Shape isomers and possible shape-coexistences in Pt, Hg and Pb isotopes with N $\leq$ 126

Krzysztof Pomorski, Uniwersytet MCS, Lublin

Międzygórze, May 28th, 2020

My first Thursday seminar at Hoża *

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THE SEARCH FOR THE COMMON SYMMETRY OF PAIRING+ +QUADRUPOLE FORCES IN NUCLEAR THEORY

BY K. POMORSKI AND S. SZPIKOWSKI

Department of Theoretical Physics, M. Curie-Skłodowska University, Lublin*

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Pairing forces connected with the R_3 group and quadrupole forces with the SU_3 group were taken together to generate the common symmetry group. It has been proved that the resulting group is the symplectic group in $(N+1)(N+2)$ dimension, where N is the major shell number. The special case of $Sp(6)$ for $N = 1$ is discussed in detail.

* Warszawa, October 31st, 1968.

Results presented here were obtained in collaboration with:

- Johann Bartel, IPHC & UdS, Strasbourg, France,
- Artur Dobrowolski, UMCS, Lublin,
- Herve Molique, IPHC & UdS, Strasbourg, France,
- Bożena Nerlo-Pomorska, UMCS, Lublin,
- Costel M. Petrache, CSNSM, CNRS-IN2P3, Orsay, France
- Christelle Schmitt, IPHC & UdS, Strasbourg, France,

and has been published in:

– K.P., B.N.P., A. Dobrowolski, J. Bartel, C. M. Petrache, EPJA **56**, 107, (2020),
– K.P., B.N.P, J. Bartel, H. Molique, Bulg. J. of Physics, **46**, 269 (2019).
– K.P., B.N.P., J. Bartel, C. Schmitt, EPJA **53**, 59 (2017).
– C. Schmitt, K.P., B.N.P., J. Bartel, Phys. Rev. C **95**, 034612 (2017).

Program of my presentation:

- Short description of the theoretical model,
- Fourier expansion of nuclear shapes,
- New description of non-axial shapes of deformed nuclei,
- Test of convergence of the Fourier expansion,
- Potential energy surfaces of $^{166-204}$Pt isotopes,
- Potential energy surfaces of $^{172-212}$Hg isotopes,
- Potential energy surfaces of $^{174-220}$Pb isotopes,
- Quadrupole moments and moments of inertia in the g.s. and SD minima,
- Summary and conclusions.

Theoretical model:

- Macroscopic-microscopic approximation of nuclear energy,
- Lublin-Strasbourg-Drop [K.P., J. Dudek, PRC **67**, 044316 (2003)],
- Yukawa-folded single-particle potential [K.T.R. Davies, J.R. Nix, PRC **14**, 1977 (1976)],
- Strutinsky shell-correction method with the 6^{th} order corr. polynomial,
- BCS theory, monopole pairing, approximate particle number projection,
- Fourier parametrisation of nuclear shapes [K.P. et al. APPB Sup. **8**, 667 (2015)],
- Considered shapes: non-axial, quadrupole, octupole, hexadecapole, and higher,
- Moment of inertia evaluated within the cranking model,
- Calculations are preformed for 54 even-even isotopes of Pt, Hg and Pb.

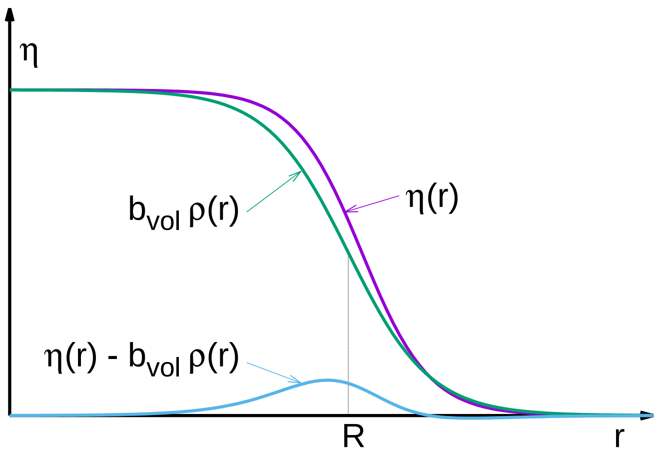
All parameters of the calculation are standard and fixed years ago.

Nuclear energy functional and the liquid drop model

The one-body density of nucleus and the corresponding energy-density are:

$$\rho = A \int\int \dots \int \Psi^\star \Psi \, d\tau_2 \cdots d\tau_A \,,$$

$$\eta = \int\int \dots \int \Psi^\star \hat{H} \Psi \, d\tau_2 \cdots d\tau_A \,.$$



The total energy (B) of nucleus can be decomposed in the following way:

$$B = \int_V \eta \, d^3\mathrm{r} = b_{\mathrm{vol}} A + \int_{\Sigma_R} d\sigma \int_0^\infty (\eta - b_{\mathrm{vol}}\,\rho)\, dr_\perp$$

$$= b_{\mathrm{vol}} \int_V d^3\mathrm{r} + \gamma^{(0)} \int_{\Sigma_R} d\sigma + \gamma'_\kappa a \int_{\Sigma_R} \kappa d\sigma + \frac{1}{2}\gamma''_{\kappa\kappa} a^2 \int_{\Sigma_R} \kappa^2 d\sigma + \gamma'_\Gamma a^2 \int_{\Sigma_R} \Gamma d\sigma + \dots$$

$$= b_{\mathrm{vol}} A + b_{\mathrm{surf}} A^{2/3} + b_{\mathrm{curv}} A^{1/3} + b_{\mathrm{curG}} A^0 + \dots \longrightarrow \quad \mathrm{LSD} \,.$$

Here κ and Γ are the first order and the second order (Gauss) curvatures respectively:

$$\kappa = \frac{1}{R_1} + \frac{1}{R_2} \quad \text{and} \quad \Gamma = \frac{1}{R_1 \cdot R_2} \,,$$

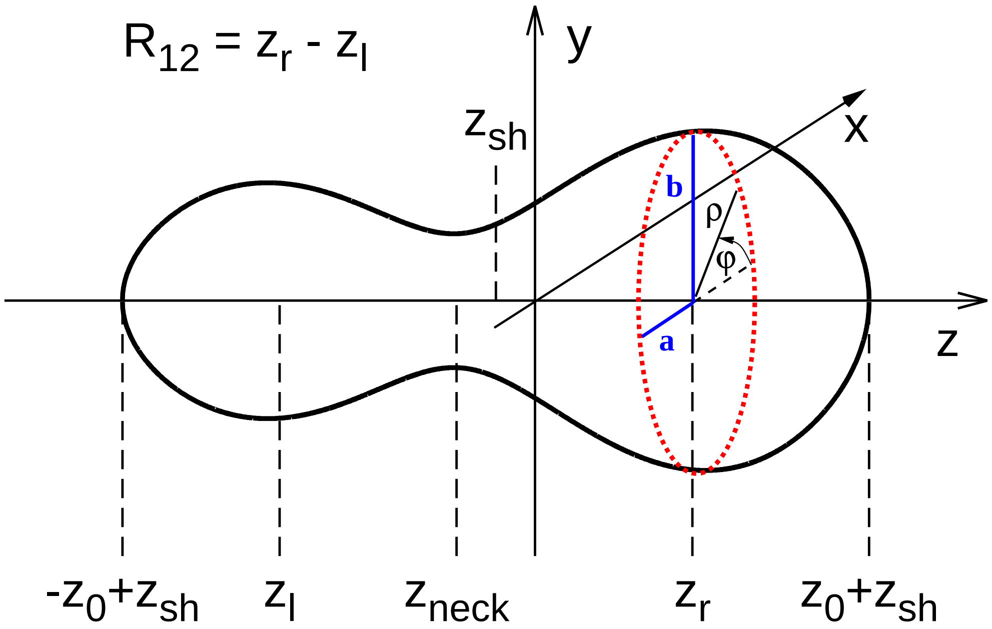
where R_1 and R_2 are the local main radii of the surface.

Fourier expansion of nuclear shapes *

The shape of nucleus in the cylindrical coordinates can be expressed as:

$$\frac{\rho_s^2(z)}{R_0^2} = \sum_{n=1}^{\infty} \left[a_{2n} \cos\left(\frac{(2n-1)\pi}{2} \frac{z - z_{sh}}{z_0} \right) + a_{2n+1} \sin\left(\frac{2n\pi}{2} \frac{z - z_{sh}}{z_0} \right) \right] ,$$

Here R_0 is radius of spherical nucleus and $2z_0$ is length of deformed nucleus.



Optimal coordinates:

$$\begin{cases} q_2 = a_2^{(0)}/a_2 - a_2/a_2^{(0)} , \\ q_3 = a_3 , \\ q_4 = a_4 + \sqrt{(q_2/9)^2 + (a_4^{(0)})^2} , \\ q_5 = a_5 - (q_2 - 2)a_3/10 , \\ q_6 = a_6 - \sqrt{(q_2/100)^2 + (a_6^{(0)})^2} . \end{cases}$$

$a_{2n}^{(0)} = (-1)^{n-1}\frac{32}{\pi^3 (2n-1)^3}$ - expansion coefficients of a sphere.

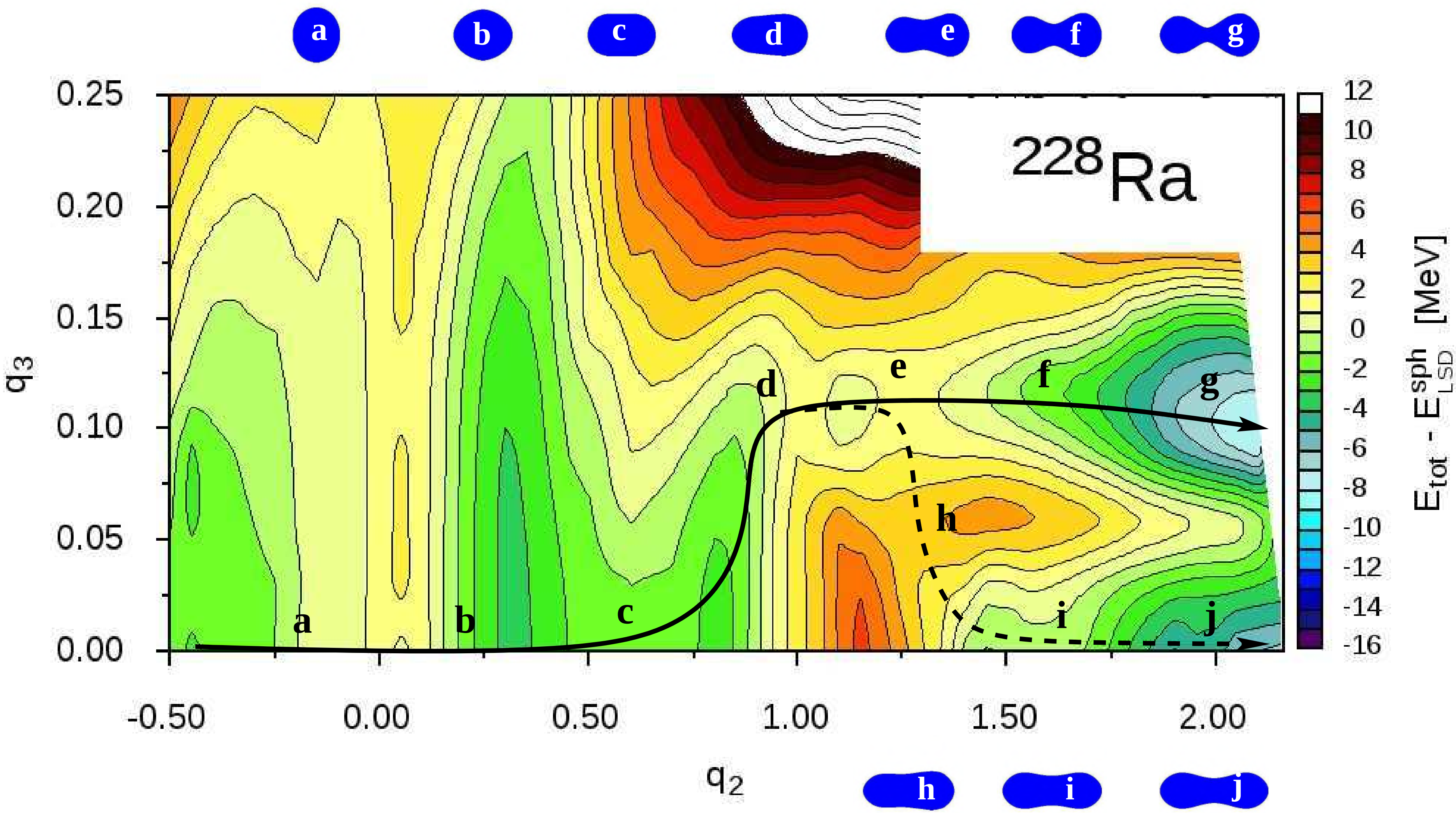
Non-axial shapes:

$\eta = (b-a)/(a+b)$

$\rho_s^2(z) = a(z)b(z)$

*K. Pomorski, B. Nerlo-Pomorska, J. Bartel, and C. Schmitt Acta Phys. Pol. B Supl. **8** (2015) 667,
C. Schmitt, K. Pomorski, B. Nerlo-Pomorska, and J. Bartel, Phys. Rev. C **95** (2017) 034612.

