Rozszczepienie jądrowe – główne problemy, czasy życia jąder nieparzystych i K-izomerów oraz metoda instantonów

W. Brodziński , J. Skalski (NCBJ)

- **Rozszczepienie - obserwable i sposoby ich opisu**
  - **stan teorii**
- **Rozszczepienie spontaniczne jąder nieparzystych i K-izomerów :**
  - **dane eksperymentalne**
  - **załamanie przybliżenia adiabatycznego, próby traktowania problemu**
- **Metoda instantonów & jej uproszczone formy**
  - **a) rozwiązania bez pairingu dla potencjału W-S;**
  - **b) zachowanie czy zmiana konfiguracji – model hybrydowy ad hoc**

- E.Fermi, Nature 133, 898 (1934) – possible production of elements of Z>92;
- I.Noddack, Z. Angew. Chem. 47, 653 (1934) – krytyka Fermiego i hipoteza rozszczepienia;

- Nagroda Nobla w za 1938 r. dla E.Fermiego:

„for his demonstrations of the existence of new radioactive elements produced by neutron irradiation, and for his related discovery of nuclear reactions brought about by slow neutrons.”

Fermi z zespołem wierzyli, że wytworzyli pierwiastki Z=93 i 94; mieli dla nich nazwy: „ausonium” i „hesperium” (grecka i poetycka nazwy Italii)

[Właściwe odkrycie 239Np: E.M.McMillan, P. Abelson - (1940)]

- O. Hahn, F.Strassman – Ba i La wśród produktów reakcji n + U – praca wysłana 22.XII.1938 r.

  Nagrodę Nobla za to (za 1944 r.) dostał sam O.Hahn (rok później).

- L.Meitner, O.R.Frisch, Nature 143, 239, 11 Feb. 1939 r. „nuclear fission”, użyli model kropli G.Gamowa (Proc. R. Soc. London A 126, 632 (1930)

- N.Bohr, J.A.Wheeler, Phys. Rev. 56, 426 (1939) - model rozszczepienia: deformacja kropli + idea jądra złożonego N.Bohra [Nature 137, 344 (1936)]

- G.N.Flerov, K.A.Petrzhak (Pietrzak?) (1941) znaleźli spontaniczne rozszczepienie 238U (pomiary za pomocą komory jonizacyjnej na stacji metra w Moskwie, 50 m pod ziemią).

^{236}U rozszczepia się; może to się stać na wiele sposobów, n.p.:

$^{236}_{92}\mathrm{U} \rightarrow ^{137}_{53}\mathrm{I} + ^{96}_{39}\mathrm{Y} + 3\mathrm{n}$

$^{137}\mathrm{I} \rightarrow ^{137}\mathrm{Xe}\ \mathrm{e}\ \bar{\nu}_e\ (T_{1/2} = 24{,}5\ \mathrm{s})$

$^{137}\mathrm{Xe} \rightarrow ^{137}\mathrm{Cs}\ \mathrm{e}\ \bar{\nu}_e\ (T_{1/2} = 3{,}818\ \mathrm{m})$

$^{137}\mathrm{Cs} \rightarrow ^{137}\mathrm{Ba}\ \mathrm{e}\ \bar{\nu}_e\ (T_{1/2} = 30{,}07\ \mathrm{lat})$

$^{96}\mathrm{Y} \rightarrow ^{96}\mathrm{Zr}\ \mathrm{e}\ \bar{\nu}_e\ (T_{1/2} = 5{,}34\ \mathrm{s})$

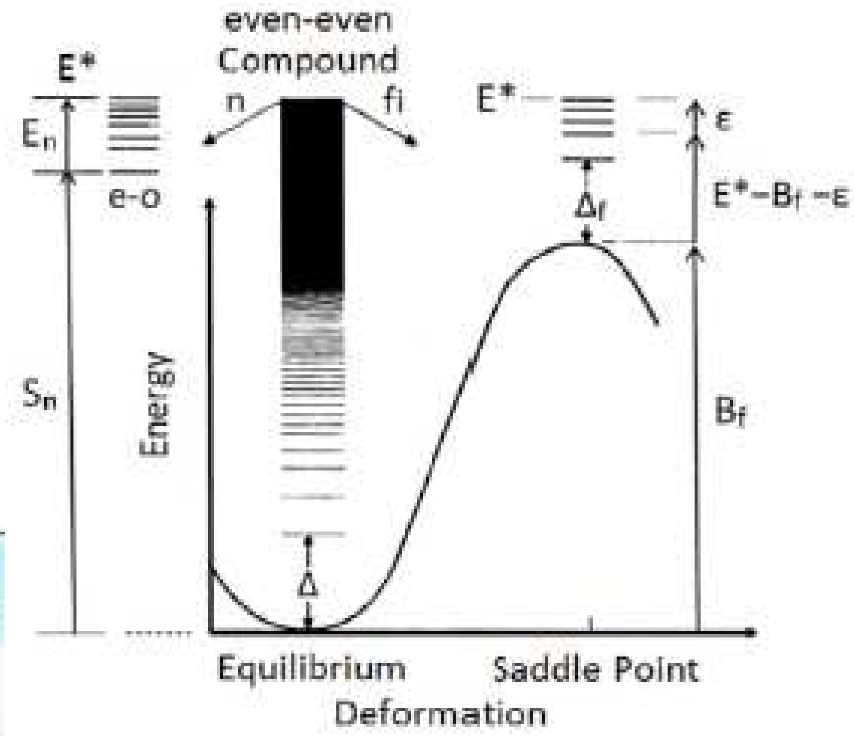
co można zapisać sumarycznie

$^{236}_{92}\mathrm{U} \rightarrow ^{137}\mathrm{Ba} + ^{96}\mathrm{Zr} + 3\mathrm{n} + 4\mathrm{e} + 4\bar{\nu}_e$.

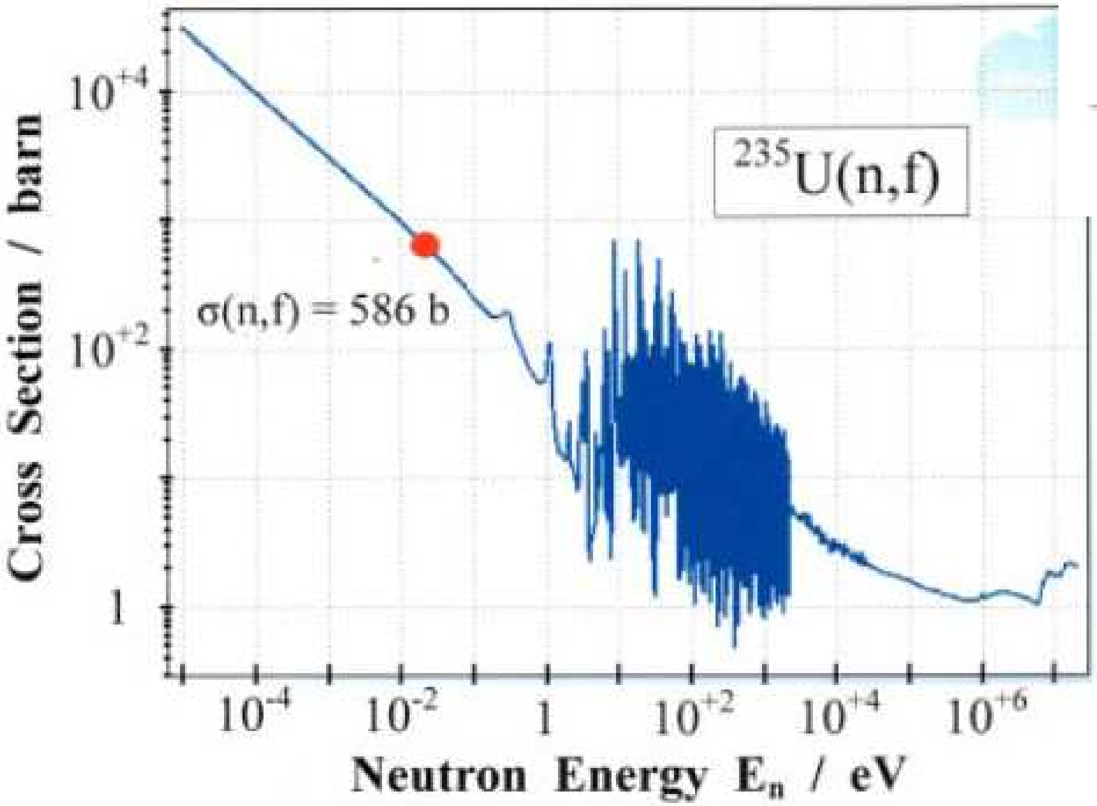
Około 200 MeV; ~ 10 MeV – neutrina;
wynikiem może być jedna z ~ 500 par fragmentów

Fragmenty pierwotne mają podobne Z/N jak j. złożone
- dlatego następują ich beta-rozpady.

F. Gönnenwein
Neutron-induced Fission (FG-NIF)

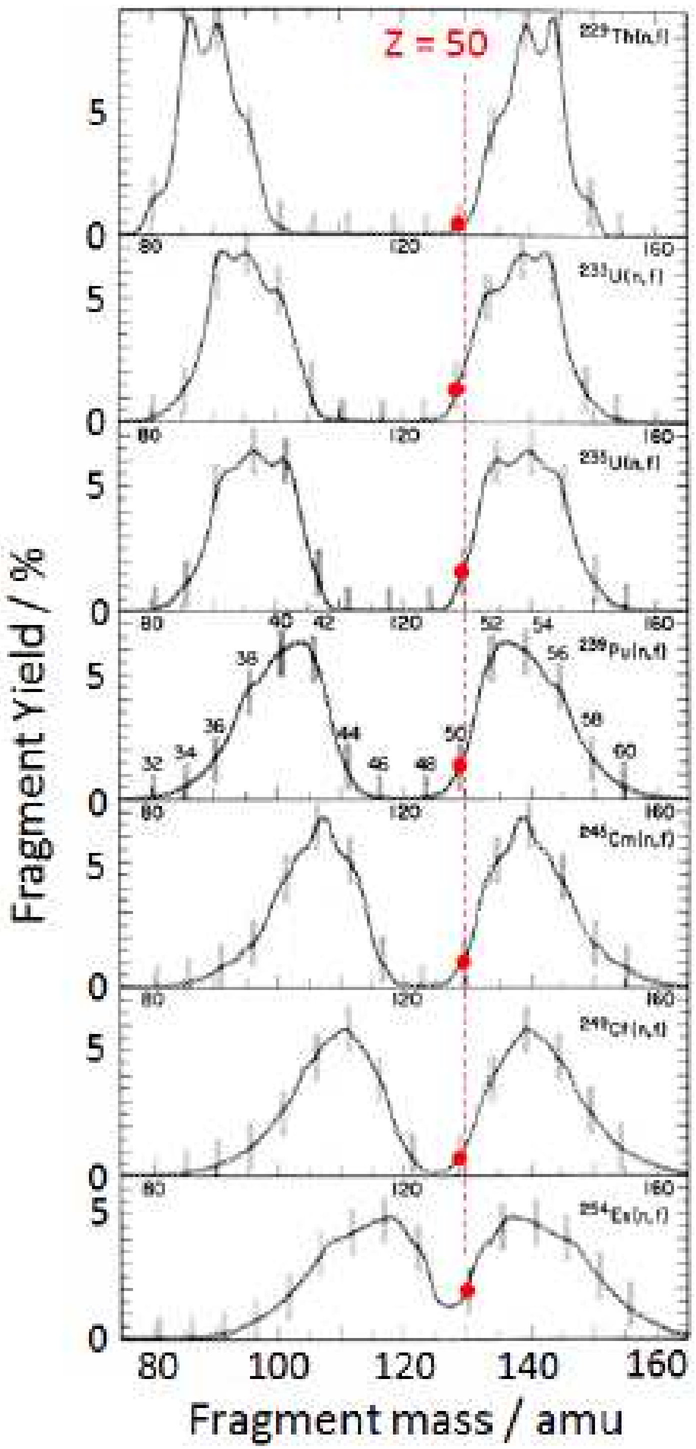


At 25 meV - 586 b = 340 geom. cross section;

