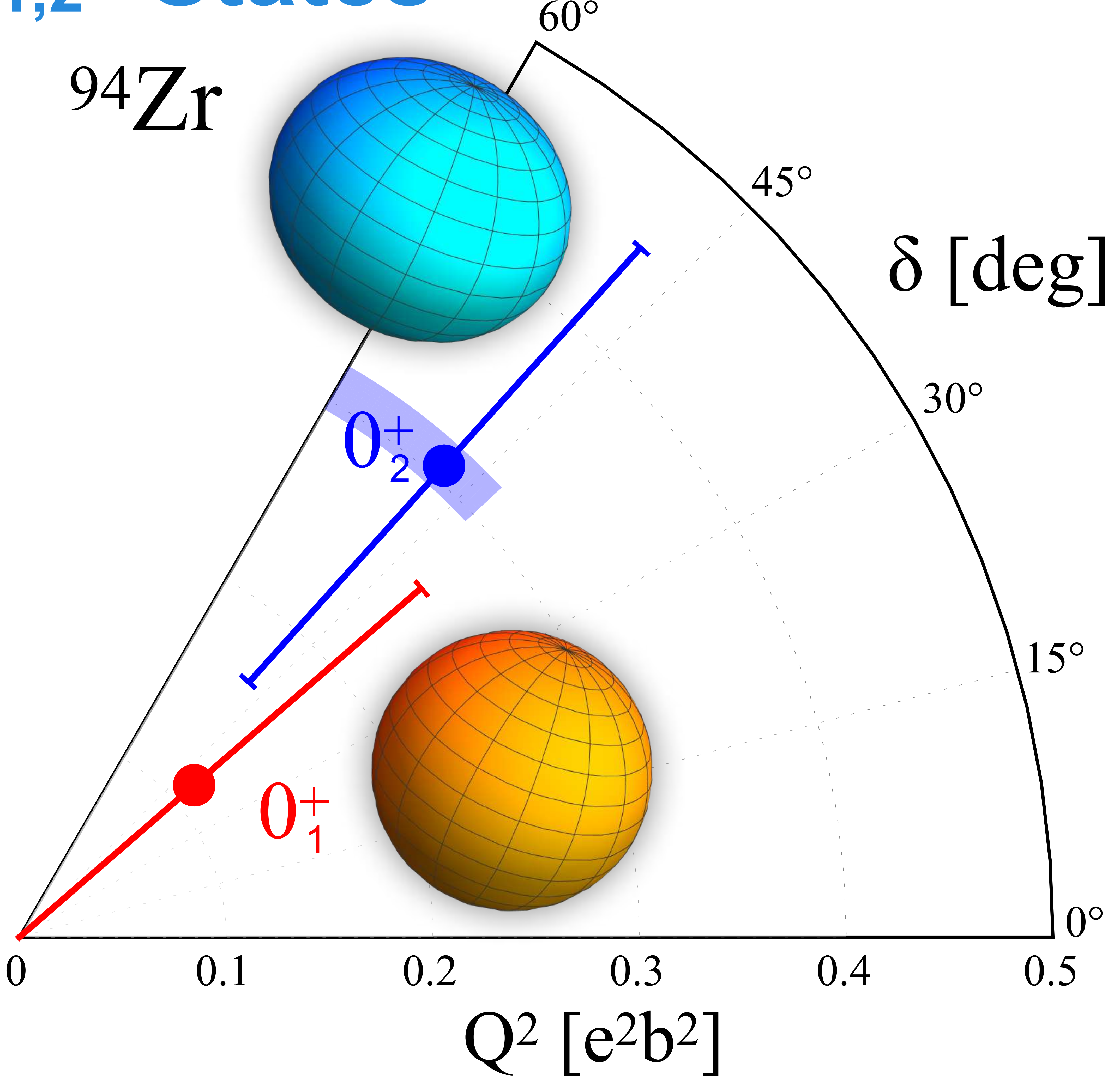

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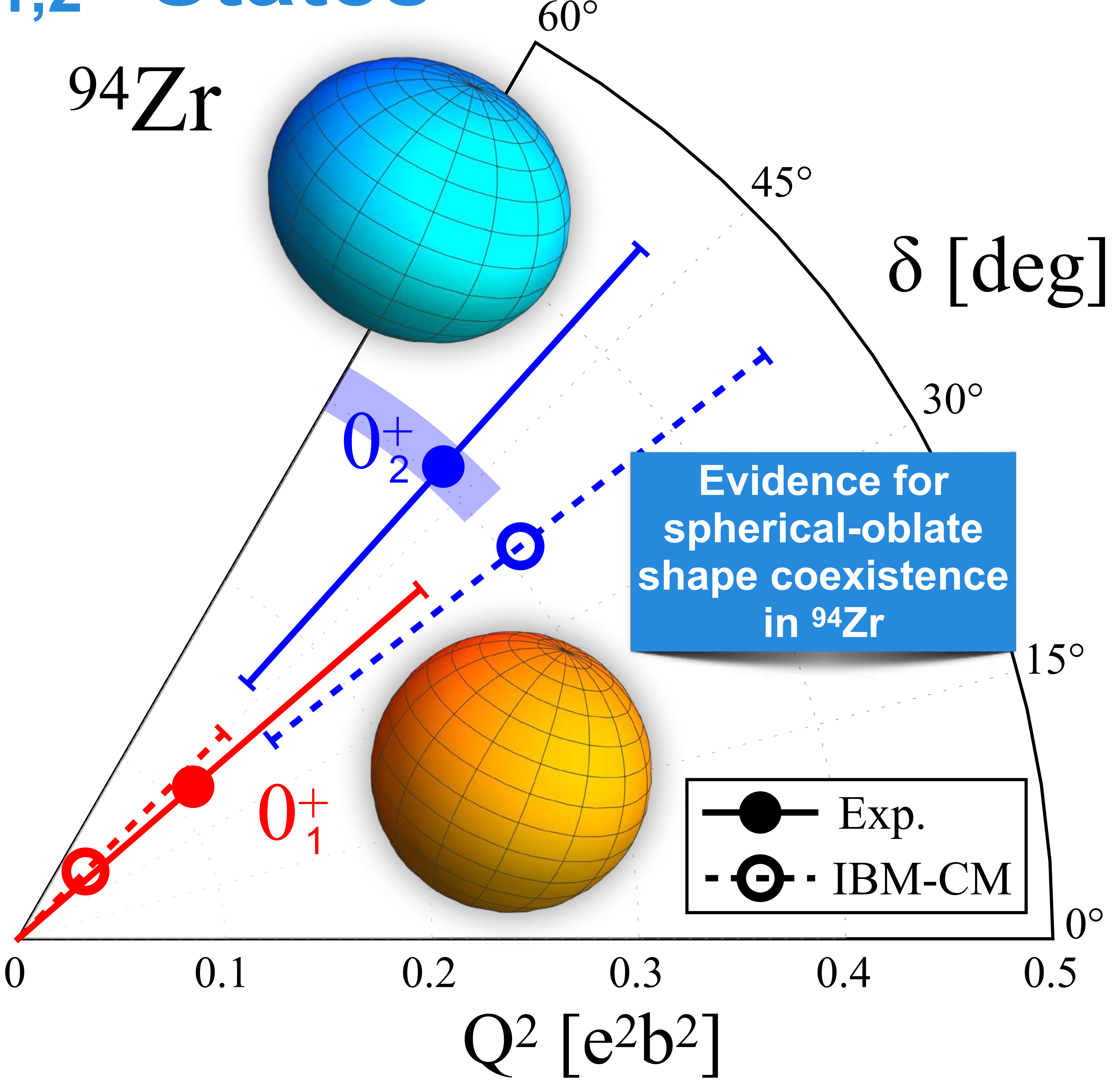
	Exp.	Th.	Exp.	Th.	Exp.	Th.
0^+_1	0.112(4)	0.046	0.143(4)	0.094	−0.37(7)	−0.72
0^+_2	0.305(12)	0.308	0.14(3)	0.153	−0.8(2)	−0.4