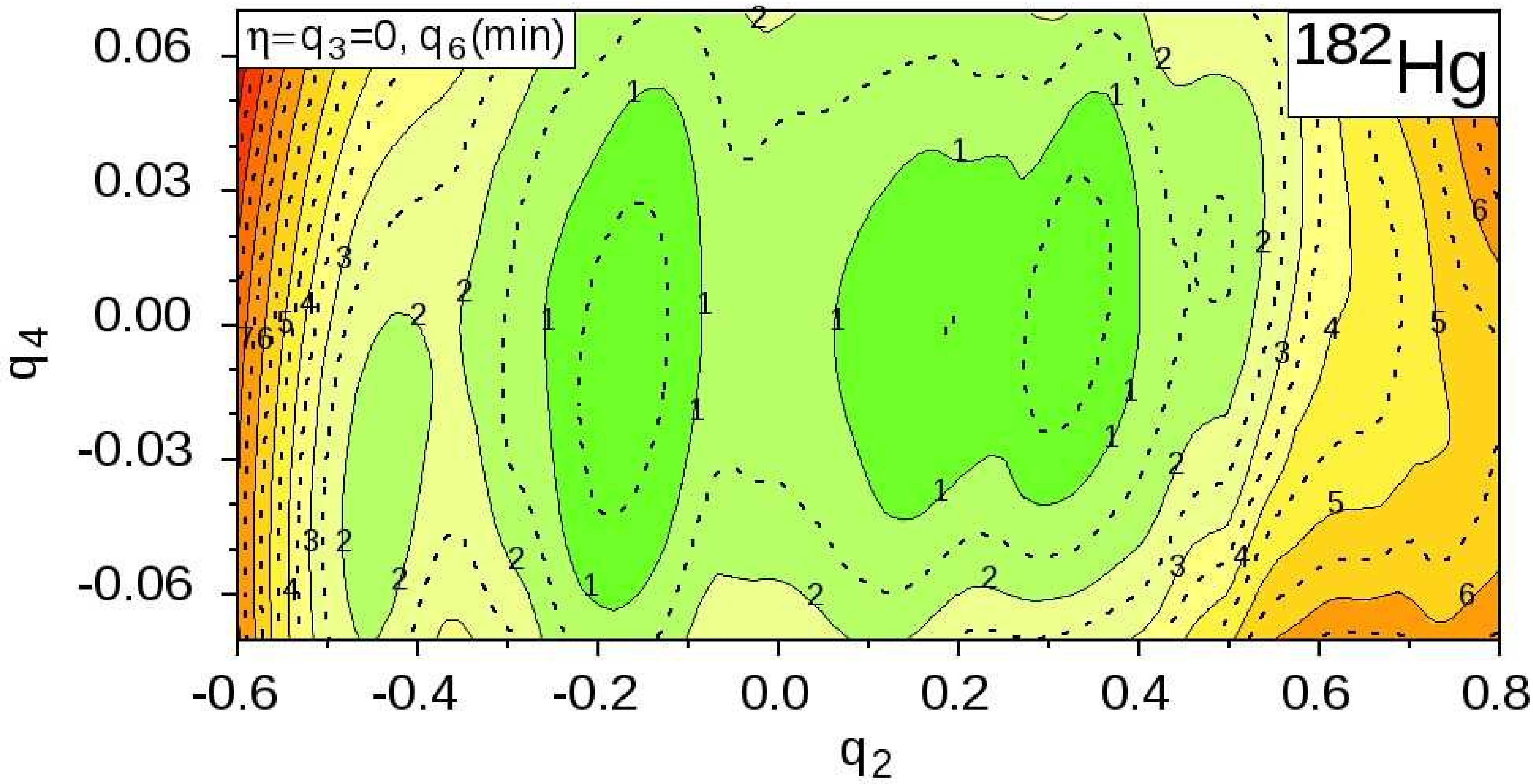
Potential energy surface of ^{228}Ra on the (q_2, q_3) plane*



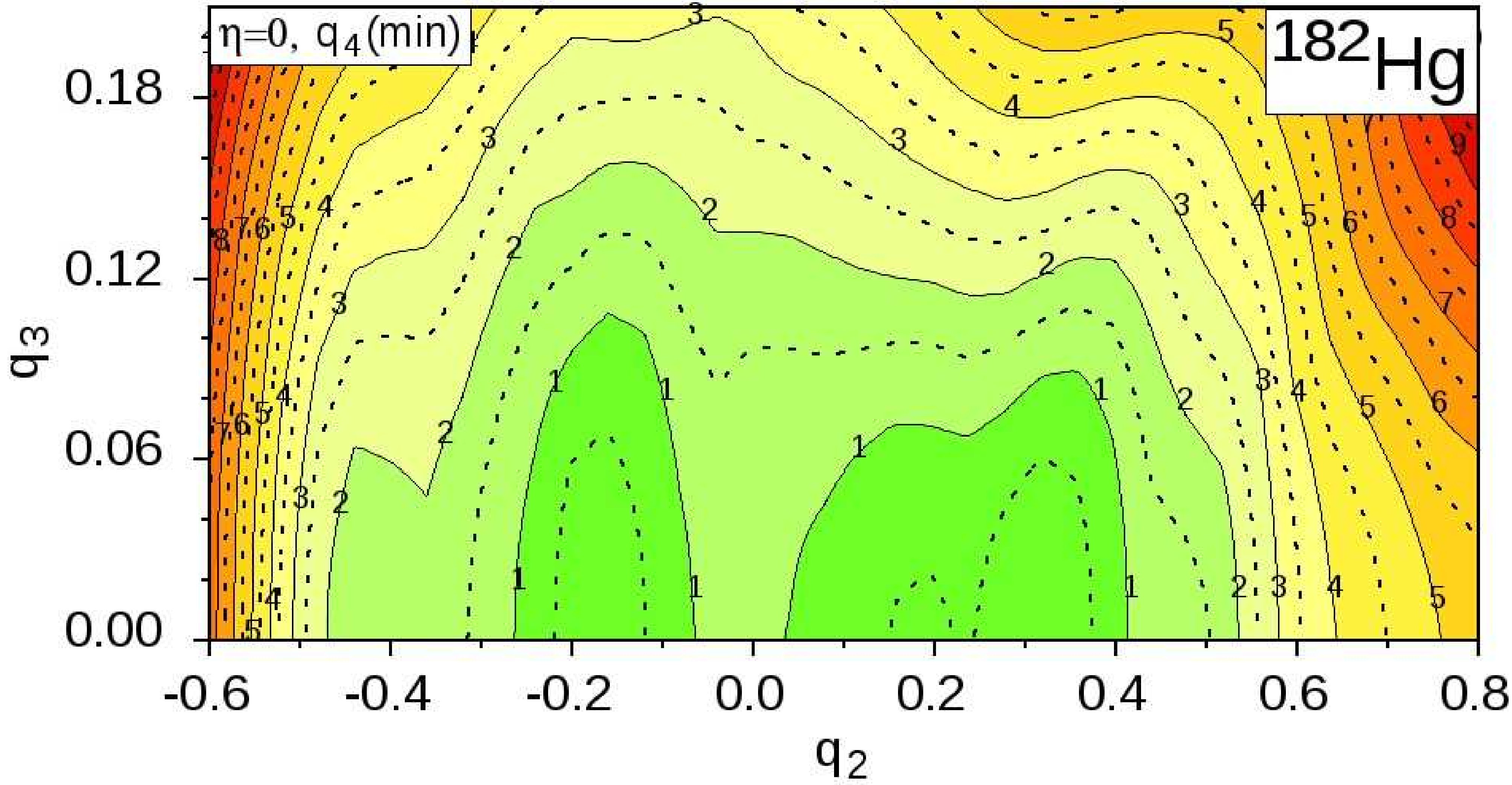
Here: q_2 – elongation ; q_3 – left-right asymmetry

*C. Schmitt, K. Pomorski, B. Nerlo-Pomorska, and J. Bartel, Phys. Rev. C **95** (2017) 034612.

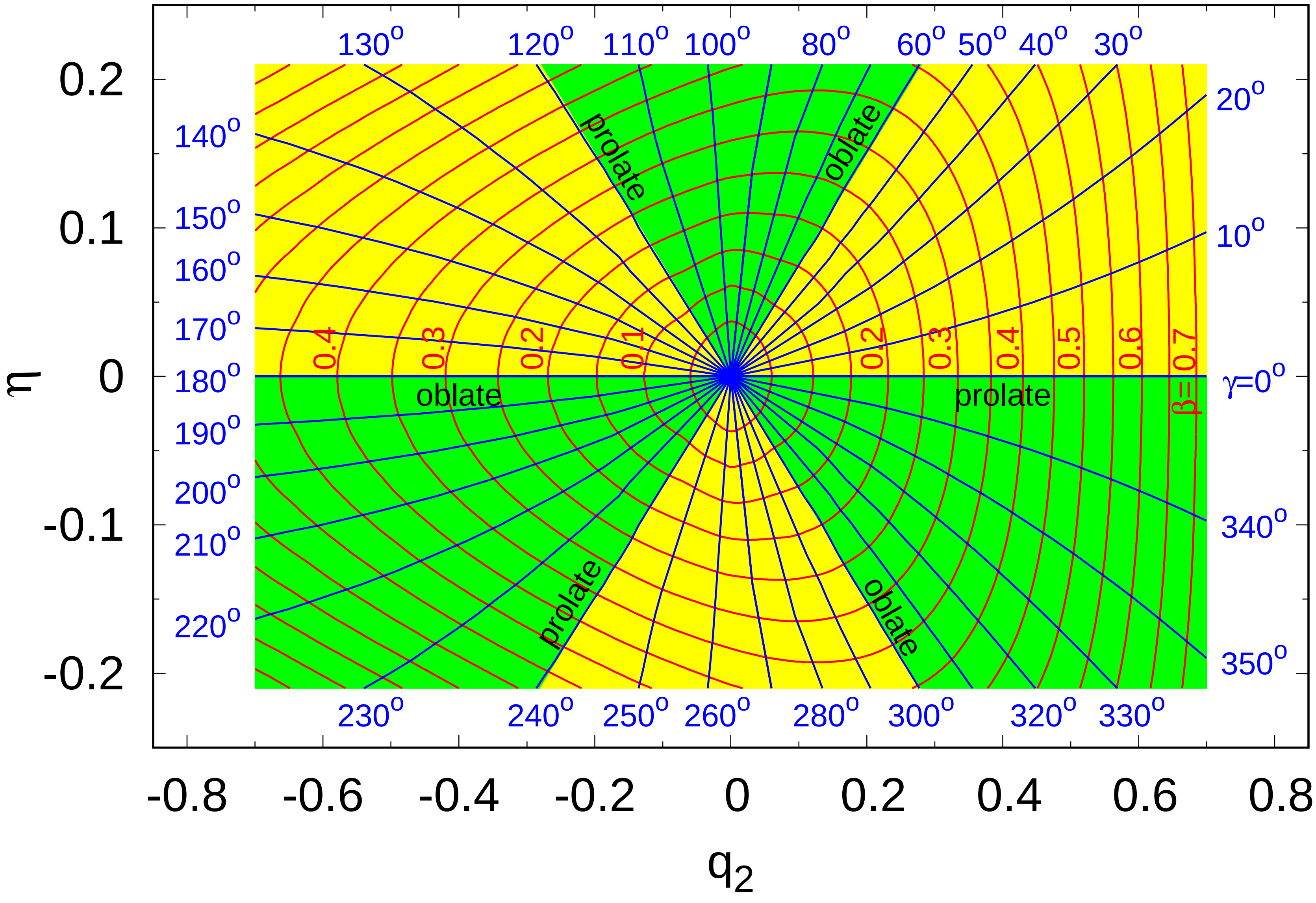
Potential energy surface of ^{182}Hg on the (q_2, q_4) plane