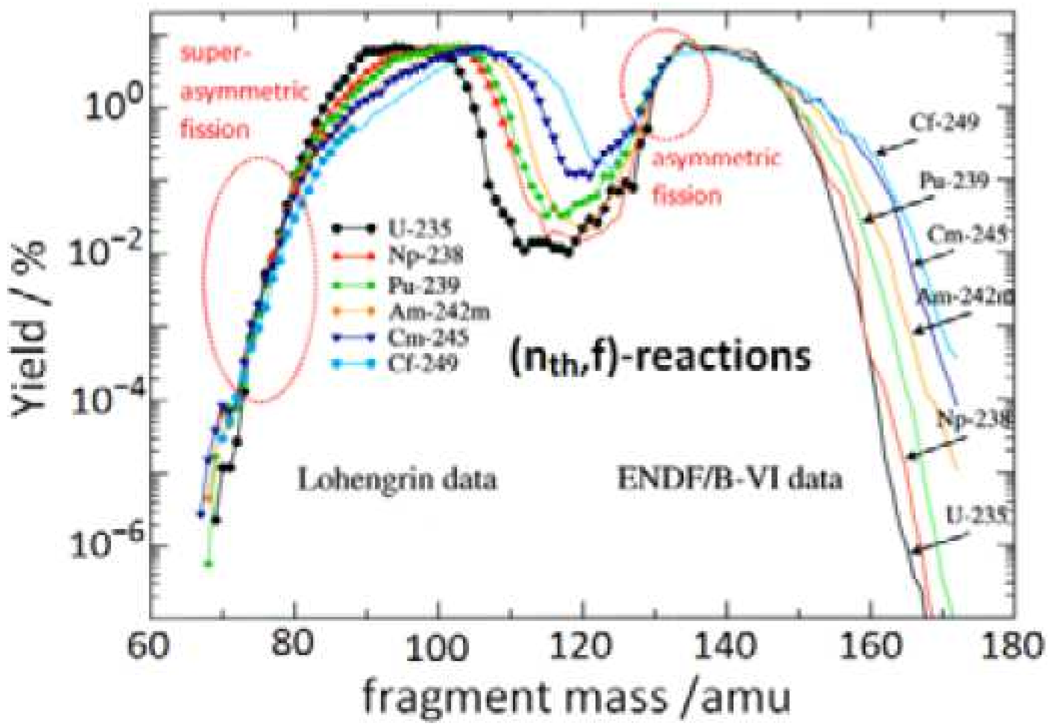


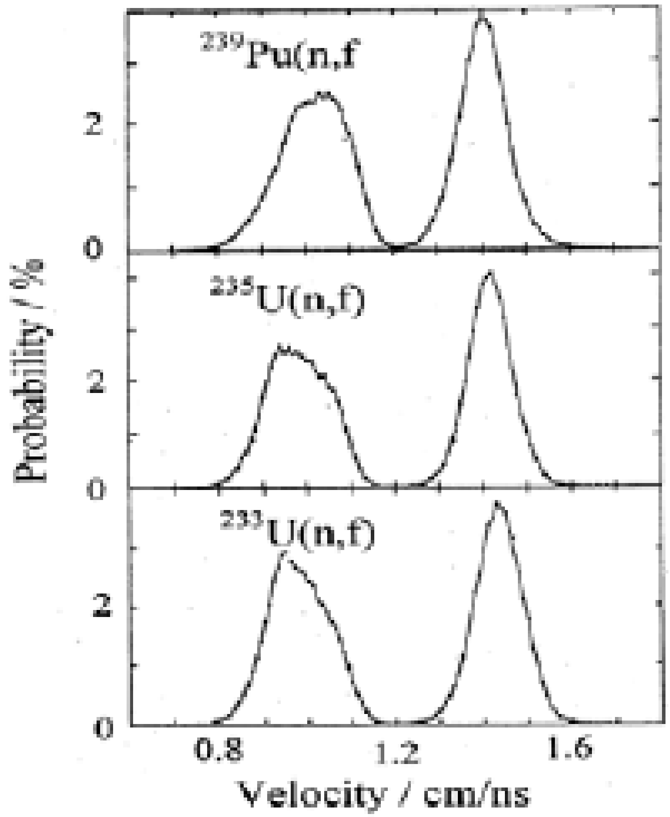
Fissile isotope:
S_n - B_f>0;

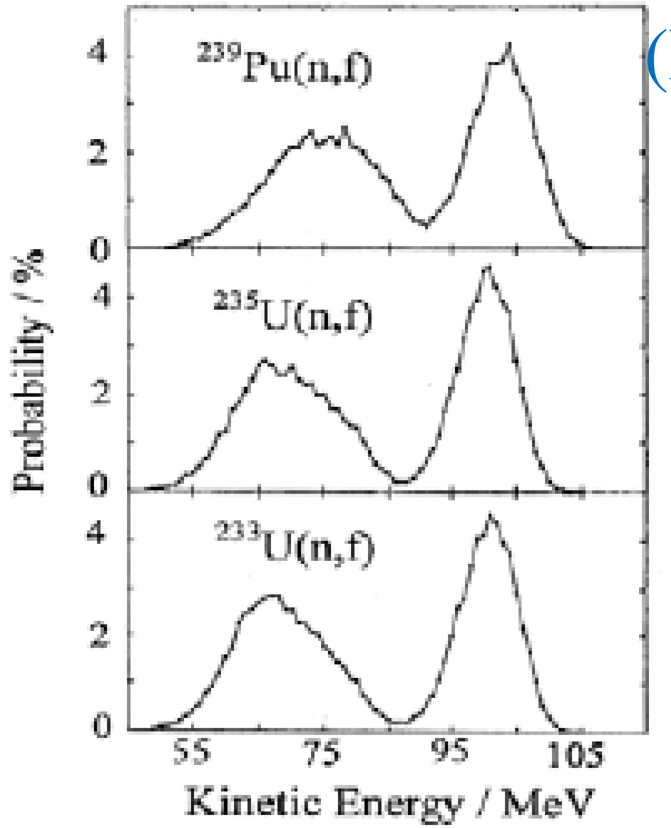
233U, 235U, 239Pu


Z = 50
Fragment Yield / %
Fragment mass / amu

(FG-NIF)


super-asymmetric fission
asymmetric fission
U-235
Np-238
Pu-239
Am-242m
Cm-245
Cf-249
(nth,f)-reactions
Lohengrin data
ENDF/B-VI data
Yield / %
fragment mass /amu





(FG-NIF)

P. Geltenbort: PHD thesis, Univ. Tübingen, 1985 unpublished

$\langle V_L^* \rangle$	$\langle V_H^* \rangle$	$\langle M_L^* \rangle$	$\langle M_H^* \rangle$	$\langle E_{KL}^* \rangle$	$\langle E_{KH}^* \rangle$
1.420(5)	0.983(5)	96.4(2)	139.6(2)	100.6(5)	69.8(5)

235U(n_th,f)
V in cm/ns

Reaction	233U(n,f)	235U(n,f)	239Pu(n,f)	252Cf(sf)
TKE*/MeV	170.1(5)	170.5(5)	177.9(5)	184.0(13)

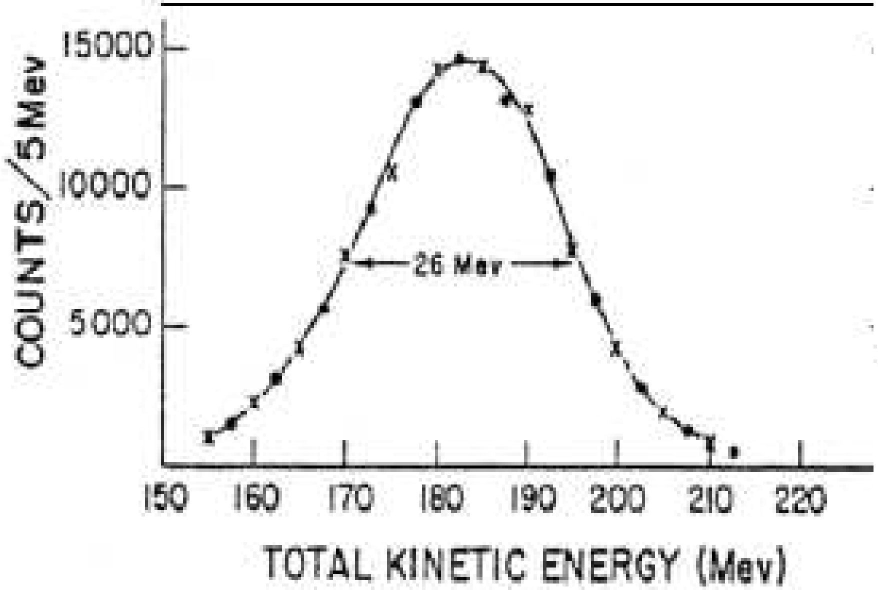
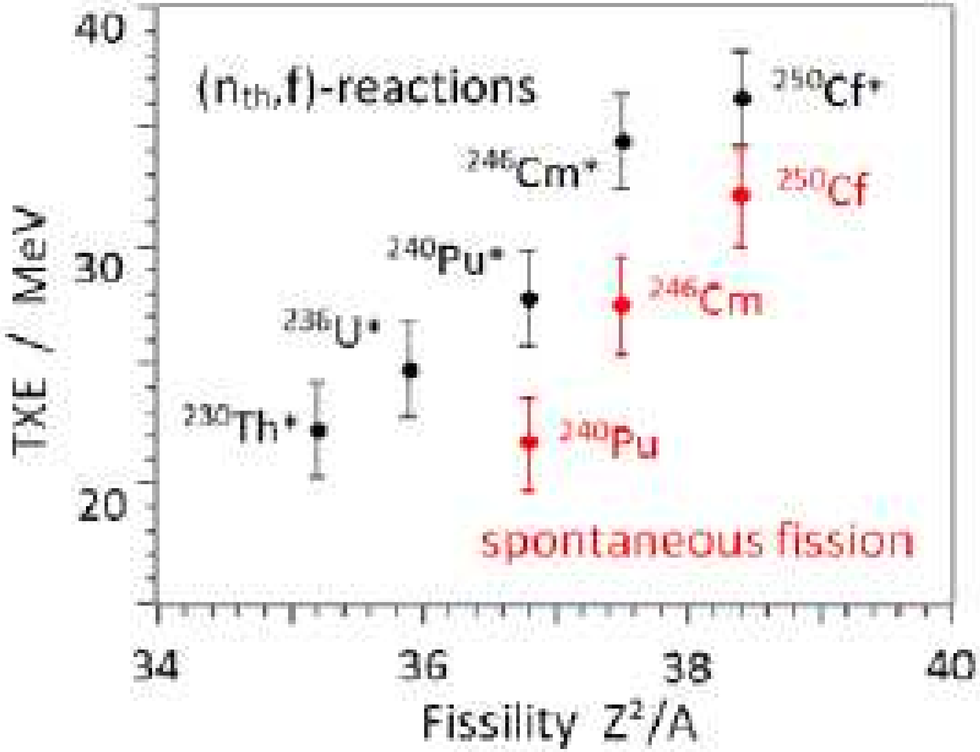


Fig. 17: Width of TKE in ^{252}Cf(sf). [17]



(FG-NIF)

$$x = E_C^0 / 2E_S^0.$$

$$x = (1/50.13)Z^2/A.$$

TKE + TXE = Q

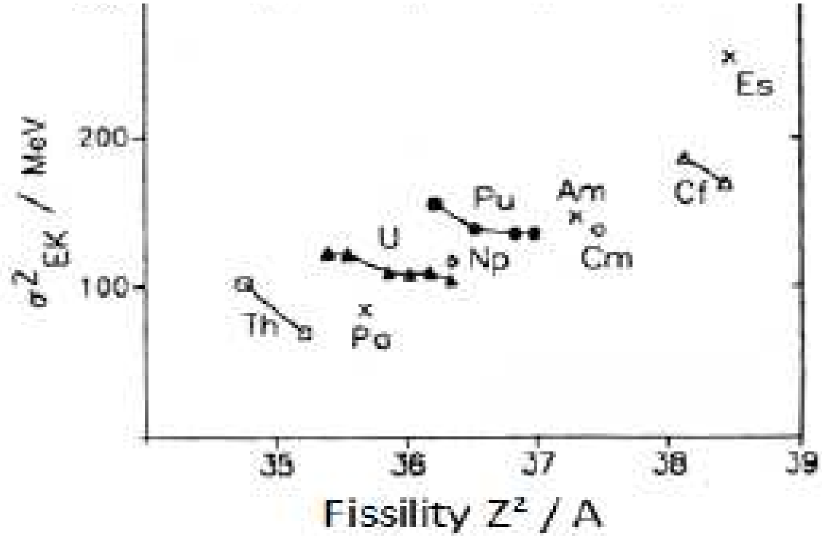
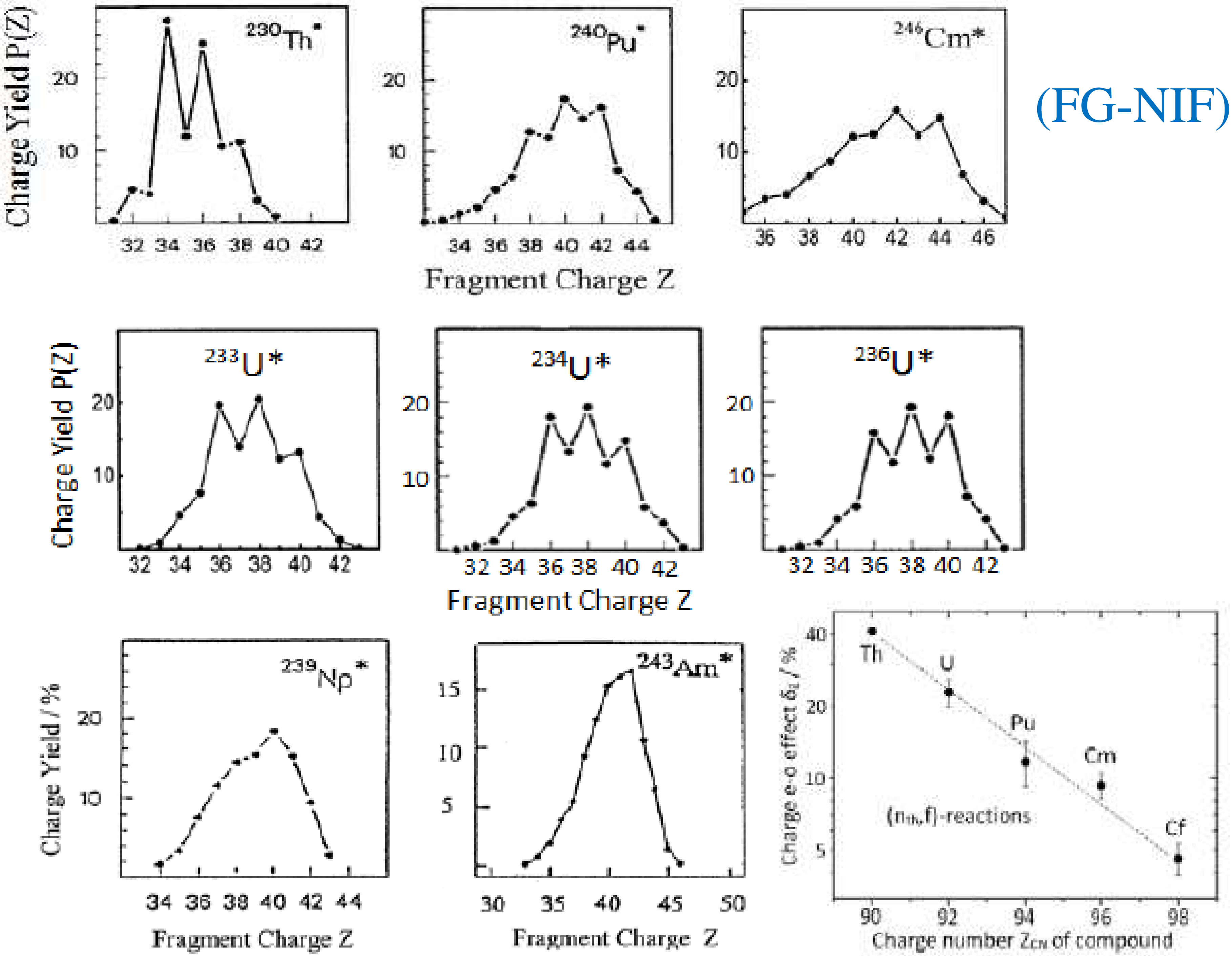