Evidence for spherical-oblate shape coexistence in ^{94}Zr

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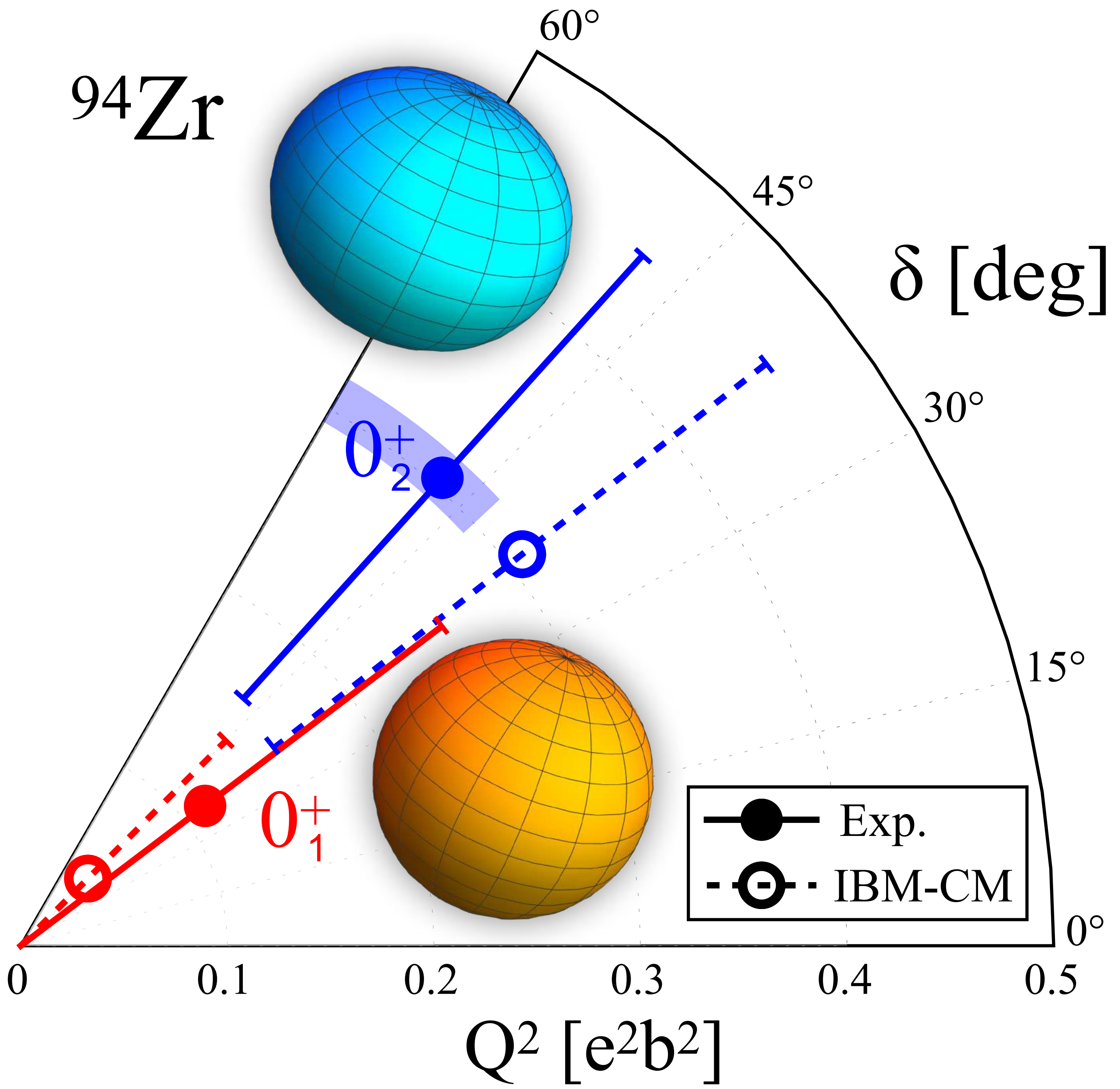
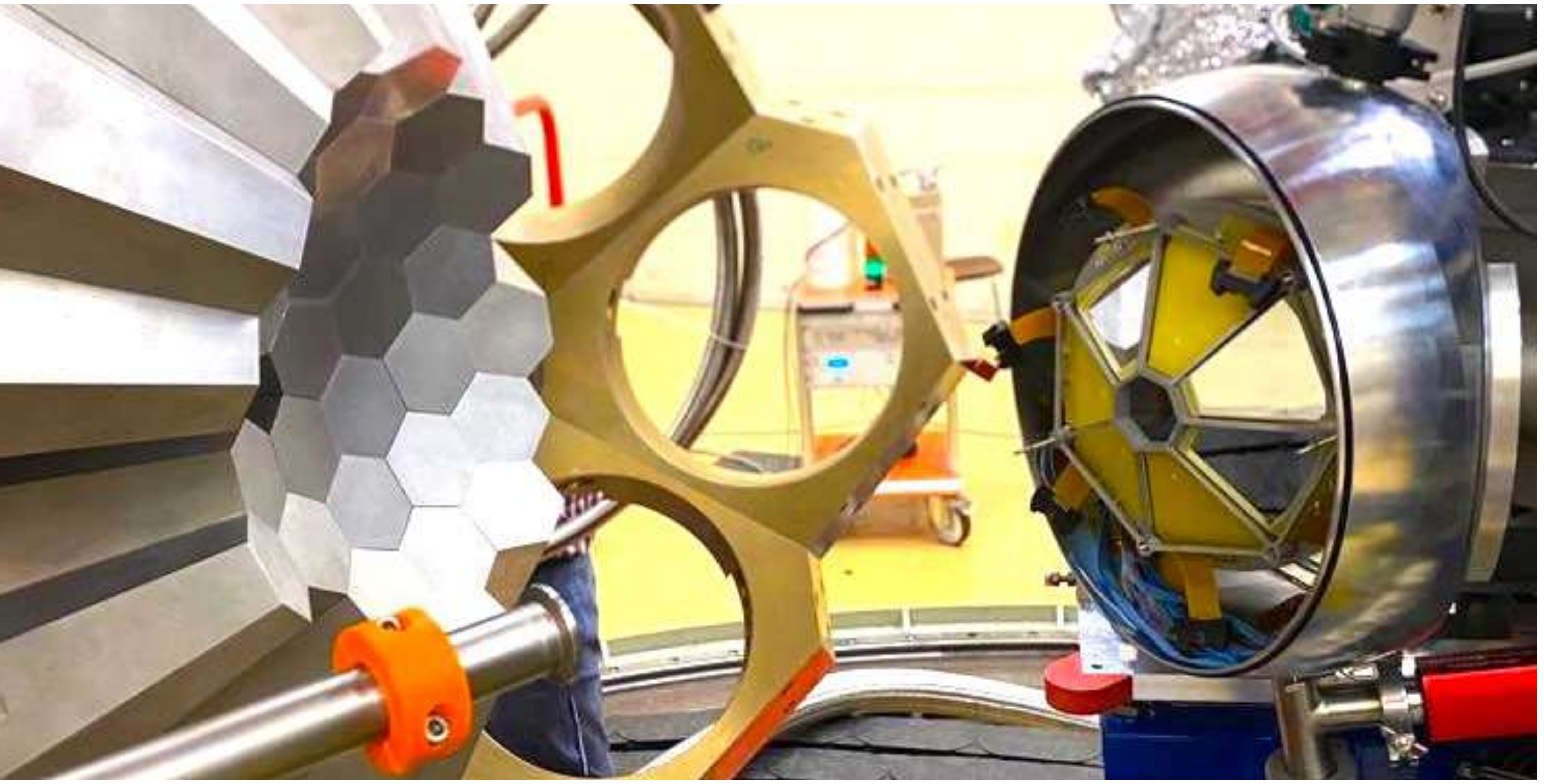
Summary



Low-energy Coulomb excitation at LNL

- Well-established activity which involves many users and institutions
- Currently exploiting AGATA and SPIDER with stable beams
- New possibilities will be offered soon by SPES



Coulex of ^{94}Zr

- A high-statistics Coulomb-excitation experiment was performed at LNL
- 17 transitional matrix elements with relative signs and 2 diagonal matrix elements with absolute signs extracted
- Model-independent determination of spherical-oblate shape coexistence from the obtained quadrupole moments and rotational invariants

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FIRENZE DIVISION

THANK YOU FOR THE ATTENTION