Potential energy surface of ^{182}Hg on the (q_2, q_3) plane

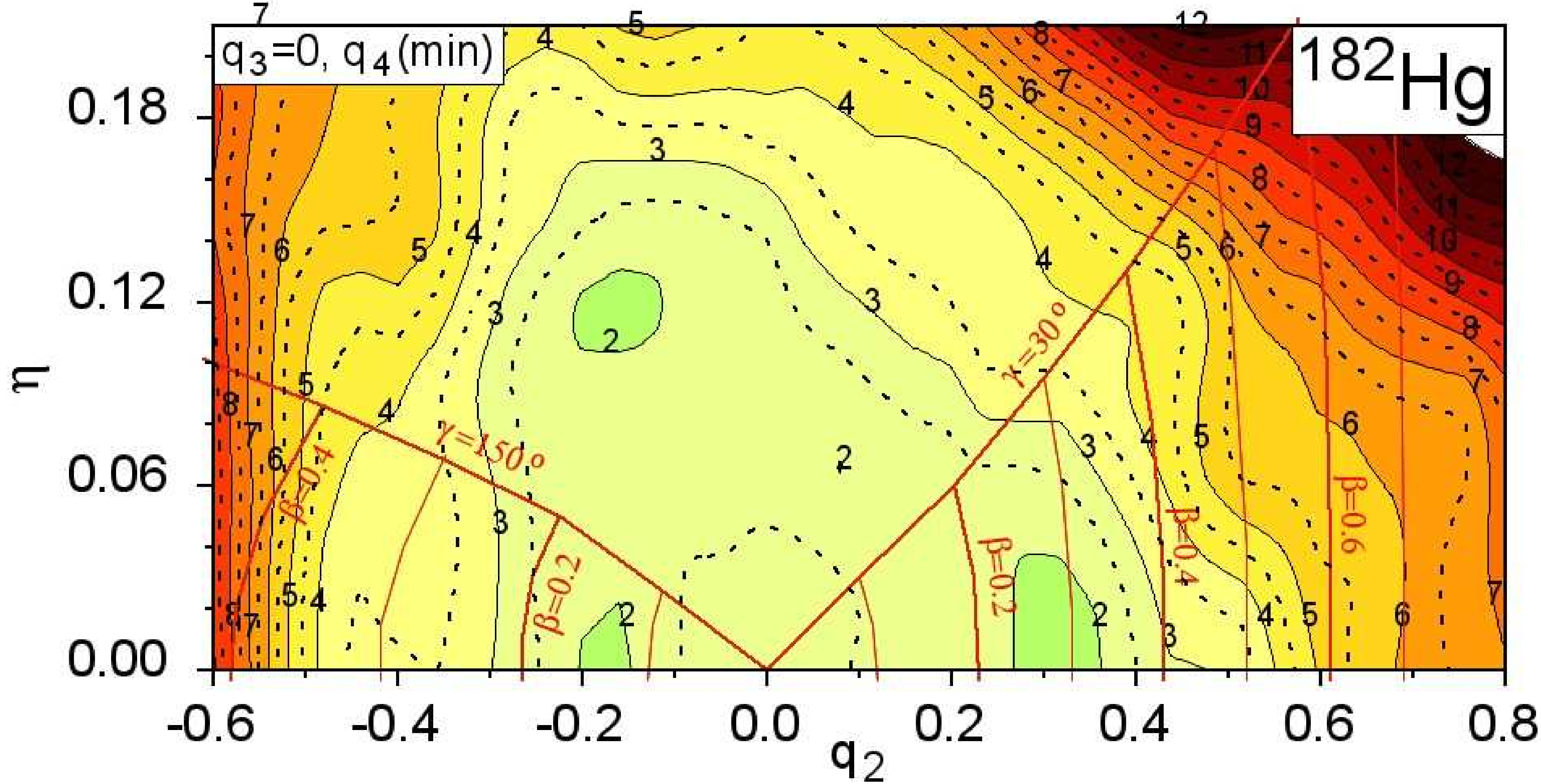


Relation between (q_2, η) and (β, γ) for spheroid

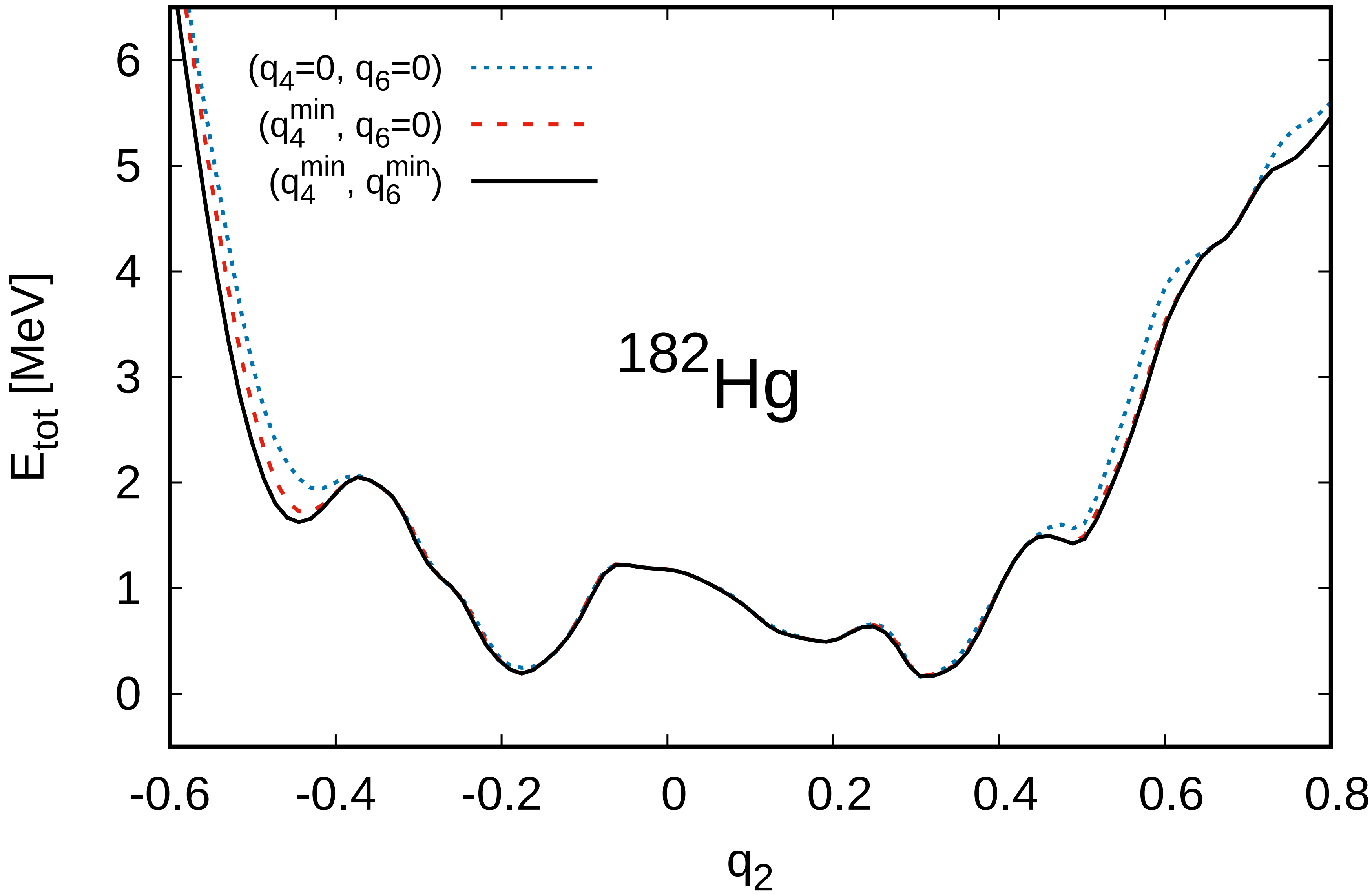


Here $\eta = (b - a)/(a + b)$ and $q_2 \approx (c/R_0 - R_0/c)$, where $a \cdot b \cdot c = R_0^3$.

Potential energy surface of ^{182}Hg on the (q_2, η) plane

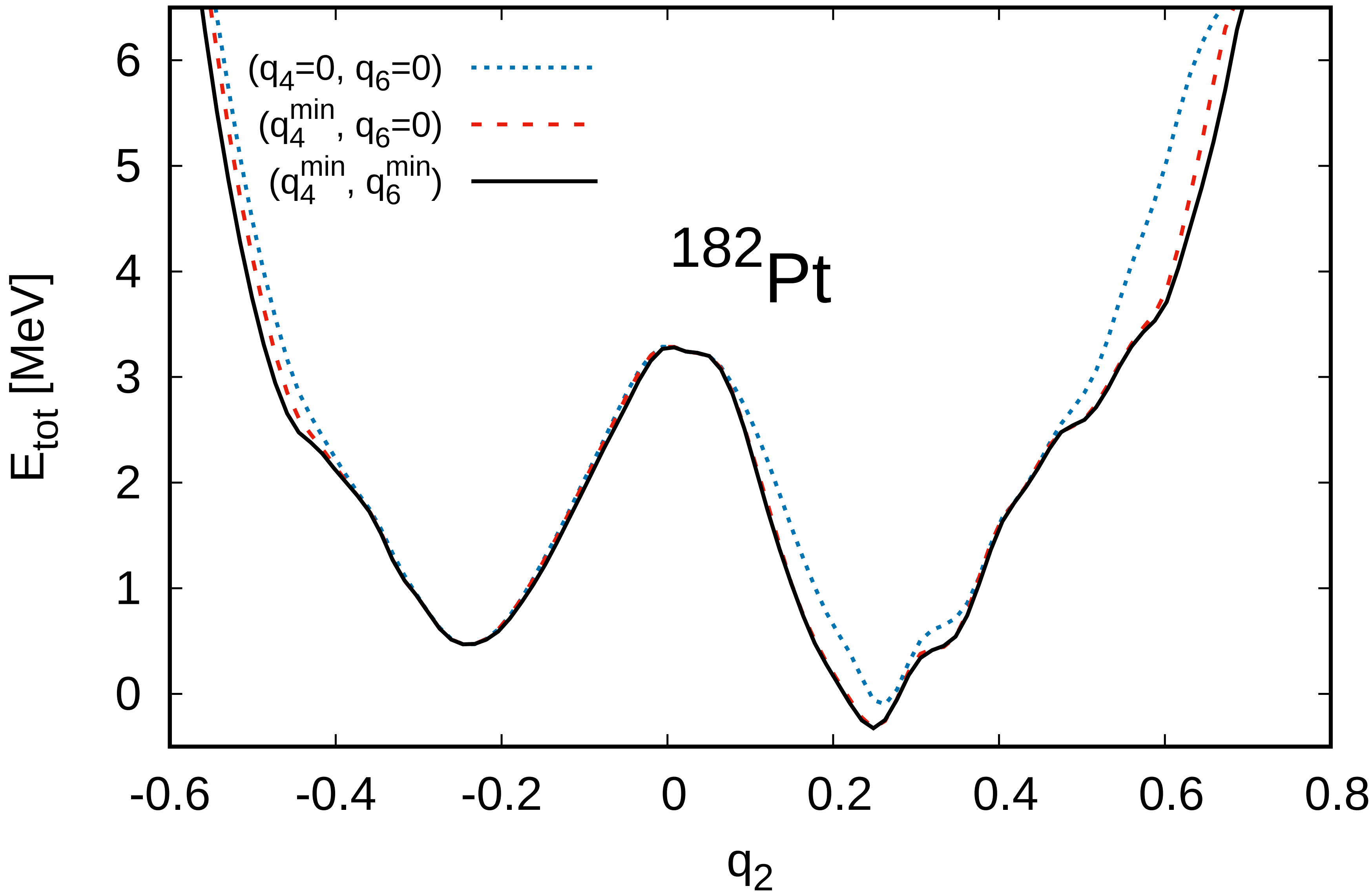


Role of higher order deformations

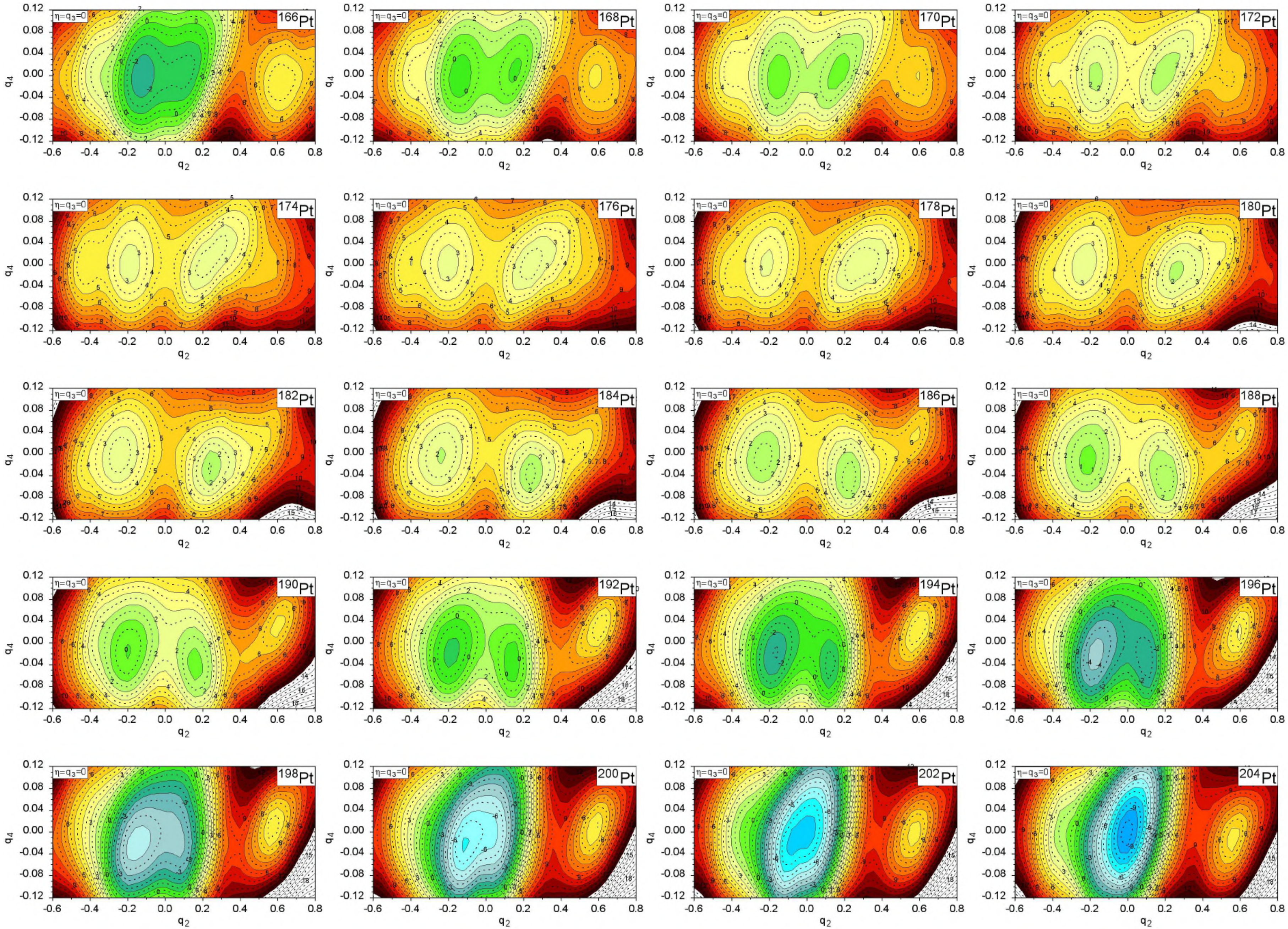


Similar effect is also observed in other isotopes.