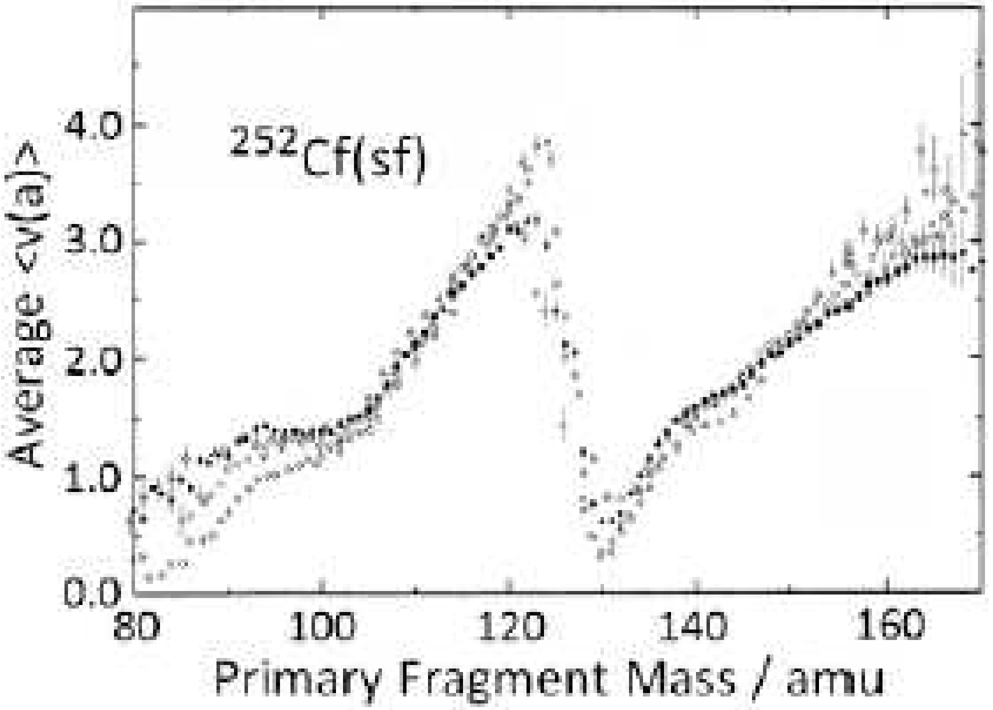
Fig. 18: Variance versus fissility in (n_{th},f) reactions.[18]



Charge Yield P(Z)
230Th*
240Pu*
246Cm*
Fragment Charge Z
233U*
234U*
236U*
Charge Yield P(Z)
Fragment Charge Z
239Np*
Charge Yield / %
Fragment Charge Z
243Am*
Fragment Charge Z
Th
U
Pu
Cm
Cf
(nth,f)-reactions
Charge e-o effect δZ / %
Charge number ZCN of compound

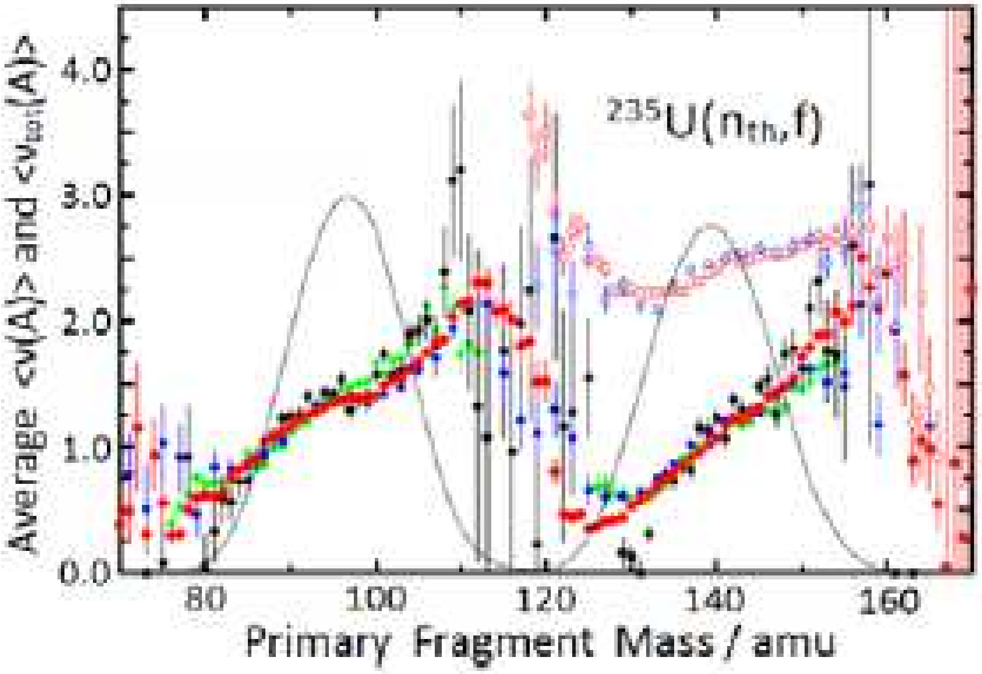
(FG-NIF)

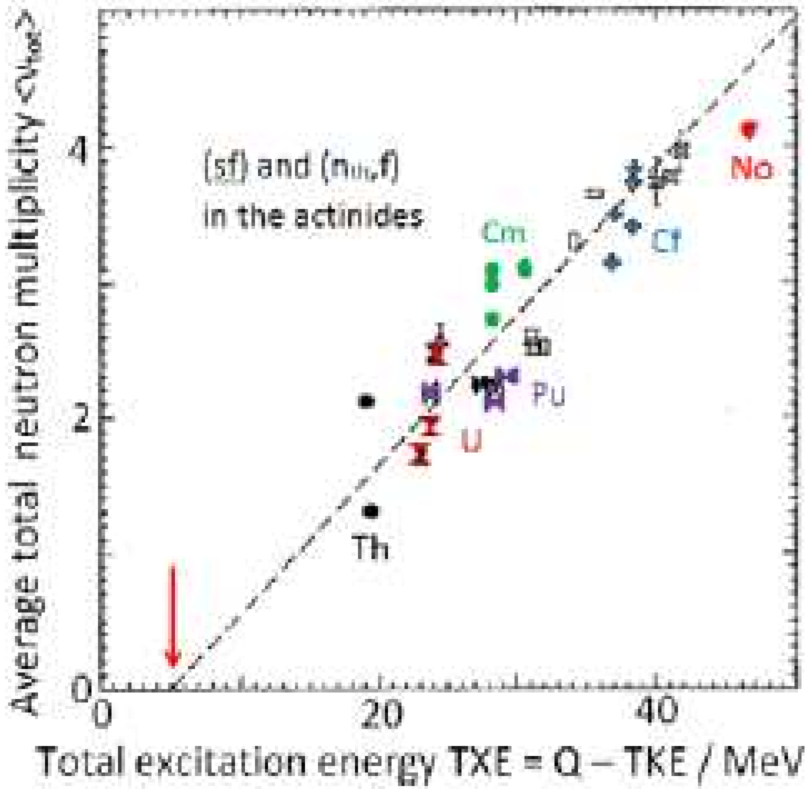
(FG-NIF)



CN nucleus	230Th	234U	236U	240Pu	246Cm	250Cf
$<\nu_{tot}>$	2.08	2.50	2.43	2.89	3.83	4.08

Reaction	$^{233}U(n_{th},f)$	$^{235}U(n_{th},f)$	$^{252}Cf(sf)$
ν_L / ν_H	1.395/1.100	1.390/1.047	2.056/1.710





(FG-NIF)

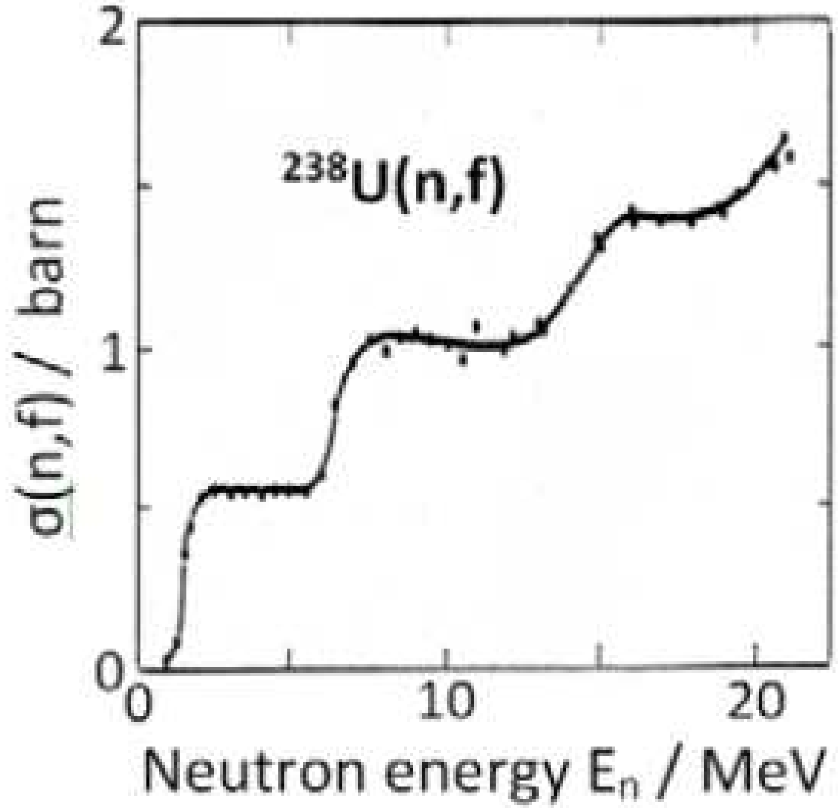


Fig. 49:: σ(n,f) vs E_n for ^{238}U

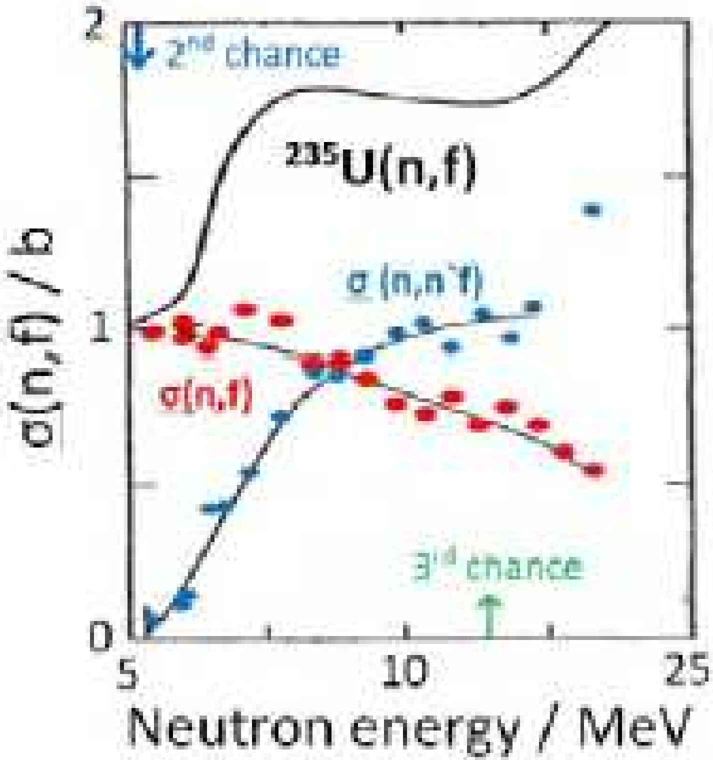


Fig. 50: σ(n,f) vs E_n for ^{235}U

Second, third,… chance fission

Delayed neutron emission improtant for reactor operation

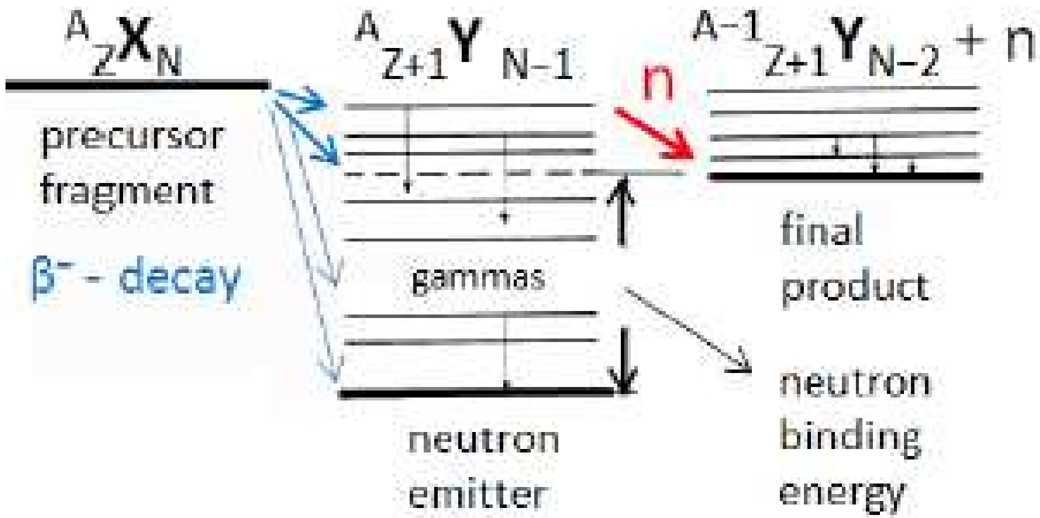


Fig. 51: Level schemes in delayed neutron emission

(FG-NIF)

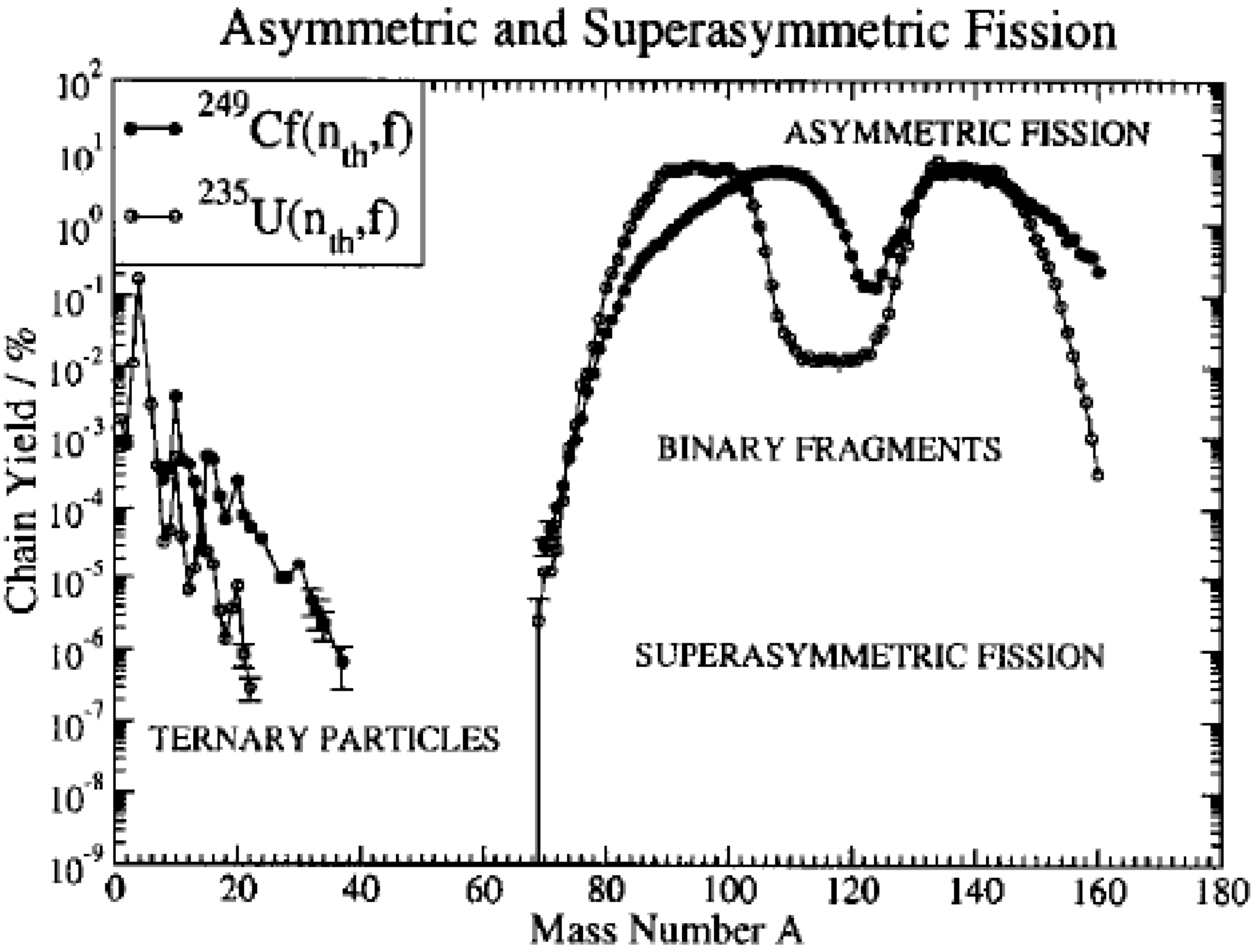
235U(n,f):

T_av = 9 s

k	$T_{1/2}$ /s	E_n /MeV	P_k /%
1	53.0	0.41	3.5
2	21.6	0.47	18.1
3	5.3	0.44	17.3
4	2.3	0.56	38.7
5	0.83	0.52	15.6
6	0.25	0.54	6.6

N_del/N_tot:
235U - 0.65%
239Pu - 0.24%

F. Gönnenwein / Nuclear Physics A734 (2004) 213–21(



Opis rozszczepienia: zwykle oddzielnie tunelowanie i ruch z E>V;

Zainteresowanie odtworzeniem własności fragmentów;
zastosowania: nukleosynteza, reaktory.

1) E>V: a) TDHF(niechętnie się rozszczepia),
TDGCM(GOA), TDHFB (z pairingiem);
punkt wyjścia: funkcjonał gęstości;
dużo problemów do rozwiązania.

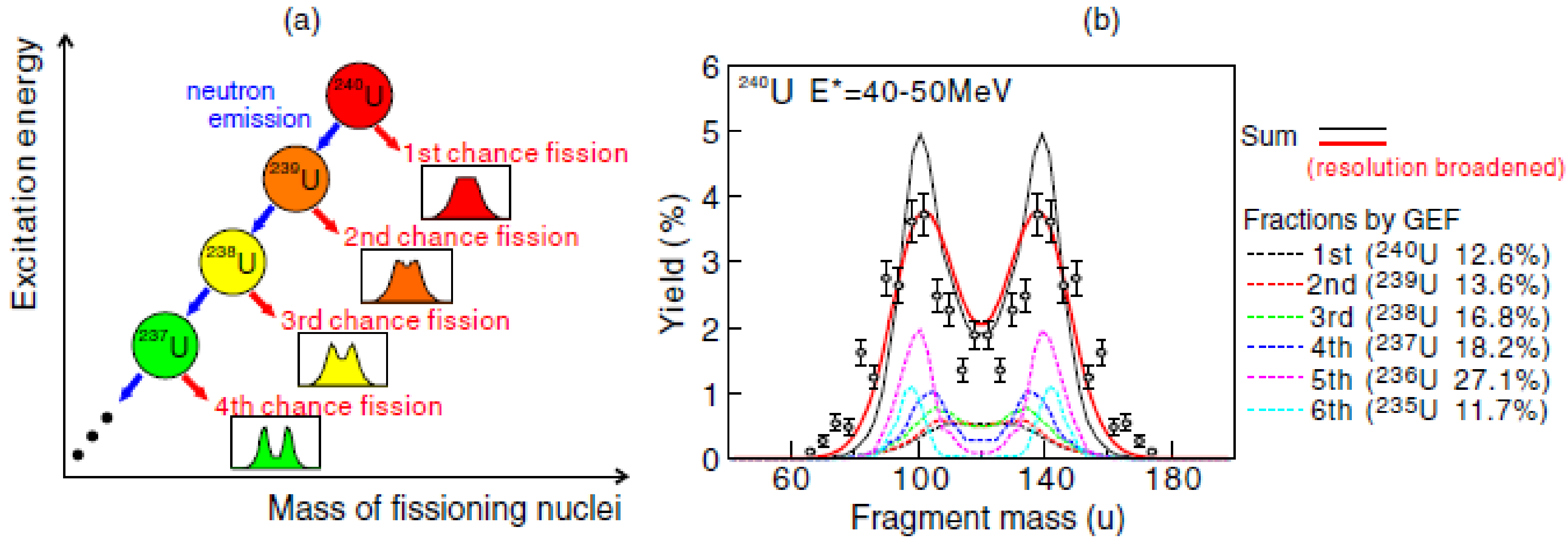
b) Metody fenomenologiczne – dynamika Langevina,
+ mic-mac + Monte Carlo

2) E<V: ~WKB, całka działania, HFB z funkcjonałem
lub mic-mac, parametry masowe.

Dalej: 4 przykłady ostatnich wyników teorii.

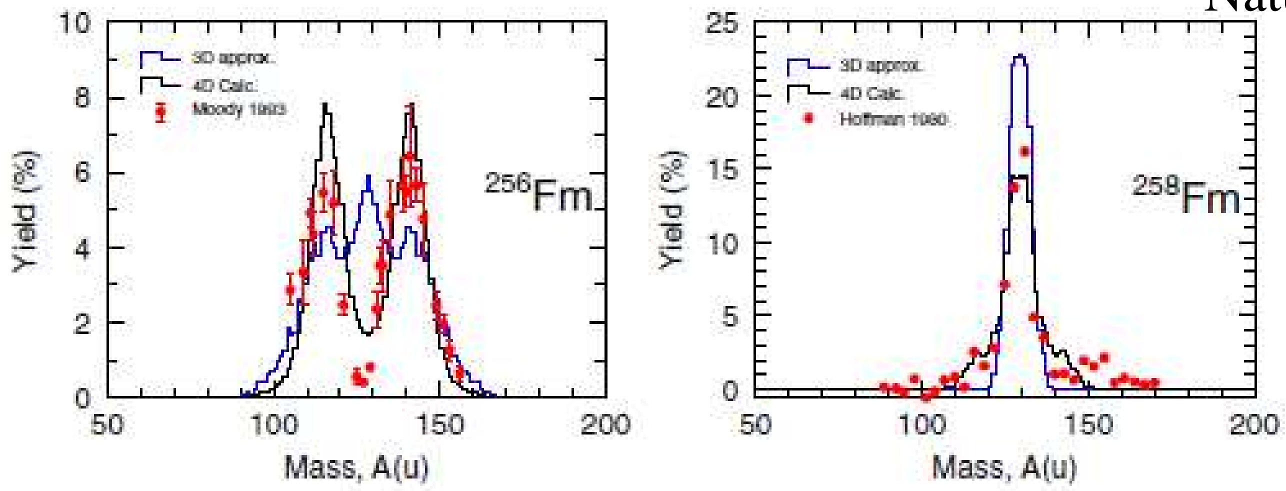
3D&4D Langevin equations, mic-mac energy + Monte Carlo

K. Hirose et al., PRL 119, 222501 (2017)

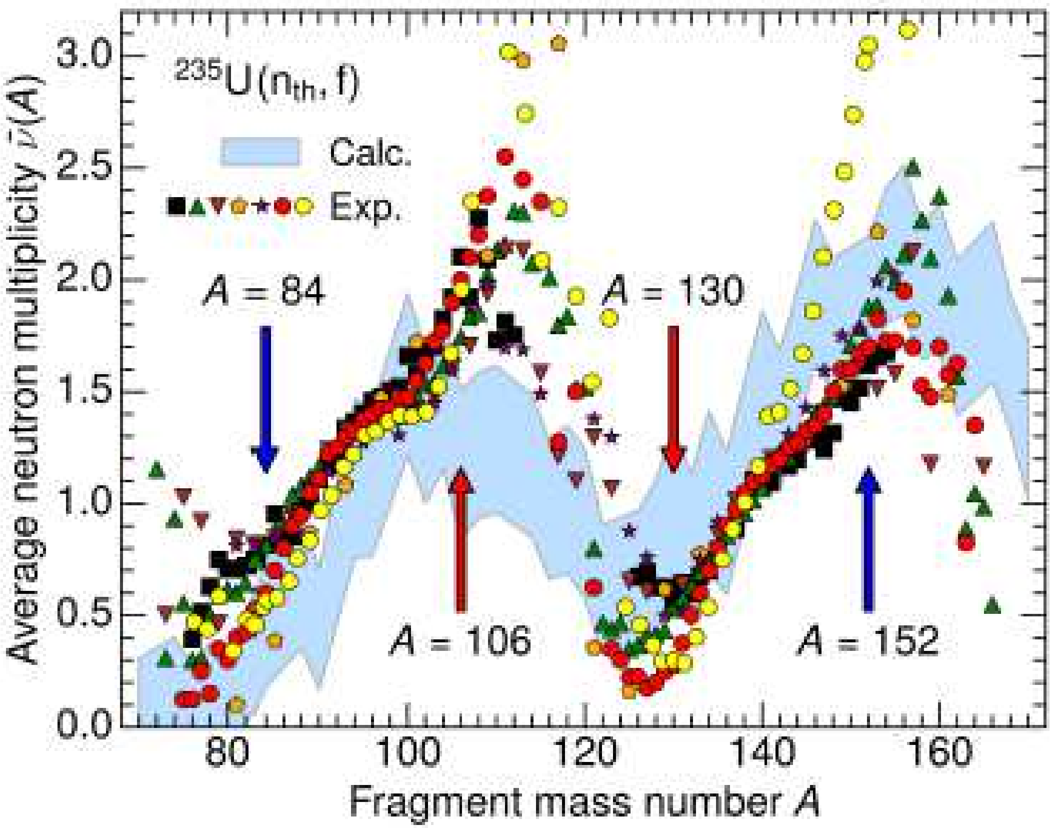


Mark Dennis Usang[1,2], Fedir A. Ivanyuk[1,3], Chikako Ishizuka[1] & Satoshi Chiba[1,4] SCIENTIFIC REPORTS | (2019) 9:1525