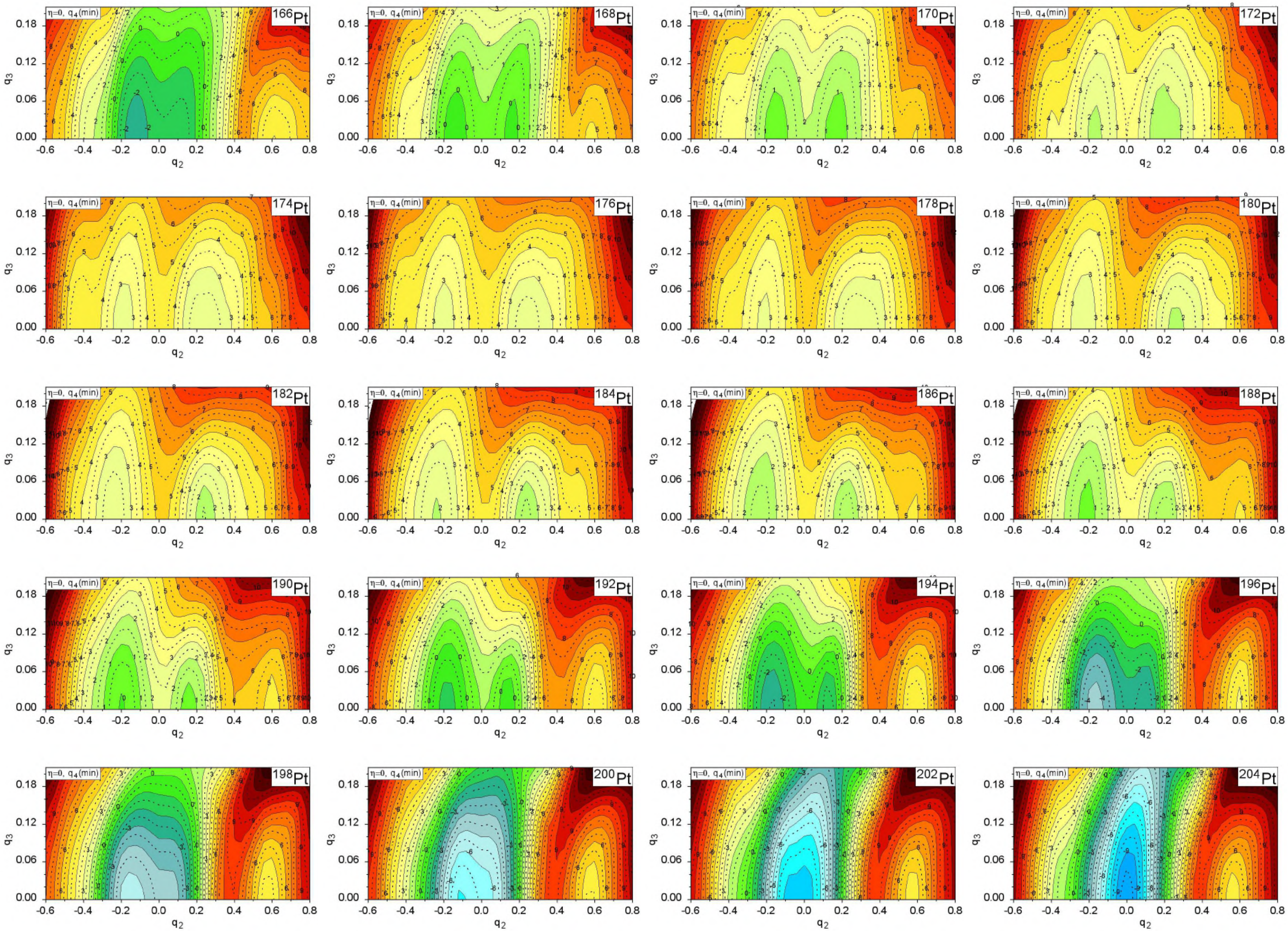
Role of higher order deformations



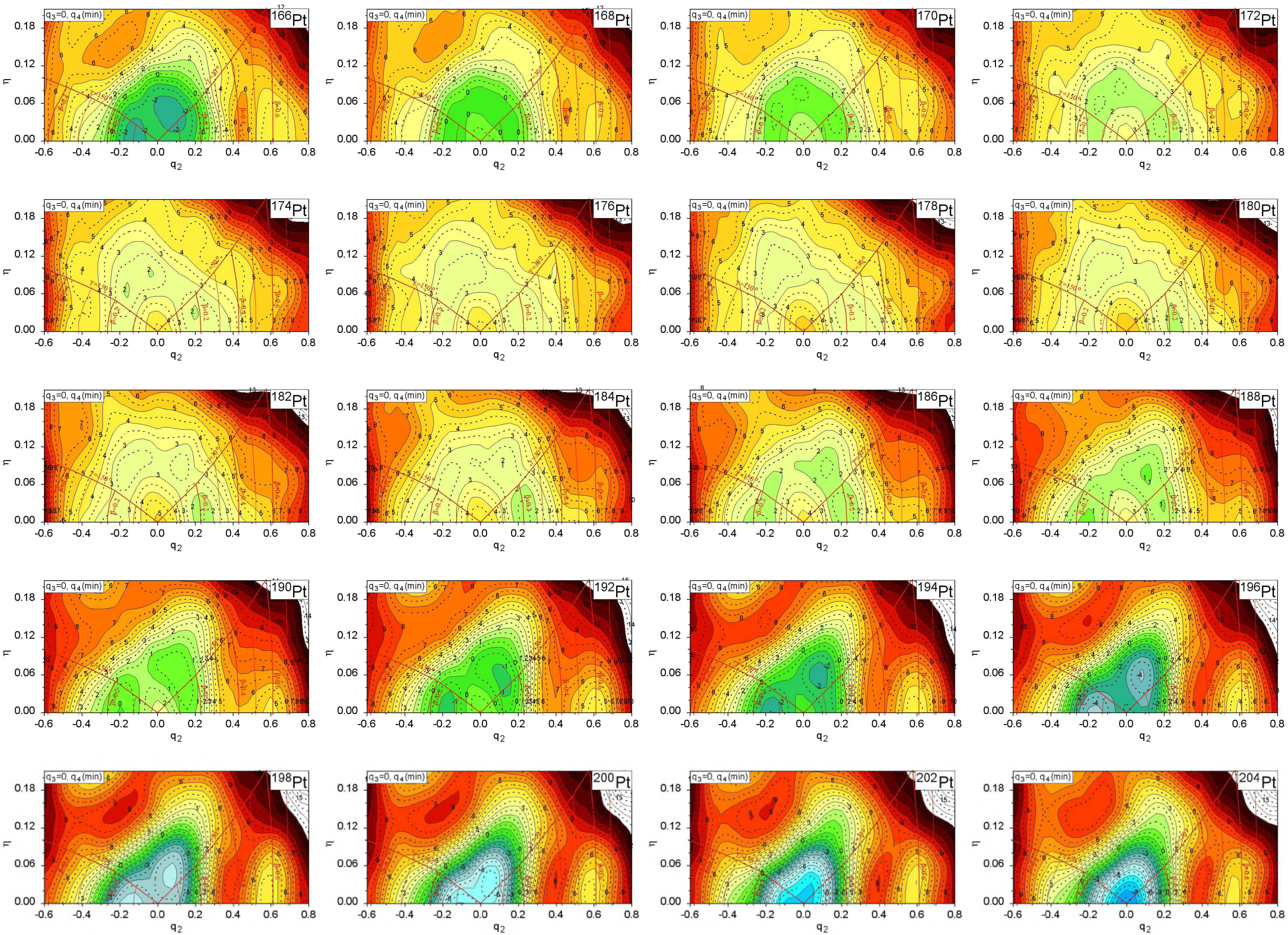
In all investigated cases the influence of higher than q_4 deformations is negligible in the vicinity of local minima.



Potential energy surface of $^{166-204}$Pt isotopes on the (q_2, q_4) plane, when $q_3 = 0$ and $\eta = 0$.

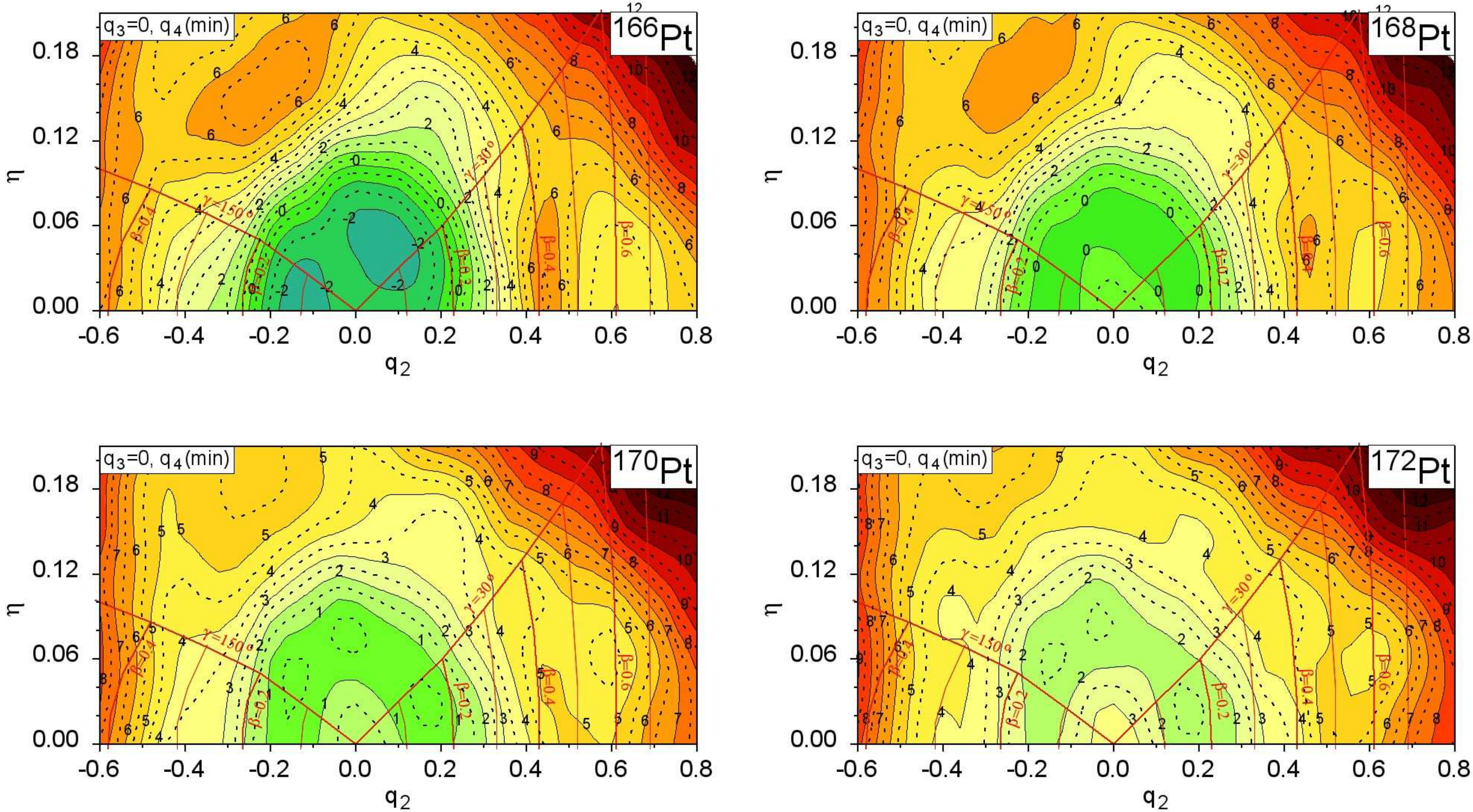


Potential energy surface of $^{166-204}$Pt isotopes on the (q_2, q_3) plane for $\eta = 0$ and $q_4(min)$.