Nature



M. Albertsson,[1] B.G. Carlsson,[1] T. Døssing,[2] P. Möller,[1,3,4] J. Randrup,[5] and S. Åberg[1]



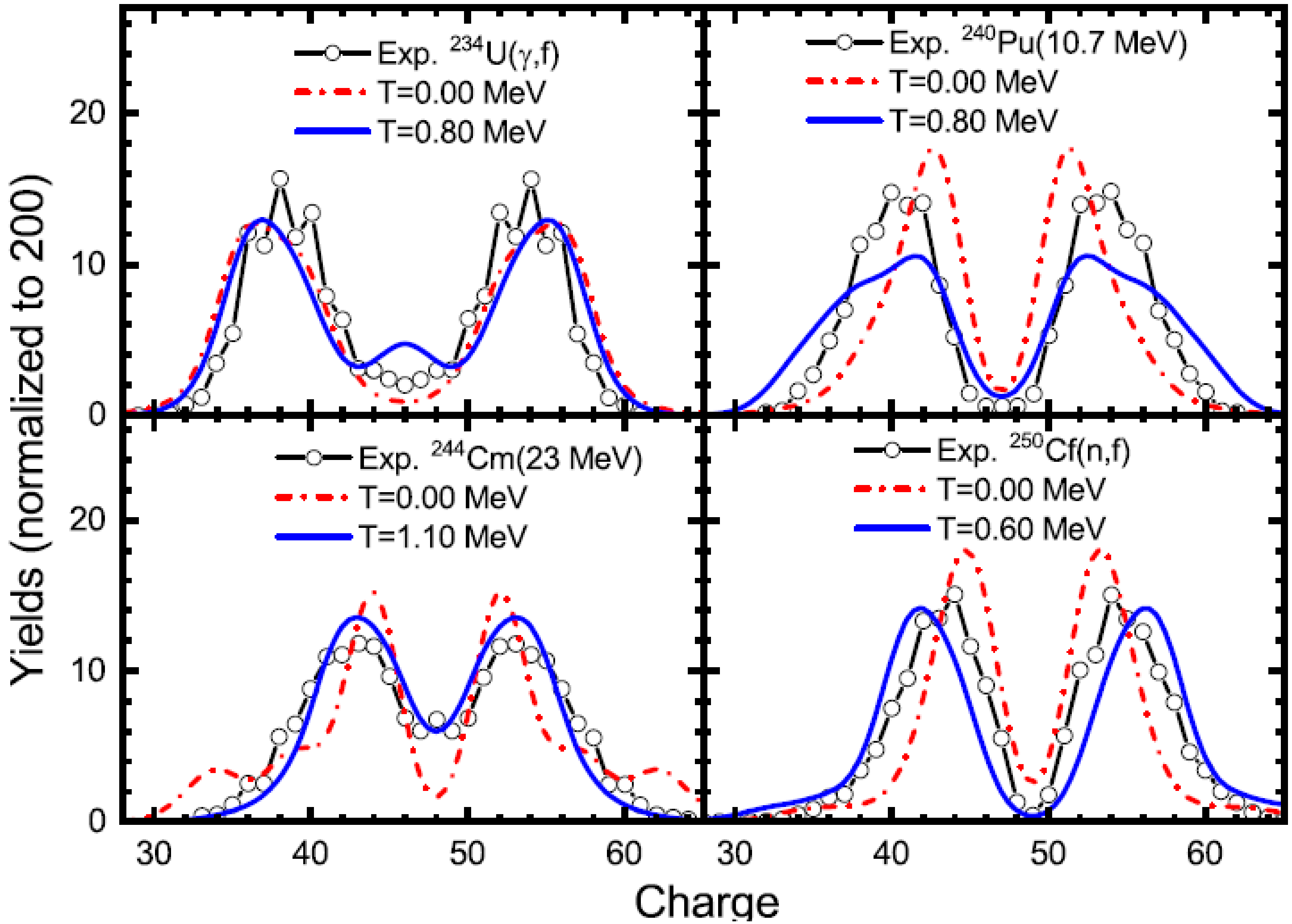
Wyniki obliczeń: szary obszar (z uwzgl. niepewności)

Statistical model for energy partition between the Fragments according to level densities at scission

JIE ZHAO *et al.* PHYSICAL REVIEW C 99, 054613 (2019)

Rozkłady ładunku

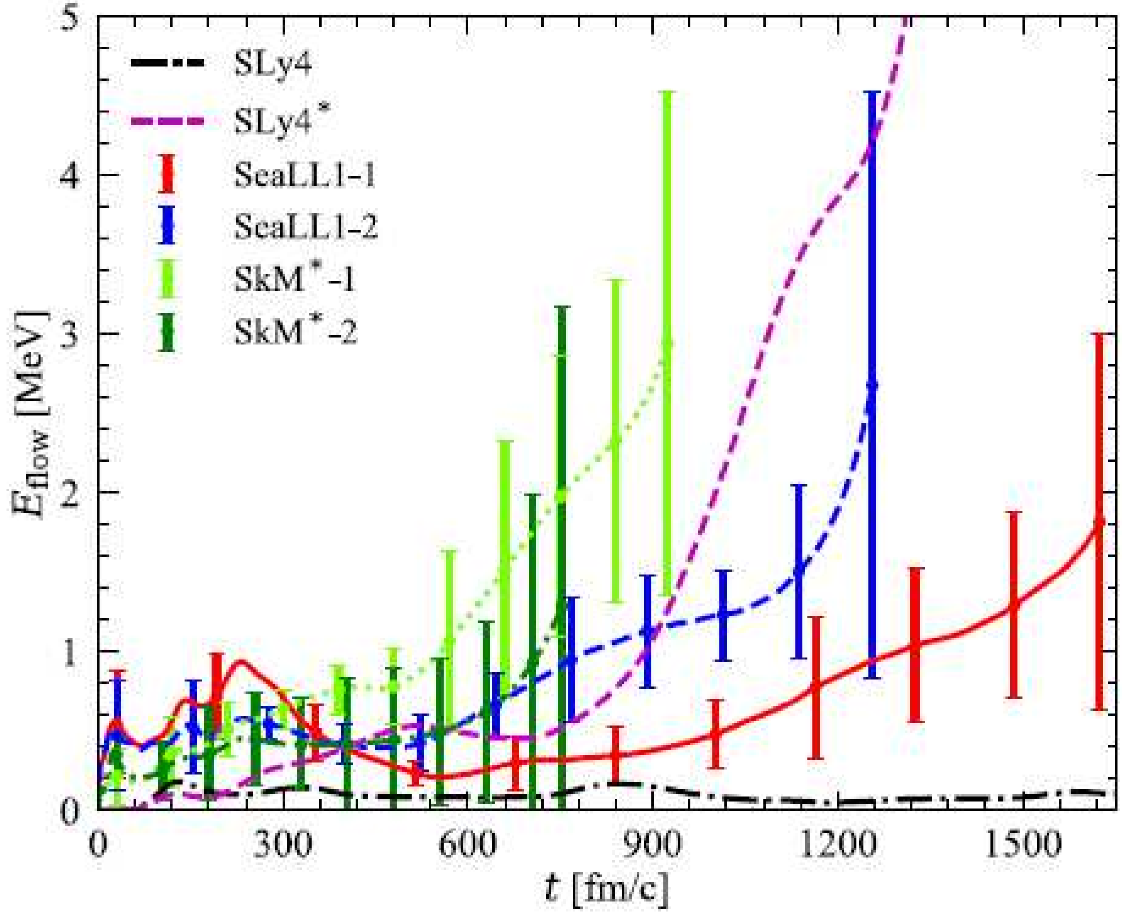
TDGCM+GOA



BULGAC, JIN, ROCHE, SCHUNCK, AND STETCU

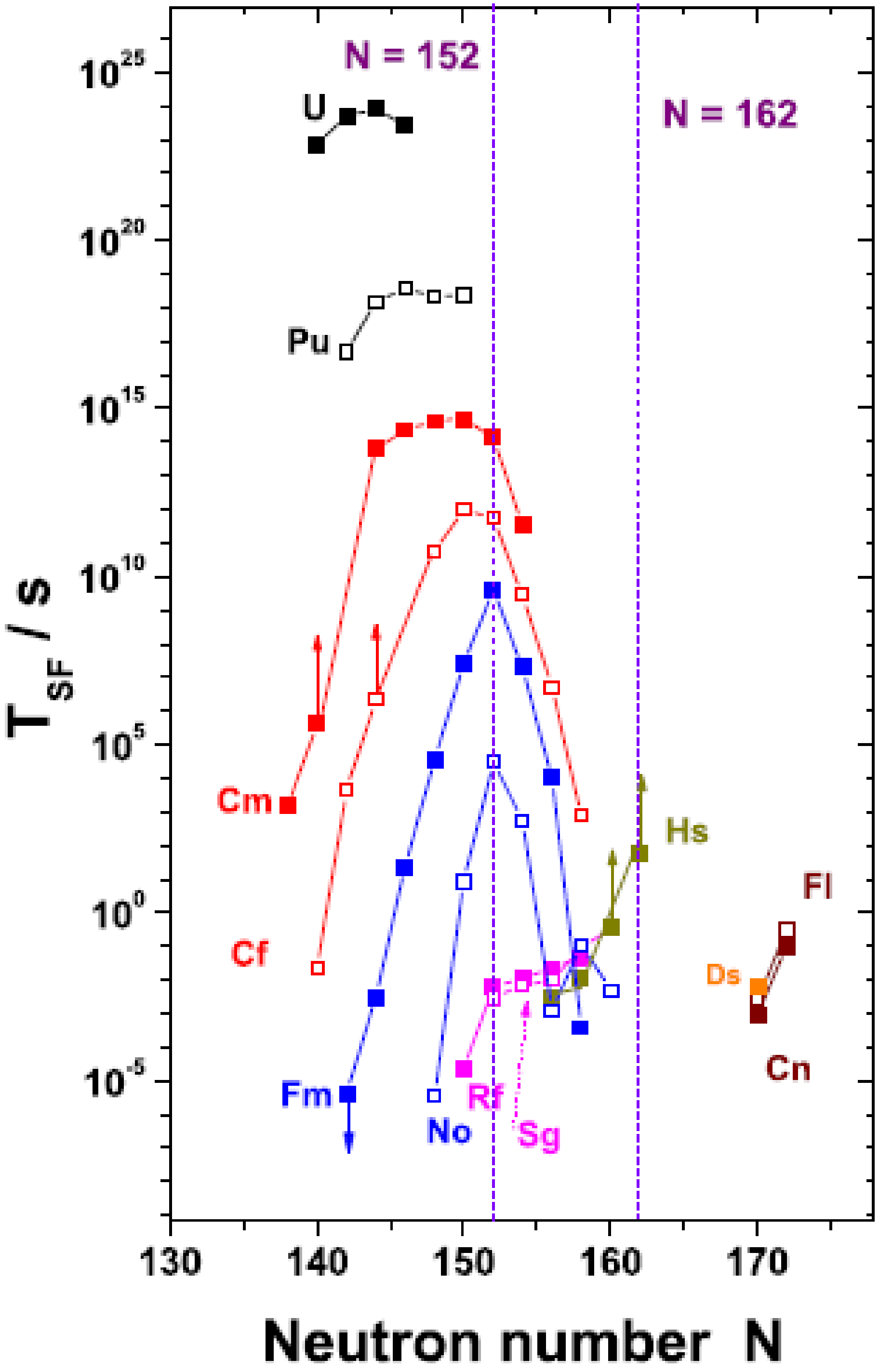
Fission dynamics of ^{240}Pu from saddle to scission and beyond

PHYSICAL REVIEW C **100**, 034615 (2019)



TD HFB;
Energia kolektywna
w funkcji czasu dla
różnych funkcjonałów.

Very large one – body dissipation, small collective energy, large excitation energy (thermal – like, non–adiabatic motion)



Experimental sf half-lives of even-even nuclei

F.P. Heßberger

Eur. Phys. J. A (2017) 53: 75

Description of spontaneous fission within adiabatic approximation

- Spontaneous fission of a nucleus: collective quantum tunneling process
- Set of collective variables $\{q_i\}$ usually chosen as parameters describing the shape of the fissioning nucleus
- Assumption that variations of the collective degrees of freedom are much slower comparing with the oscillations of individual nucleons $\Rightarrow$ **adiabatic approximation**
- Adiabatic mass parameter:

$$B_{ij} = 2\hbar^2 \sum_k \frac{\langle k|\partial/\partial q_i|0\rangle \langle 0|\partial/\partial q_j|k\rangle}{E_k - E_0}$$

- Calculation of the action corresponding to a trajectory L:

$$S(L, E_0) = \int_L \sqrt{2B_L(q(s))[V(q(s)) - E_0]}\, ds$$

- Minimizing the action in the space of collective coordinates and estimating the SF half-lives as:

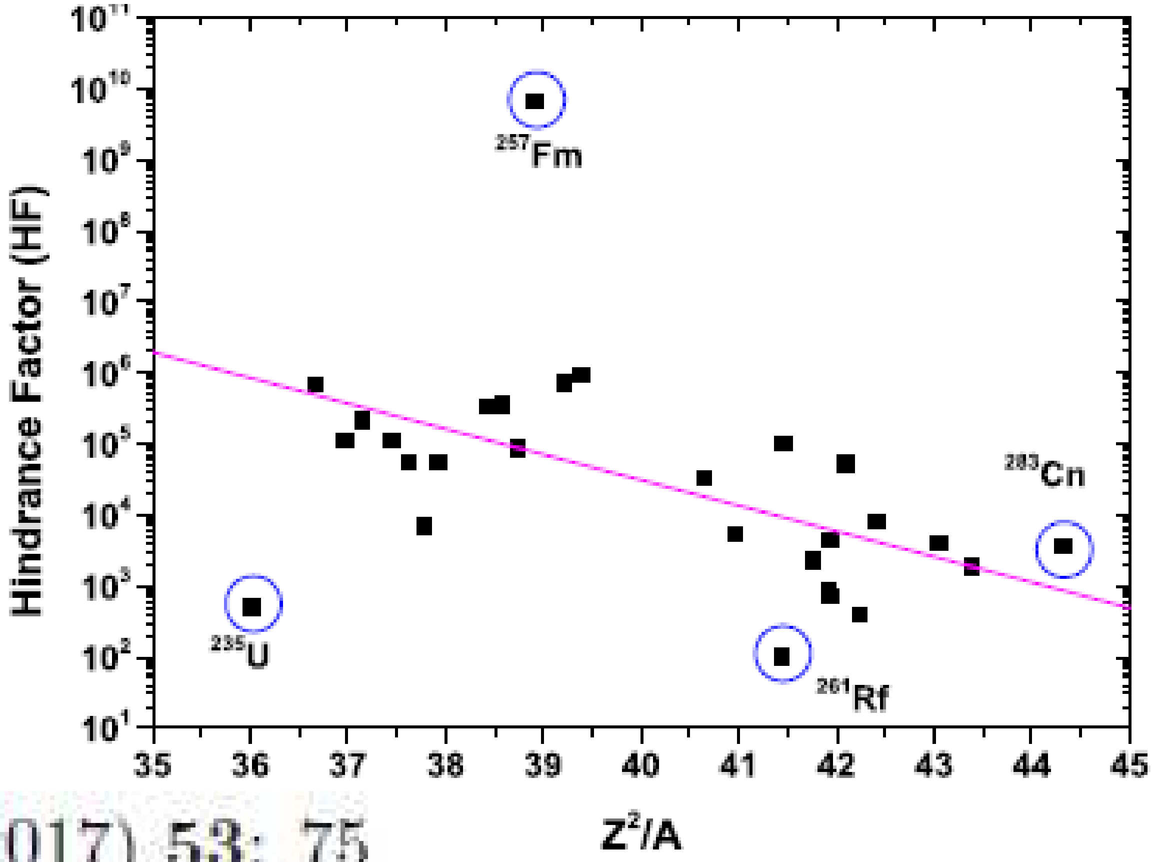
$$\Gamma \propto exp\left[-\frac{2}{\hbar} S_{min}\right] \qquad T_{1/2} = \frac{ln2}{\Gamma}$$

$$HF(Z,N) = T_{SF,exp}(Z,N)/T_{ee}(Z,N),$$

$$T_{ee}(Z,N) = (T_{SF}(Z,N-1) \times T_{SF}(Z,N+1))^{1/2}$$

$$T_{ee}(Z,N) = (T_{SF}(Z-1,N) \times T_{SF}(Z+1,N))^{1/2}$$

Odd nuclei- a hindrance of fission



F.P. Heßberger
Eur. Phys. J. A (2017) 53: 75

Fig. 19. Spontaneous fission hindrance as a function of the fissility of the fissioning nucleus, expressed by Z^2/A. The line is to guide the eyes.

Invoked reasons for Tsf increase:

- for odd-Z or odd-N (vs. even), a smaller pairing gap causes an increase in the fission barrier and in the mass parameter (as given by a cranking expression);
- blocking a specific configuration additionally rises the barrier (provided it is conserved in fission) – specialization.

In calculations: the effect of keeping high-K number may be huge; if one does not suppress it, it seems the resulting half-lives in odd-A nuclei must come out too large.

F.P. Heßberger

Eur. Phys. J. A (2017) 53: 75

Larger Omega does not give larger HF

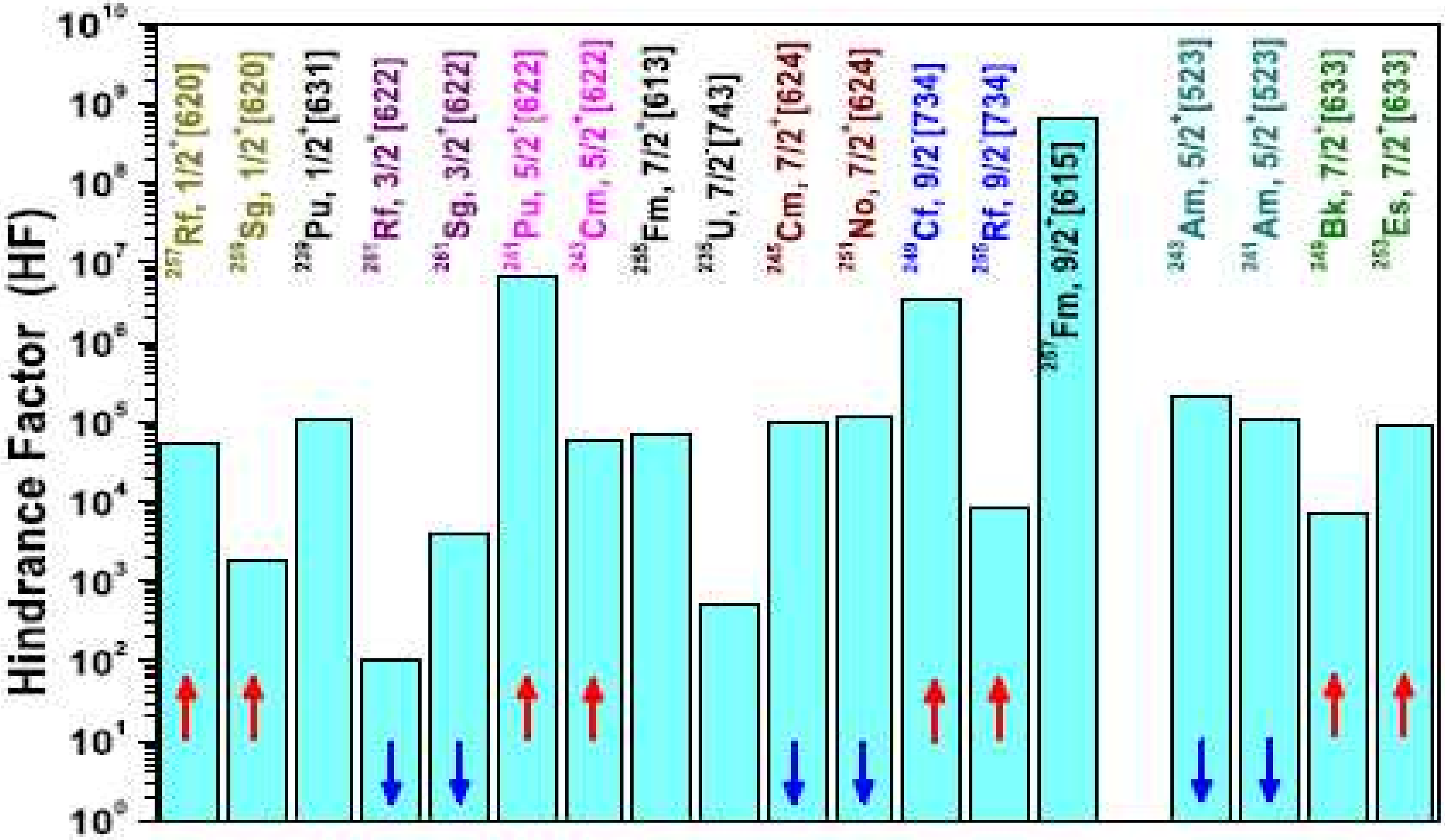


Fig. 17. Fission hindrance factors of odd-mass isotopes with experimentally assigned configuration (spin and parity) of the fissioning states

$$HF(Z,N) = T_{SF,exp}(Z,N)/T_{ee}(Z,N),$$

TABLE I: Fission halflives and hindrance factors for the K-isomers and ground states in the first well.

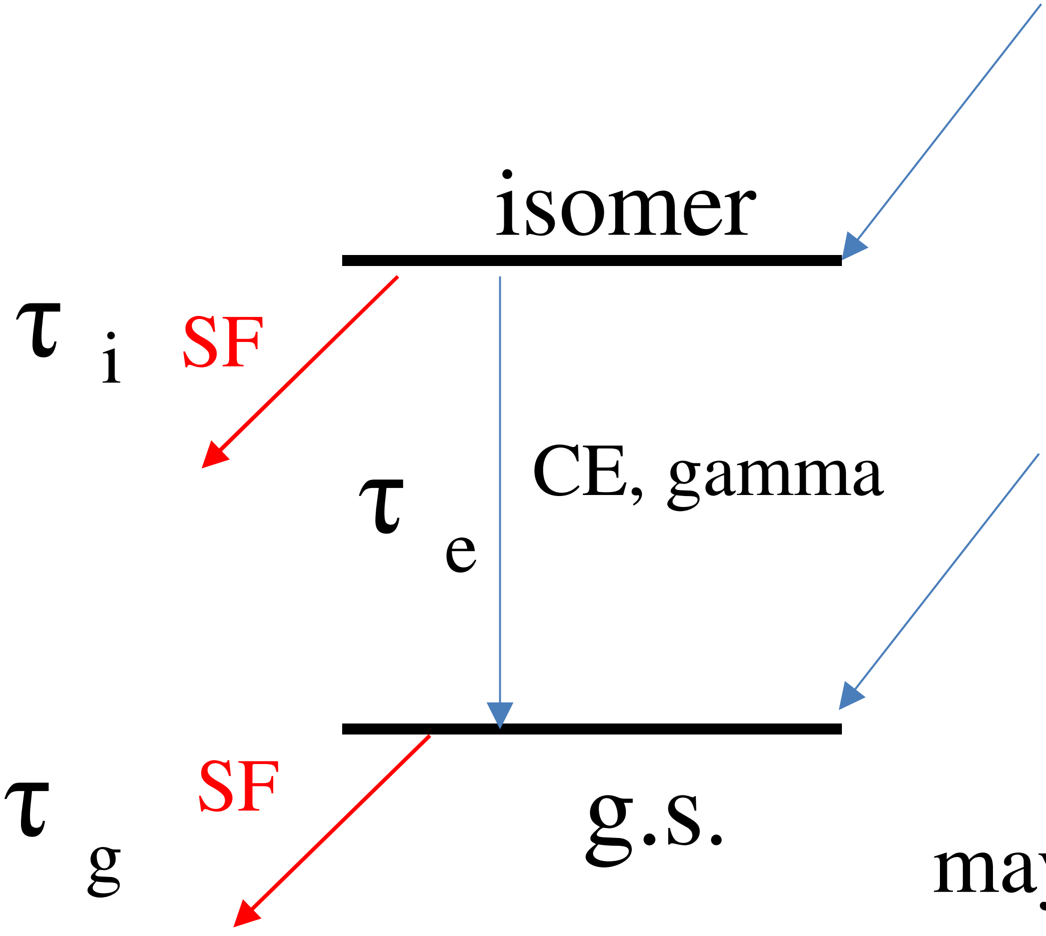
Nucleus	K^π	T_{sf}(g.s.)	T_{sf}(izo)	HF= T_{sf}(izo)/T_{sf}(g.s.)
^{250}No [a]	(6^+)	3.7 μs	$> 45\mu$s	> 10
^{254}No [b]	8^-	3×10^4 s	1400 s	$\approx \frac{1}{20}$
^{254}Rf [c]	(8^-)	23 μs	$> 50\mu$s	> 2
	(16^+)		$> 600\mu$s	> 25

[a] D. Petersen et al., Phys. Rev. C 73, 014316 (2006), F. P. Hessberger, Eur. Phys. J. A 53.
[b] F. P. Hessberger et al., Eur. Phys. J. A 43, 55 (2010).
[c] H. M. David et al., PRL 115, 132502 (2015).