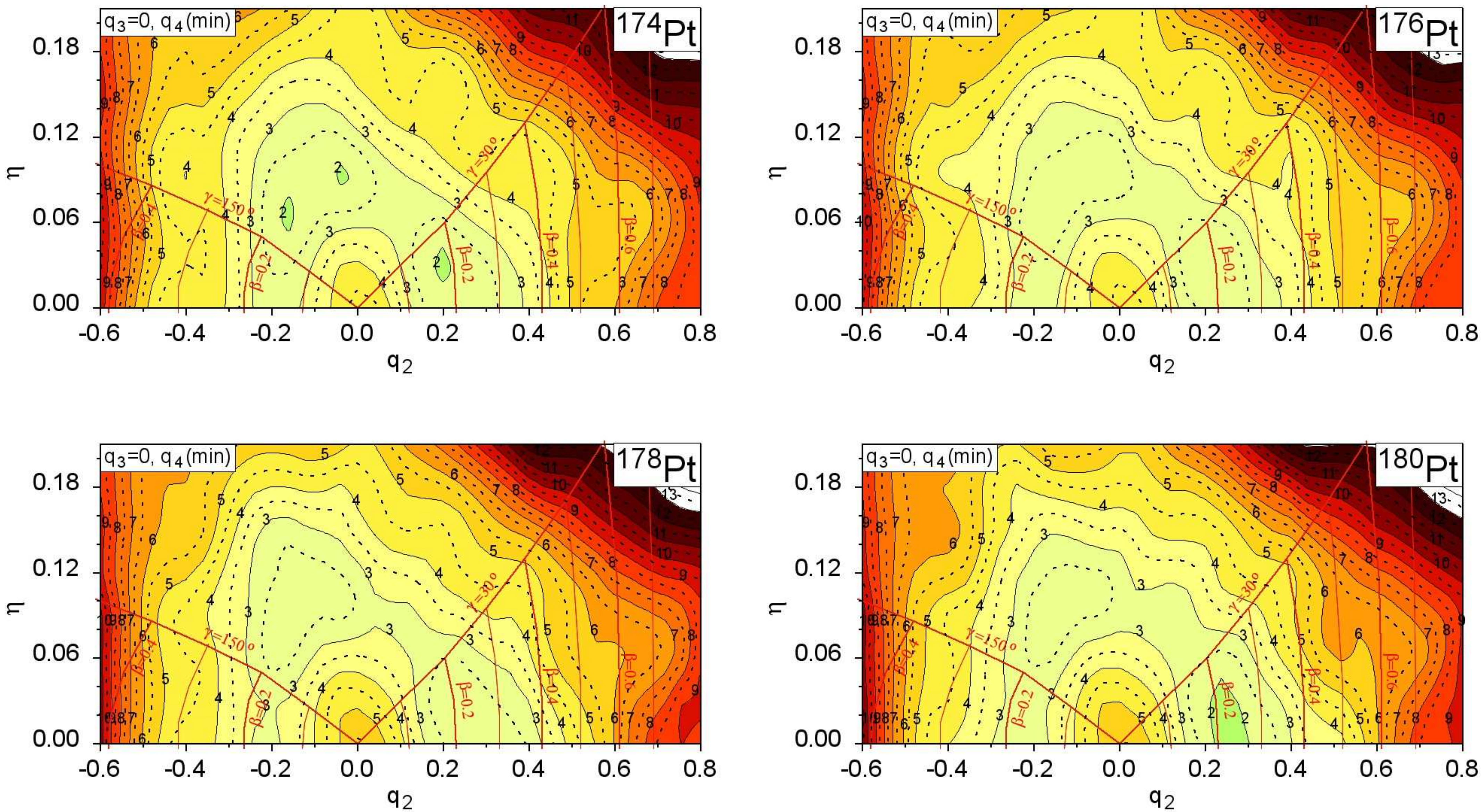
Potential energy surface of $^{166-204}$Pt isotopes on the (q_2, η) plane for $q_3 = 0$ and $q_4(min)$.

PES of $^{166-172}$Pt on the (q_2, η) plane minimized with respect q_3 and q_4



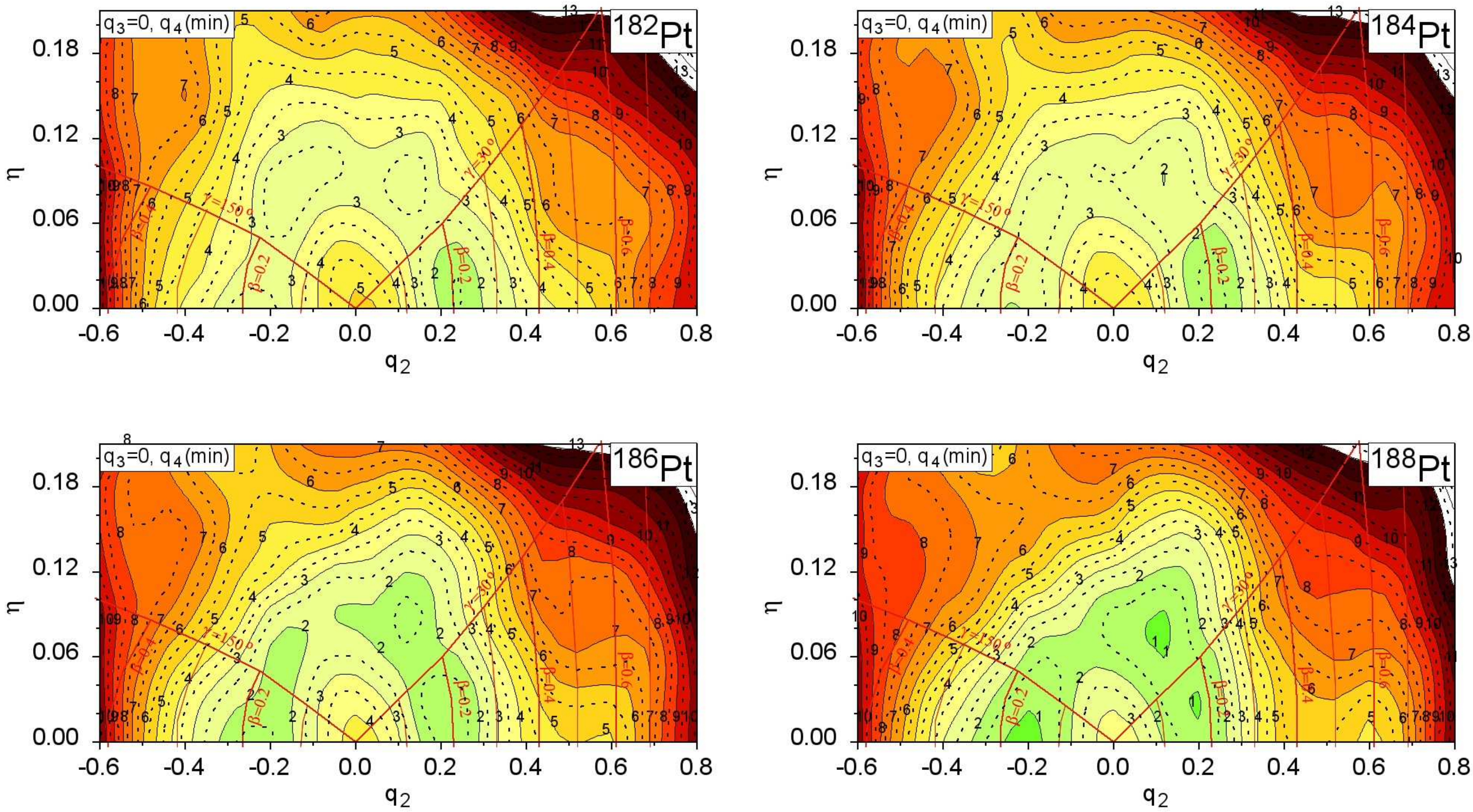
Notice: ^{170}Pt and ^{172}Pt are triaxial in the ground state.

PES of $^{174-180}$Pt on the (q_2, η) plane minimized with respect q_3 and q_4



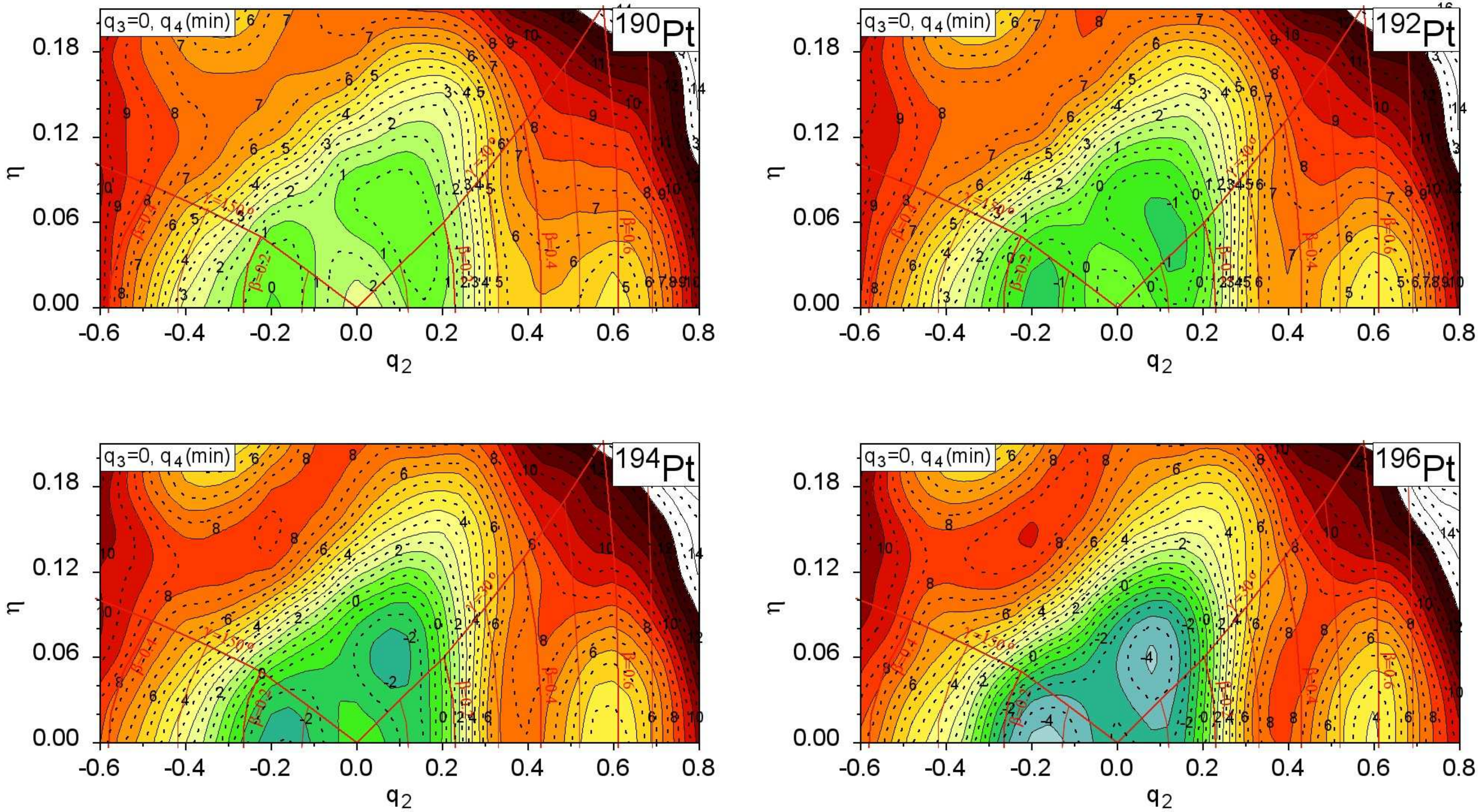
Notice: ^{174}Pt is triaxial in the ground state.