Isomers in the first well

In theoretical models:
odd nucleus – one blocked state
isomer – at least two blocked states

Experimental difficulty for isomers



If:

$\tau_e \ll \tau_i$ &

only fission observed

$\tau_e + \tau_g$

may be taken for τ_i

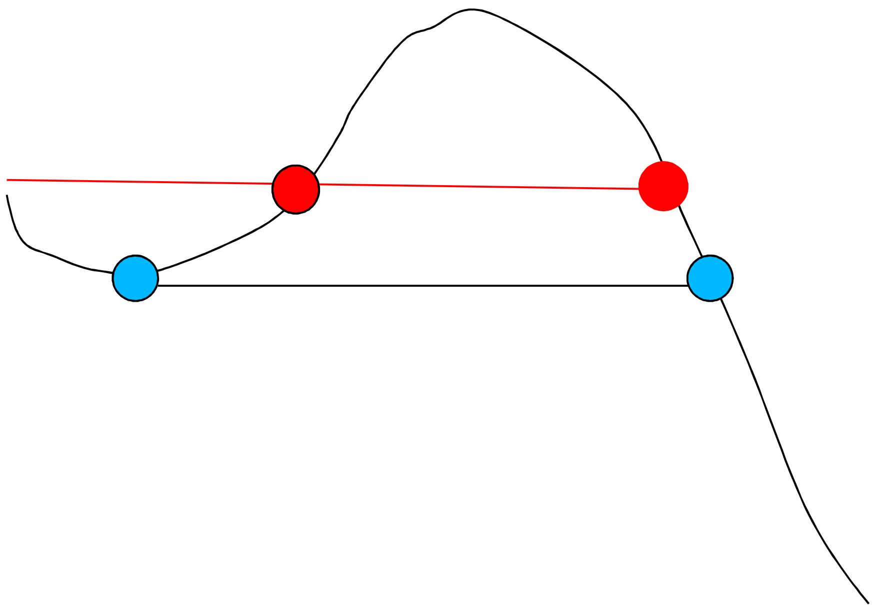
TABLE II: Excitation energies and fission halflives of shape isomers (ground states in the second well), of the excited (probably K-isomeric) states there [a] and the hindrance factors HF= T_{sf}(izo)/T_{sf}(g.s.).

Nucleus	$E(g.s.)$	T_{sf}(g.s.)	E_{izo}	T_{sf}(izo)	HF
^{236}Pu	3.0	37 ns	4.0	34 ns	≈ 1
^{237}Pu	2.6	85 ns	2.9	1.1 μs	
^{238}Pu	2.4	0.6 ns	3.5	6 ns	10
^{239}Pu	3.1	7.5 μs	3.3	2.6 ns	
^{240}Pu	2.2(?)	37 ns			
^{241}Pu	2.2	21 μs	2.3	32 ns	
^{242}Pu	~2.0	3.5 ns	?	28 ns	8
^{243}Pu	1.7	45 ns			
^{244}Pu	?	0.4 ns			
^{245}Pu	2.0	90 ns			

Isomers in the second well

^{237}Am	2.4	5 ns			
^{238}Am	~2.5	35 μs			
^{239}Am	2.5	163 ns			
^{240}Am	3.0	0.94 ms			
^{241}Am	~2.2	1.2 μs			
^{242}Am	2.2	14 ms			
^{243}Am	2.3	5.5 μs			
^{244}Am	2.8	0.9 ms	?	~6.5 μs	
^{245}Am	2.4	0.64 μs			
^{246}Am	~2.0	73 μs			
^{240}Cm	~ 2.0	10 ps	~3.0	55 ns	550
^{241}Cm	~ 2.3	15.3 ns			
^{242}Cm	~ 1.9	40 ps	~2.8	180 ns	4500
^{243}Cm	1.9	42 ns			
^{244}Cm	~ 2.2	$<$ 5 ps	~3.5	$>$ 100 ns	$>$ 20000
^{245}Cm	2.1	13.2 ns			

[a]B. Singh, R. Zywina, and R. Firestone, Nuclear Data Sheets 97 241 (2002).



Fission half-lives for isomers do not shorten as suggested by this picture, so the barrier for an isomer must probably rise with respect to that for the g.s.

Odd nucleus

Ground state of an odd nucleus in the form of BCS state:

$$|0> = a^+_{\nu_0} \prod_{\mu \neq \nu_0} (u_\mu + v_\mu a^+_\mu a^+_{\bar{\mu}})|vac>$$

Calculating adiabatic mass parameter for this state we obtain the following formula:

$$\begin{aligned} B_{q_i q_j} &= 2\hbar^2 \Big[\sum_{\mu,\nu \neq \nu_0} \frac{\langle \mu | \partial_{q_i} \hat{H} | \nu \rangle \langle \nu | \partial_{q_j} \hat{H} | \mu \rangle}{(E_\mu + E_\nu)^3} (u_\mu v_\nu + u_\nu v_\mu)^2 \\ &+ \frac{1}{8} \sum_{\nu \neq \nu_0} \frac{(\tilde{\varepsilon}_\nu (\partial_{q_i} \Delta) - \Delta(\partial_{q_i} \tilde{\varepsilon}_\nu)) (\tilde{\varepsilon}_\nu (\partial_{q_j} \Delta) - \Delta(\partial_{q_j} \tilde{\varepsilon}_\nu))}{E_\nu^5} \Big] \\ &+ 2\hbar^2 \sum_{\nu \neq \nu_0} \frac{\langle \nu | \partial_{q_i} \hat{H} | \nu_0 \rangle \langle \nu_0 | \partial_{q_j} \hat{H} | \nu \rangle}{(E_\nu - E_{\nu_0})^3} (u_\nu u_{\nu_0} - v_\nu v_{\nu_0})^2 \end{aligned}$$

- if another state comes close to the blocked state ν_0 then mass parameter explodes!
- if the blocked state ν_0 lies higher in energy than other state ν one gets negative values of mass parameter!

The main point: for odd Z or/and N or for a K-isomer adiabatic tunneling is a nonsense:

Adiabatic B for such states can be huge at close crossings and this implies a vanishingly small collective velocity there.

But: there is no reason to assume adiabatic motion; at any collective velocity there will be some, usually non – adiabatic, tunneling.

However, then there is no (general) expression for the mass parameter.

Ideas:

-Partial release of K quantum number – first barriers are often triaxial;

-Consider the minimization of S allowing the pairing gap to vary freely [L.G. Moretto and R.P. Babinet Phys. Lett. 49B, 147 (1974)]. A. Staszczak, A. Baran, Pomorski & K. Boning found that this decreases the action in Phys. Lett. 161, 227 (1985). Then Yu. A. Lazarev showed in a simple model [Phys. Scripta 35, 255 (1987)] that the action minimization with respect to the gap would reduce (a desired outcome) fission hindrance for odd-A nuclei and isomers.

Caveats:

-The cranking inertia was used in S;

- The gap is related to the Hamiltonian and should be determined by the dynamics before the action is calculated.

Instanton method

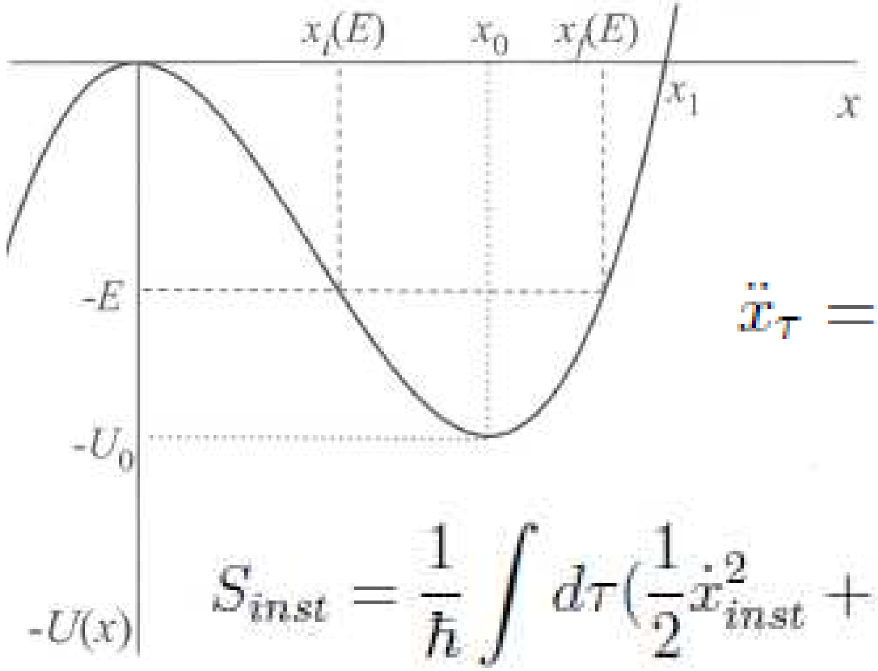
In field theory: S. Coleman, Phys. Rev. D 15 (1977) 2929
In nuclear mean-field theory:
S. Levit, J.W. Negele and Z. Paltiel, Phys. Rev. C22 (1980) 1979

Reformulation & Connection to other approaches to the Large Amplitude Collective Motion:
J. Skalski, PRC 77, 064610 (2008).

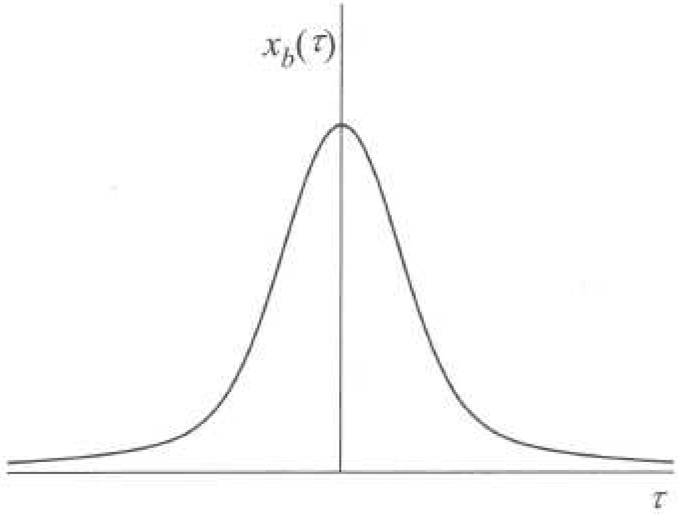
The main idea: even if there is no mass, there is action.

A consequence: the requantization of the collective motion may be sometimes meaningless.

Instantons – imaginary-time, periodic solutions of the mean-field equations with boundary conditions set by the metastable state. 1D analogy:



$$\ddot{x}_\tau = \frac{\partial U}{\partial x}$$



$$S_{inst} = \frac{1}{\hbar}\int d\tau\left(\frac{1}{2}\dot{x}_{inst}^2 + U(x)\right) = \frac{2}{\hbar}\int dx\,(2U(x))^{1/2}$$

$$\Gamma = \left(\frac{S_{inst}}{2\pi}\right)^{1/2}\left|\frac{det[-\partial_\tau^2 + \partial_x^2 U(x=0)]}{det'[-\partial_\tau^2 + \partial_x^2 U(x = x_{inst})]}\right|^{1/2} e^{-S_{inst}}.$$

E.M. Chudnovsky and J. Tejada "Macroscopic Quantum Tunneling of the Magnetic Moment", Cambridge University Press 1998

The selfconsistent iTDHFB equations:

Floquet exponents (periodicity)

from: $$\delta \int i \cdot d\tau \langle \Phi(\tau) \mid \hbar\partial_\tau + \hat{H} \mid \Phi(-\tau) \rangle = 0,$$

$$\hbar\partial_\tau \begin{pmatrix} A_k(\tau) \\ B_k(\tau) \end{pmatrix} + \begin{pmatrix} \hat{h}(\tau) - \lambda, & \hat{\Delta}(\tau) \\ -\hat{\Delta}^*(-\tau), & -(\hat{h}^*(-\tau) - \lambda) \end{pmatrix} \begin{pmatrix} A_k(\tau) \\ B_k(\tau) \end{pmatrix} = \zeta_k \begin{pmatrix} A_k(\tau) \\ B_k(\tau) \end{pmatrix}$$

$$\frac{\langle \Phi(\tau) \mid a_\nu^\dagger a_\mu \mid \Phi(-\tau) \rangle}{\langle \Phi(\tau) \mid \Phi(-\tau) \rangle} = \rho_{\mu\nu}(\tau) = (B^*(-\tau) B^T(\tau))_{\mu\nu},$$

$$\frac{\langle \Phi(\tau) \mid a_\nu a_\mu \mid \Phi(-\tau) \rangle}{\langle \Phi(\tau) \mid \Phi(-\tau) \rangle} = \kappa_{\mu\nu}(\tau) = (B^*(-\tau) A^T(\tau))_{\mu\nu},$$

$$\frac{\langle \Phi(\tau) \mid a_\nu^\dagger a_\mu^\dagger \mid \Phi(-\tau) \rangle}{\langle \Phi(\tau) \mid \Phi(-\tau) \rangle} = \tilde{\kappa}_{\mu\nu}(\tau) = (A^*(-\tau) B^T(\tau))_{\mu\nu}.$$

$$\rho(-\tau) = \rho^\dagger(\tau),$$

$$\kappa^T(\tau) = -\kappa(\tau),$$

$$\tilde{\kappa}(\tau) = \kappa^\dagger(-\tau).$$

$$\hat{h}(\tau) = \hat{t} + \hat{\Gamma}(\tau)$$

$$\Gamma_{\mu\nu}(\tau) = \sum_{\gamma\delta} (v_{\mu\gamma\nu\delta} - v_{\mu\gamma\delta\nu}) \rho_{\delta\gamma}(\tau),$$

$$\Delta_{\mu\nu}(\tau) = \sum_{\gamma\delta} v_{\mu\nu\gamma\delta} \kappa_{\gamma\delta}(\tau).$$

$$S/\hbar = \int_{-T/2}^{T/2} d\tau \langle \Phi(\tau) | \partial_\tau \Phi(-\tau) \rangle$$
$$= \frac{1}{2} \int_{-T/2}^{T/2} d\tau\, Tr[\partial_\tau A^\dagger(-\tau) A(\tau) + \partial_\tau B^\dagger(-\tau) B(\tau)]$$
$$= -\frac{1}{2} \int_{-T/2}^{T/2} d\tau\, Tr[A^\dagger(-\tau) \partial_\tau A(\tau) + B^\dagger(-\tau) \partial_\tau B(\tau)].$$

Another form of action integrand:

$$-\sum_{i\ obs} \frac{\zeta_i}{2} - \frac{1}{2} \sum_{\mu\nu} \left((h_{\mu\nu}(\tau) - \lambda\delta_{\mu\nu})(2\rho_{\nu\mu}(\tau) - \delta_{\mu\nu}) + \kappa_{\mu\nu}(\tau)\Delta^*_{\mu\nu}(-\tau) + \kappa^*_{\mu\nu}(-\tau)\Delta_{\mu\nu}(\tau) \right).$$

The iTDHFB equations conserve:

$$\langle \Phi(\tau) \mid \hat{H} \mid \Phi(-\tau) \rangle$$

$$\sum_\mu \left(B^*_{\mu i}(-\tau) B_{\mu j}(\tau) + A^*_{\mu i}(-\tau) A_{\mu j}(\tau) \right) = \delta_{ij},$$

$\langle \Phi(\tau) \mid \sum_\mu a^\dagger_\mu a_\mu \mid \Phi(-\tau) \rangle = Tr\ \rho$ - only at selfconsistency

Instanton is a self-consistent solution to the boundary value rather than the initial value problem. Once found, it gives some fission rate – without any mass parameter.