PES of $^{182-188}$Pt on the (q_2, η) plane minimized with respect q_3 and q_4



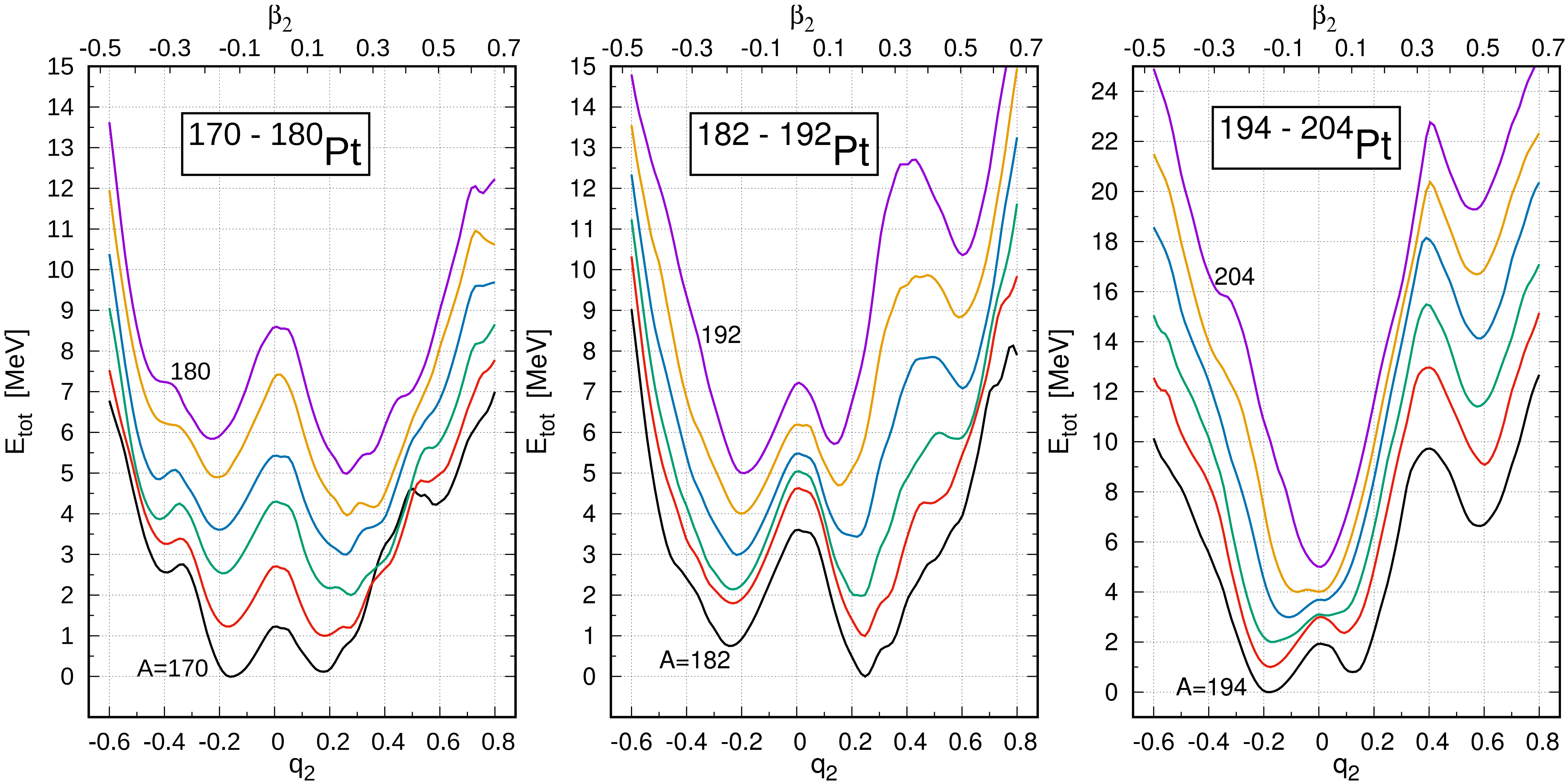
Notice: possible shape-coexistence in $^{182-188}$Pt and superdeformed isomers in $^{186-188}$Pt isotopes.

PES of $^{190-196}$Pt minimized with respect q_3 and q_4

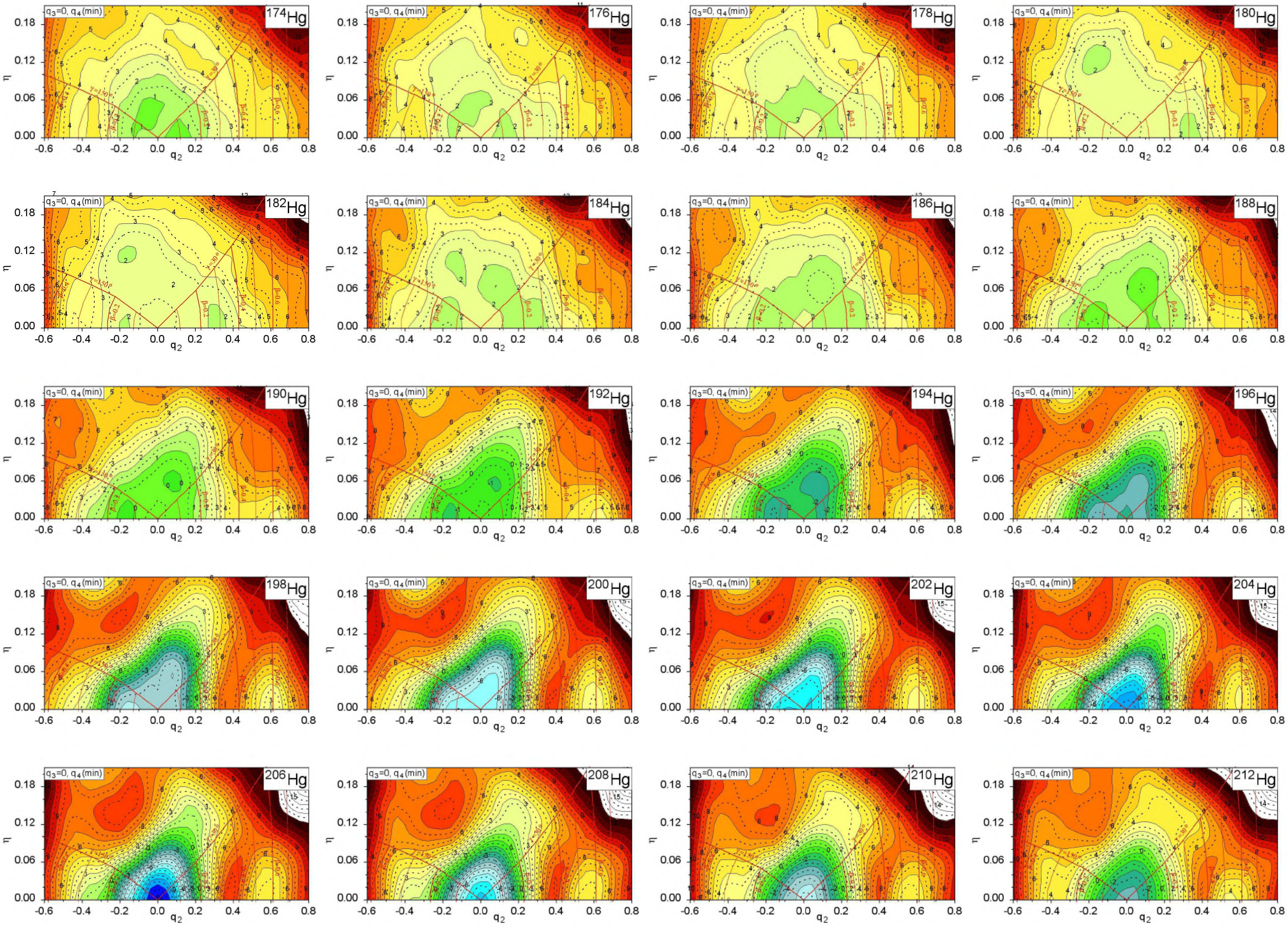


Notice: $^{190-192}$Pt isotopes are oblate in the ground states and have superdeformed isomers.

Potential energy of Pt isotopes minimized with respect of (q_3, q_4) as function of q_2

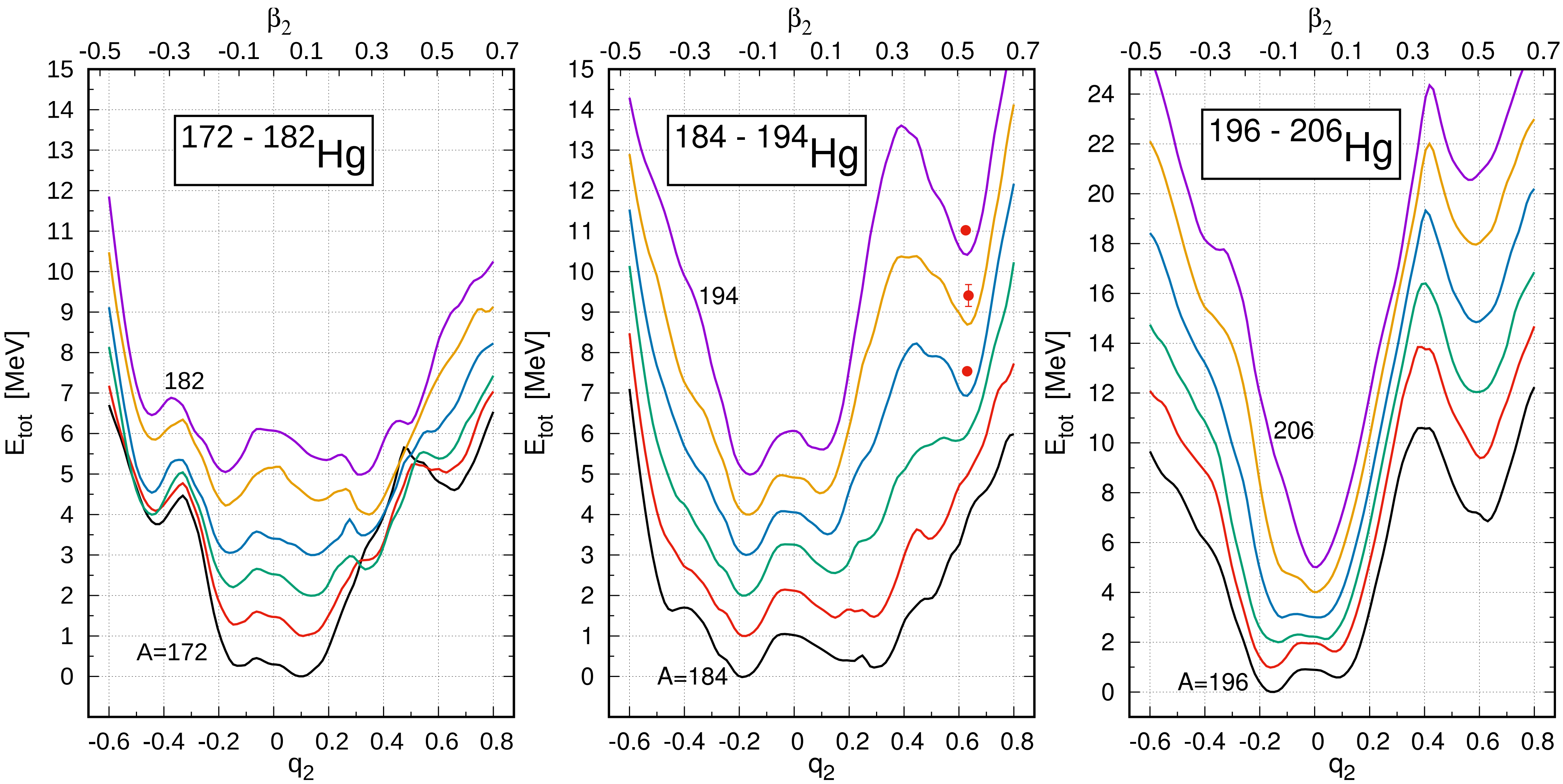


Here $\eta = 0$.



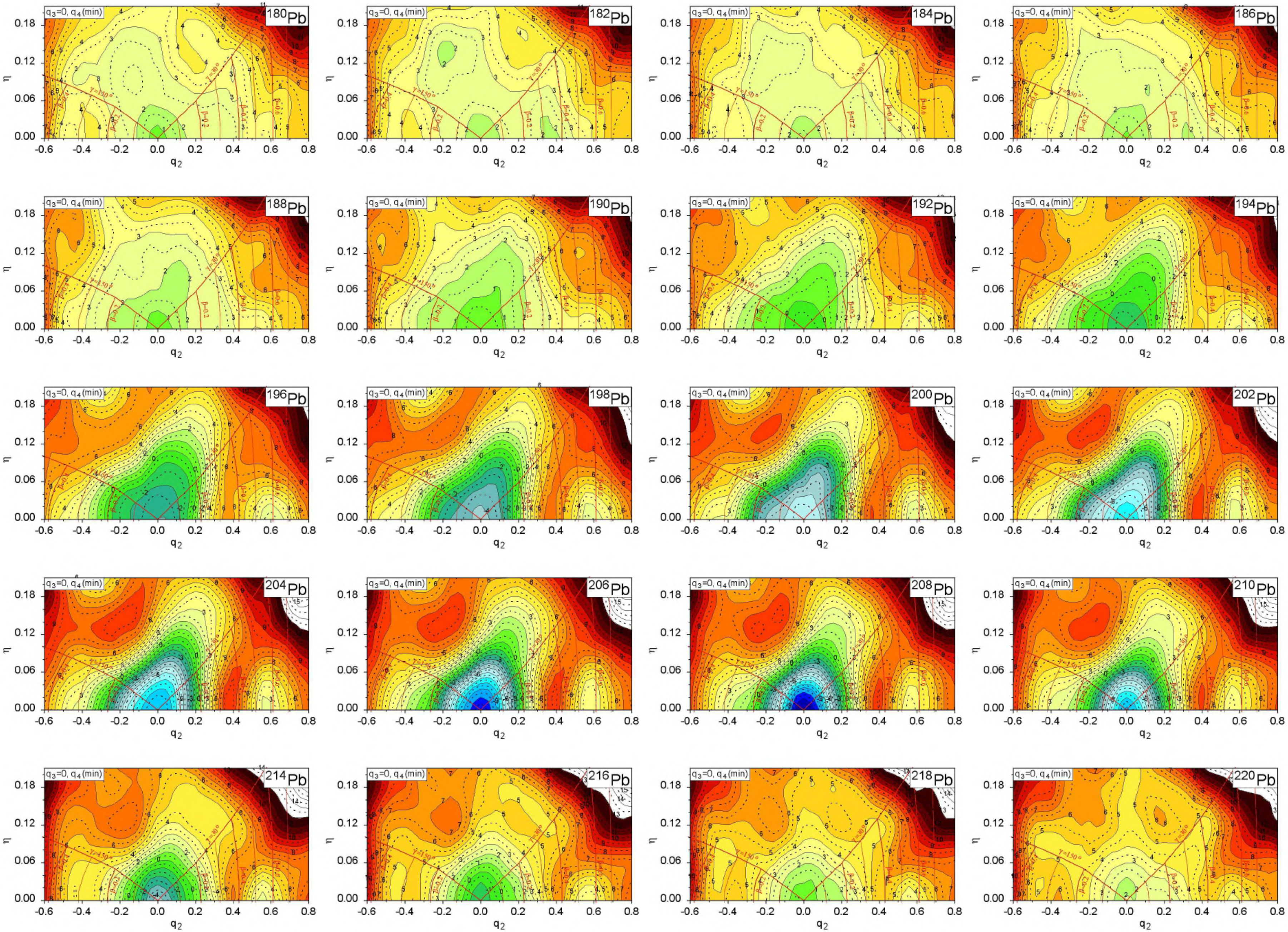
Potential energy surface of $^{174-212}$Hg isotopes on the (q_2, η) plane for $q_3 = 0$ and $q_4(min)$.

Potential energy of Hg isotopes minimized with respect of (q_3, q_4) as function of q_2



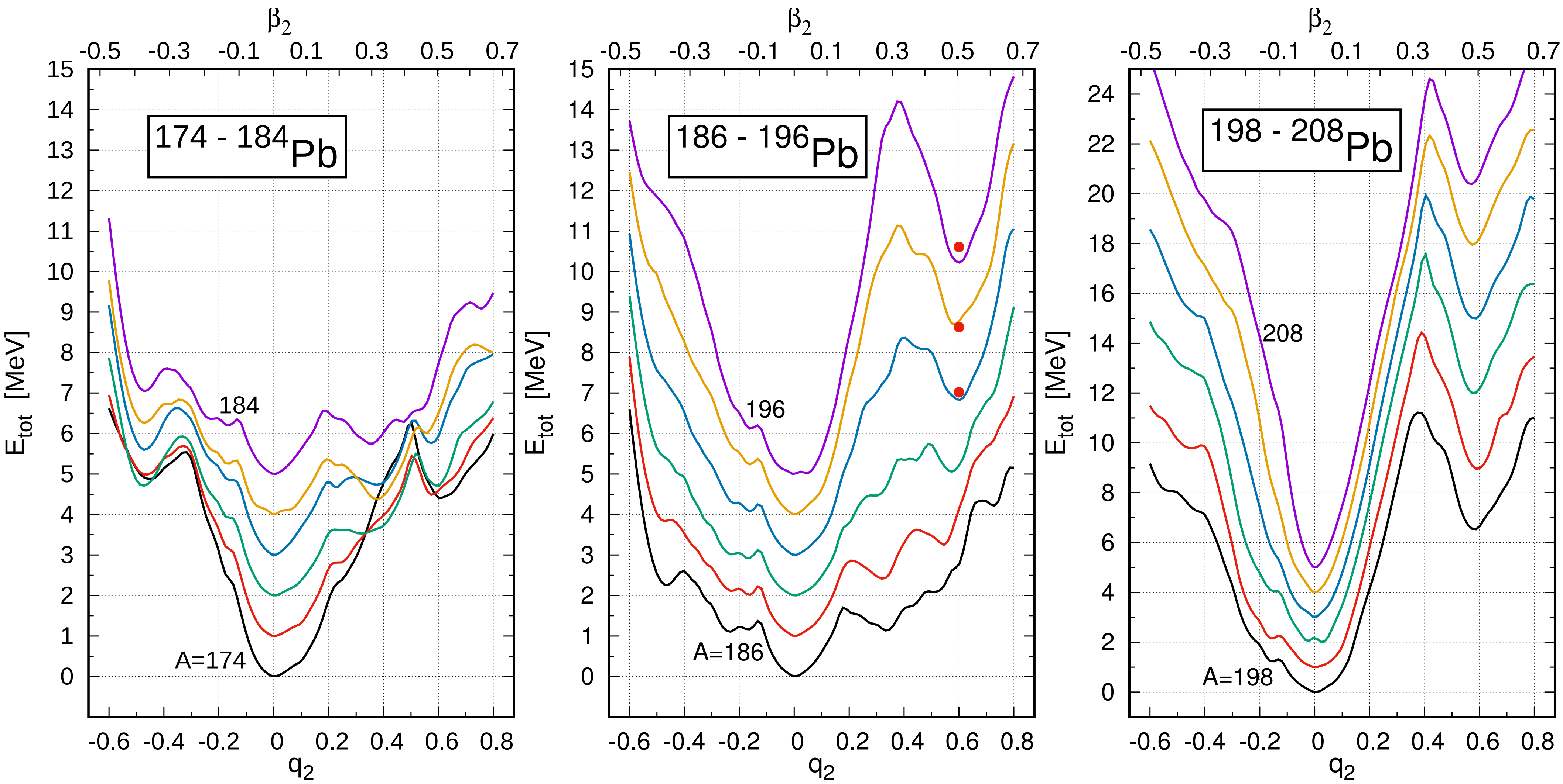
Here $\eta = 0$.

Exp. energies of the SD minima (●) are from [A. Lopez-Martens et al., Prog. Part. Nucl. Phys. **89**, 137 (2016).]



Potential energy surface of $^{180-218}$Pb isotopes on the (q_2, η) plane for $q_3 = 0$ and $q_4(min)$.

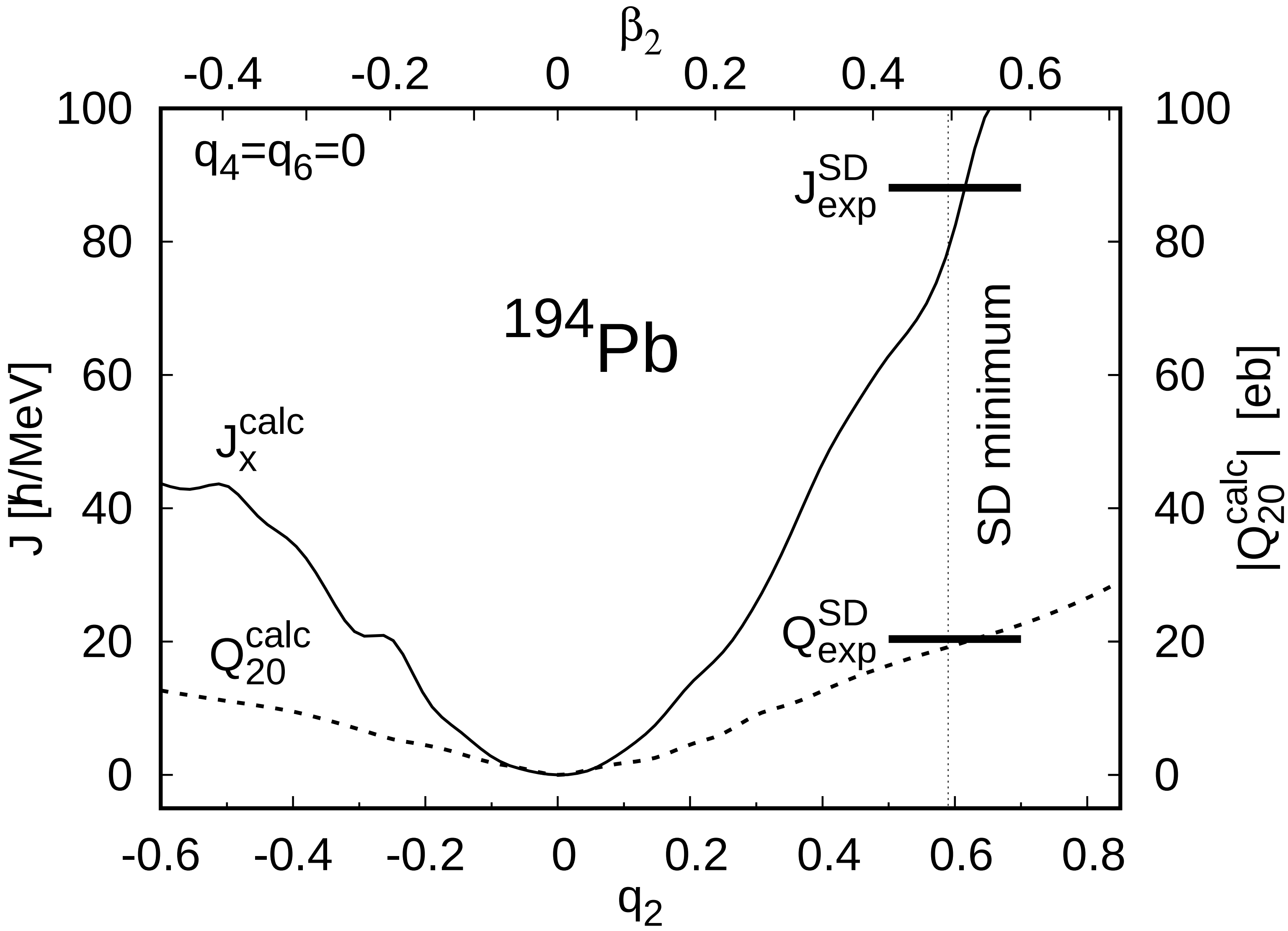
Potential energy of Pb isotopes minimized with respect of (q_3, q_4) as function of q_2



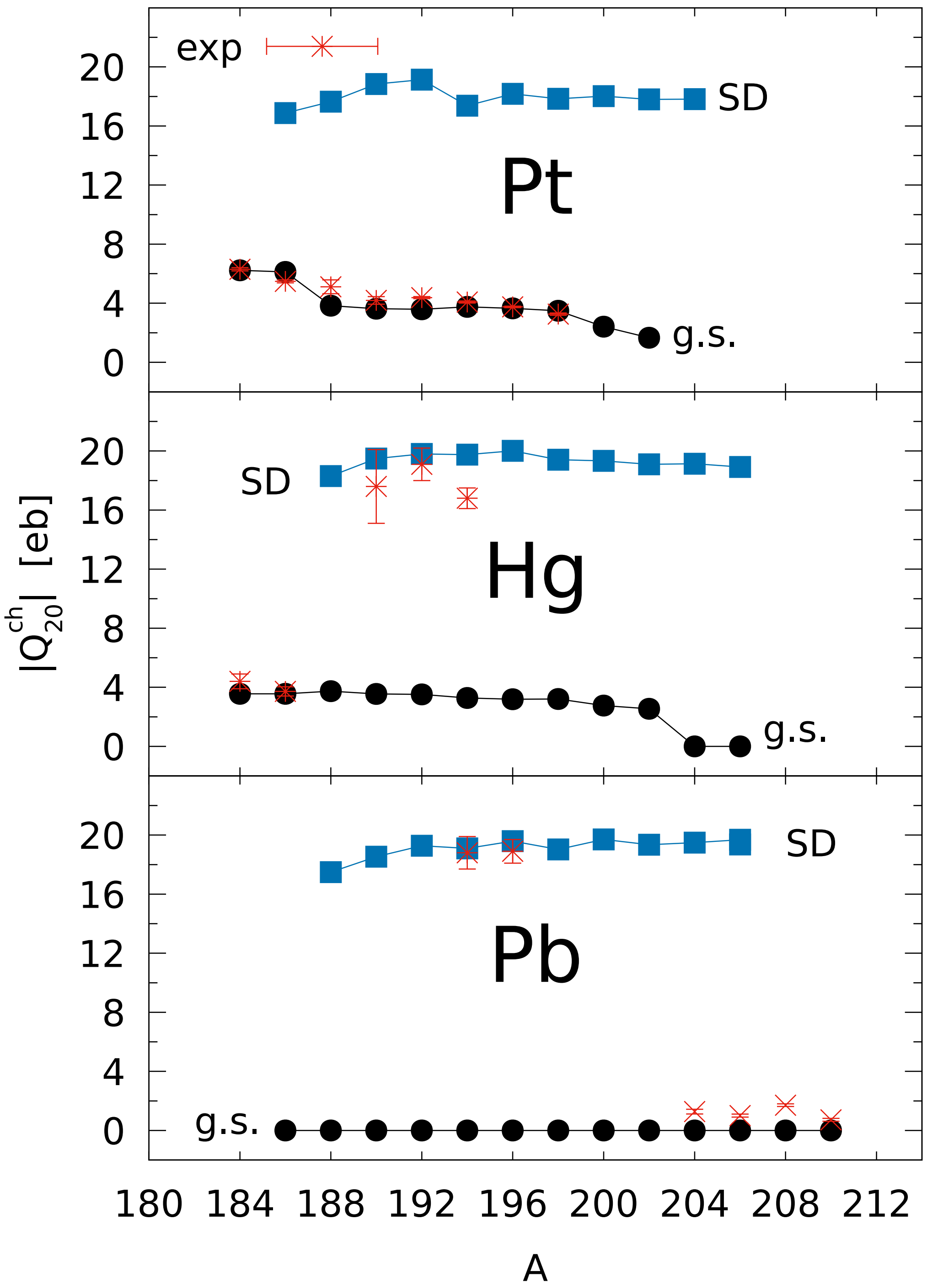
Here $\eta = 0$.

Exp. energies of the SD minima (●) are from [A. Lopez-Martens et al., Prog. Part. Nucl. Phys. **89**, 137 (2016).]

Quadrupole moment and moment of inertia of ^{194}Pb in the SD minimum



Exp. data a taken from [A. Lopez-Martens et al., Prog. Part. Nucl. Phys. **89**, 137 (2016).]



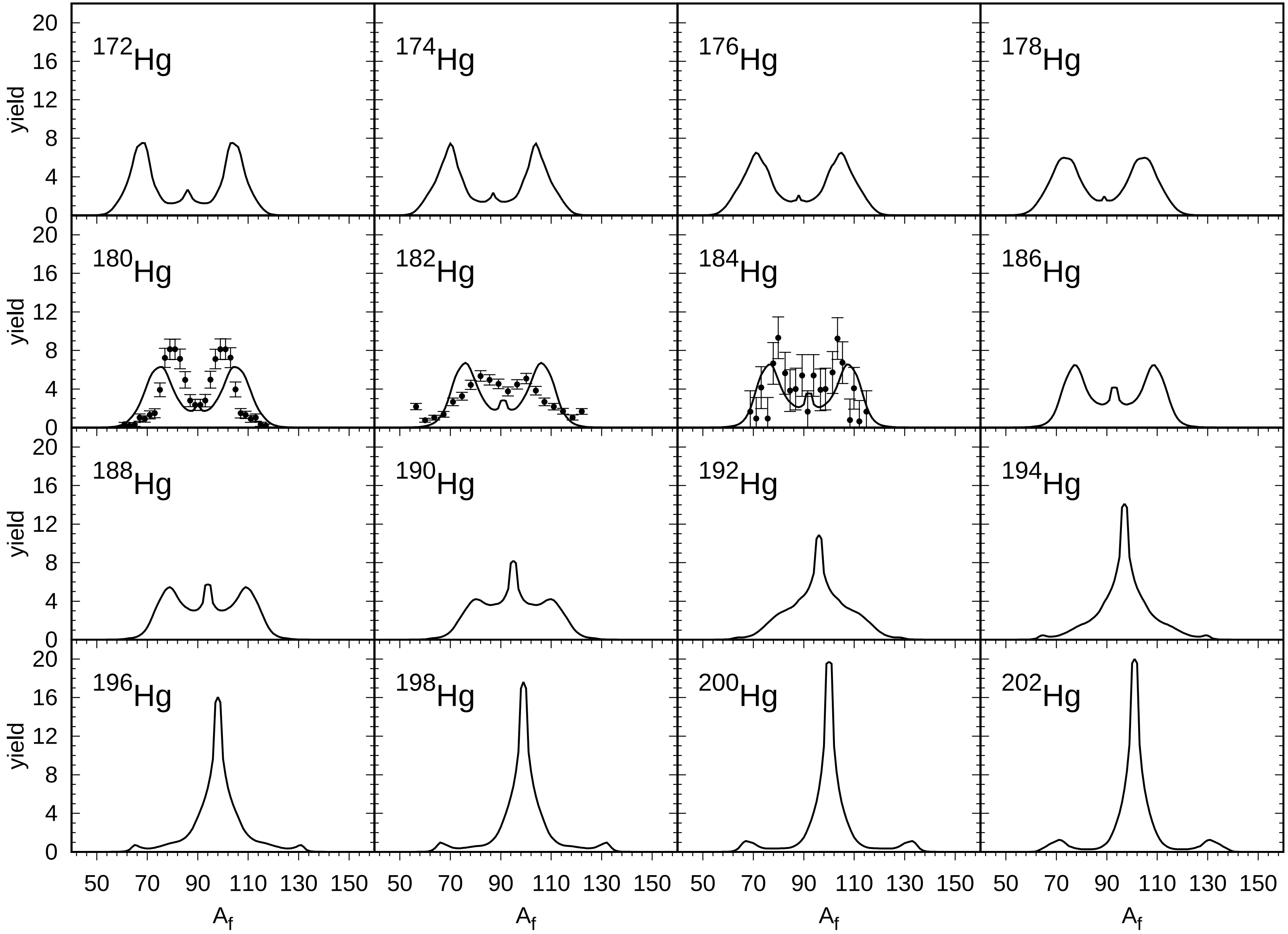
Electric quadrupole moments of Pt, Hg, Pb nuclei in the ground state (g.s.) and in the superdeformed (SD) minima

Experimental (*) data are taken from:

– A. Lopez-Martens, T. Lauritsen, S. Leoni, T. Dössing, T.L. Khoo, S. Siem, Prog. Part. Nucl. Phys. **89**, 137 (2016),

– B. Singh, R. Zywina, R.B. Firestone, Nucl. Data Sheets **97**, 241 (2002).

– https://www.nndc.bnl.gov/nudat2/



Our newest results on the fission fragment mass-yields and the PES's of Pt-Rn nuclei are in:

K.P., A. Dobrowolski, Rui Han, B. Nerlo-Pomorska, M. Warda, Z.G. Xiao, Y.J. Chen, L.L Liu, J.L. Tian, Phys. Rev. C , accepted for publication, (2020). Preprint is in arXiv:2001.08652.

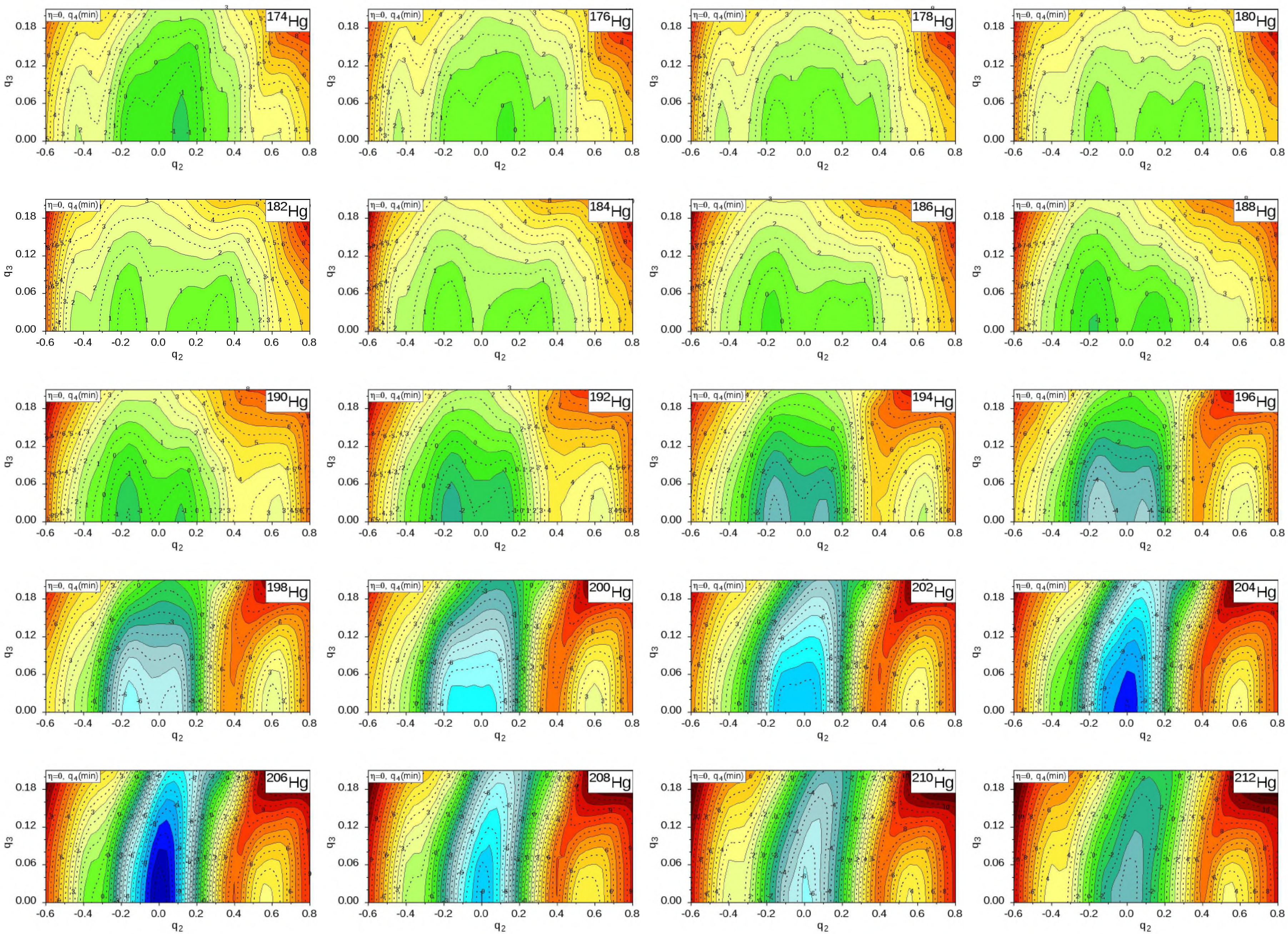
Summary and conclusions:

- New, rapidly convergent Fourier expansion of nuclear shape is used,
- An effective six dimensional set of the Fourier deformation parameters was used to describe the nuclear potential surfaces,
- The role of higher multipolarity deformations q_5 and q_6 is shown to be in practice negligible,
- Yukawa-folded mean field describes well shell structure of Pt, Hg and Pb isotopes.
- The mac-mic model with the LSD macroscopic energy reproduces quite precisely the equilibrium deformations of all investigated nuclei.
- Several shape isomers are predicted in Pt, Hg, and Pb nuclei.

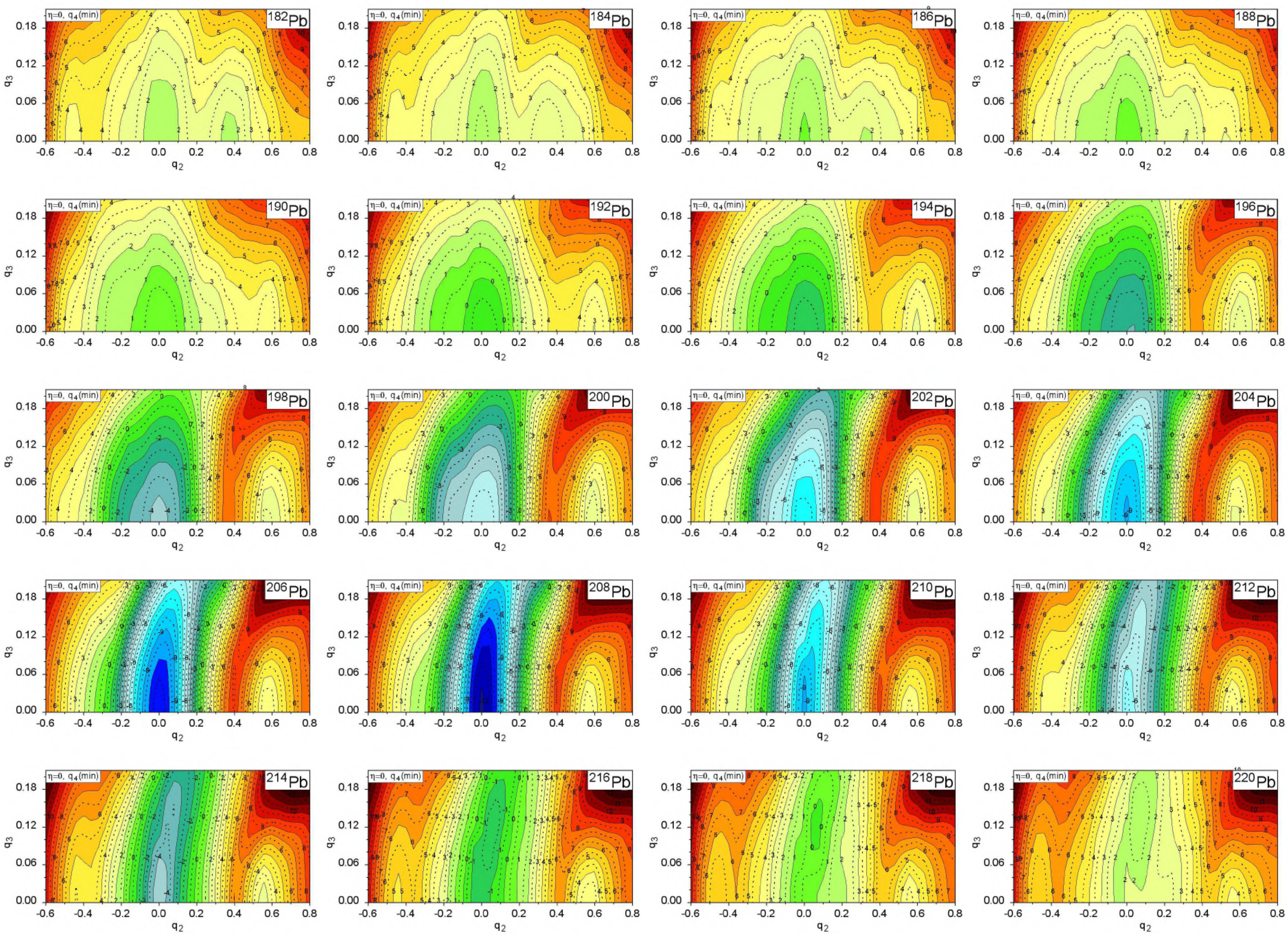
Thank you for your attention

and welcome to Międzygórze!

One of historical hotels in Międzygórze.



Potential energy surface of $^{174-212}$Hg isotopes on the (q_2, q_3) plane for $\boldsymbol{\eta = 0}$ and $\boldsymbol{q_4(min)}$.



Potential energy surface of $^{182-220}$Pb isotopes on the (q_2, q_3) plane for $\eta = 0$ and $q_4(min)$.