- The fission trajectory is defined by all coordinates of the quasiparticle vacua (Z matrices), not by a few arbitrarily chosen deformation parameters – more complicated than in chemistry.
- A priori, solution can be found for any metastable state – even or odd-A, g.s. or an isomer.
- The instanton with the lowest action gives the fission rate.
- There were ideas that pairing field (delta) could be larger in tunneling; iTDHFB eqs. describe changes in this field that follow from a well defined procedure.

At present, the selfconsistent solution seems difficult (too many variables).

Simplification (1): Woods-Saxon potential + pairing with the matrix element –G/2 in the adiabatic basis; one has two sets of equations with matrices:

$$\begin{pmatrix} \hat{\epsilon}(q)+\hat{D}, & -\Delta(\tau)\cdot\hat{I} \\ -\Delta^*(-\tau)\cdot\hat{I}, & -\hat{\epsilon}(q)+\hat{D} \end{pmatrix} \qquad \begin{pmatrix} \hat{\epsilon}(q)+\hat{D}^*, & \Delta(\tau)\cdot\hat{I} \\ \Delta^*(-\tau)\cdot\hat{I}, & -\hat{\epsilon}(q)+\hat{D}^* \end{pmatrix}$$

$\hat{\epsilon}(q)$ - diagonal

$$D_{\mu\nu} = \hbar\langle\mu \mid \frac{\partial\nu}{\partial\tau}\rangle = \hbar\dot{q}\langle\mu \mid \frac{\partial\nu}{\partial q}\rangle$$

selfconsistency: $\Delta(\tau) = G\sum_{\mu>0}\bar{\kappa}_{\mu\bar{\mu}}$

$$S = \int_{-T/2}^{T/2} d\tau \left\{ -\sum_{i>0}\zeta_i - \sum_{\mu>0}\left((2\bar{\rho}_{\mu\mu}(\tau)-1)(\epsilon_\mu(\tau)-\lambda) + \Delta(\tau)\bar{\kappa}^*_{\mu\bar{\mu}}(-\tau) + \bar{\kappa}_{\mu\bar{\mu}}(\tau)\Delta^*(-\tau)\right) \right\}$$

Required selfconsistent solution only for one function: Delta.

Simplification (2): Woods-Saxon potential, no pairing. Solution in the adiabatic basis (no selfconsistency):

$$\phi_i(\tau) = \sum_\mu c_{\mu i}(\tau)\psi_\mu(q(\tau)),$$

$$\hbar\frac{\partial c_{\mu i}}{\partial \tau} + \dot{q}\sum_\nu^N \langle \psi_\mu(q(\tau)) \mid \frac{\partial \psi_\nu}{\partial q}(q(\tau))\rangle c_{\nu i} = [\zeta_i - \varepsilon_\mu(q(\tau))]c_{\mu i}$$

quasi-occupations: $p_{\mu i}(\tau) \;=\; c^*_{\mu i}(-\tau)c_{\mu i}(\tau)$

$$S_i/\hbar = \frac{1}{\hbar}\int_{-T/2}^{T/2} d\tau \left\langle \phi_i(-\tau)\middle|\zeta_i - \hat{h}(\tau)\middle|\phi_i(\tau)\right\rangle = \frac{1}{\hbar}\int_{-T/2}^{T/2} d\tau \sum_{\mu=1}^{N}[\zeta_i - \varepsilon_\mu(q(\tau))]p_{\mu i}(\tau).$$

Simplifications (1) & (2) need extra definition of collective velocity; We tried various choices; one is: $B_{even}(q)\dot{q}^2 = 2(V(q) - E_{g.s.})$

equivalent to collective velocity depending primarily on energy.

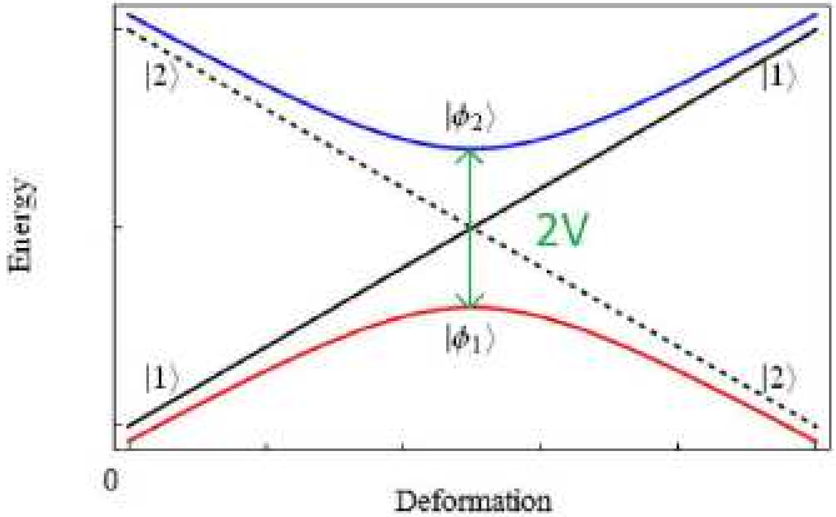
Two-level model

$$\hat{H}(q(\tau)) = \begin{pmatrix} E_1(q(\tau)) & V \\ V^* & E_2(q(\tau)) \end{pmatrix}$$

with:

$E_{1,2} = \pm E(q - q_0)$,

$V = V^*$.



Nomenclature:

$|1\rangle, |2\rangle$ – **diabatic basis**,

$|\phi_1\rangle, |\phi_2\rangle$ – **adiabatic basis**.

Eigenenergies:

$$\varepsilon_{1,2} = \frac{1}{2}\left[(E_1 - E_2) \mp \sqrt{(E_1 - E_2)^2 + 4V^2}\right].$$

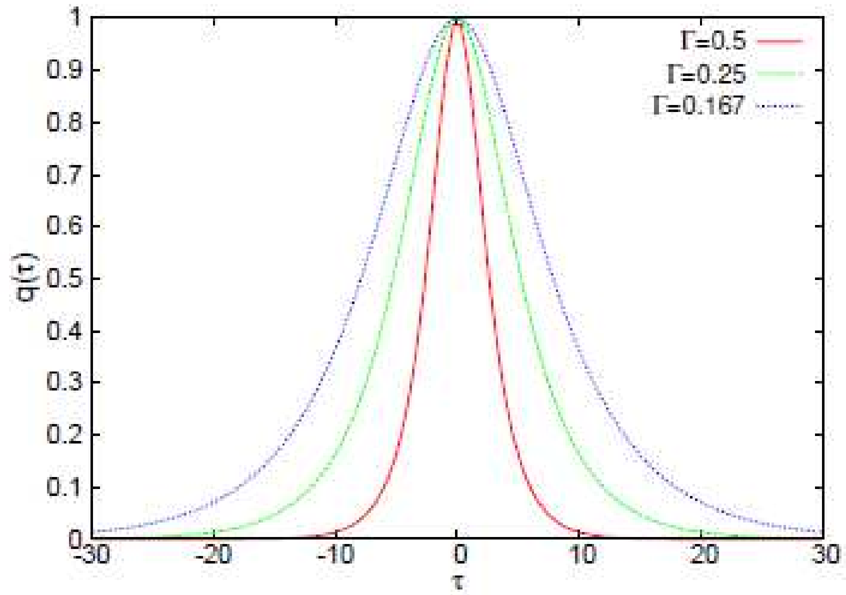
Coupling:

$$\left\langle \phi_1 \middle| \frac{d\phi_2}{dq} \right\rangle = \frac{1}{2}\frac{\alpha}{(q - q_0)^2 + \alpha^2}, \quad \alpha = V/E.$$

The function $q(\tau)$ is modelled by:

$$q(\tau) = \frac{q_{fin} - q_{ini}}{\cosh(\Gamma\tau)} + q_{ini},$$

where q_{ini}, q_{fin} are the deformations at the beginning and at the end of the fission barrier respectively.

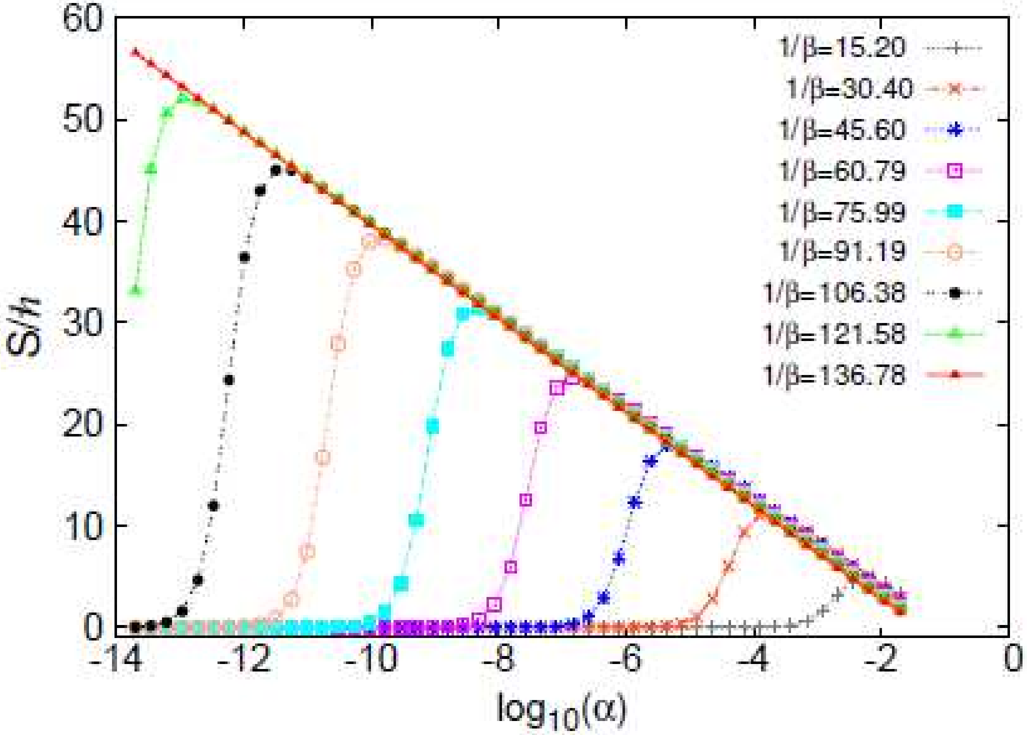


The instanton equations for this system read:

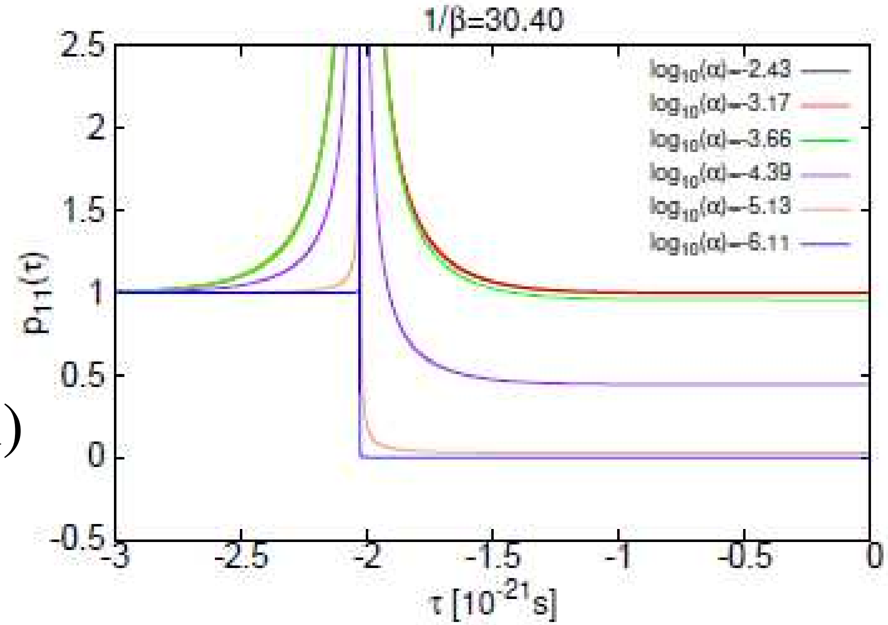
$$\frac{d}{dz}\tilde{c}_1 = \sqrt{(q-q_0)^2+\alpha^2}\,\tilde{c}_1 + \frac{1}{2}\beta\tanh(\beta z)\,(q-q_{ini})\,\frac{\alpha}{(q-q_0)^2+\alpha^2}\,\tilde{c}_2,$$
$$\frac{d}{dz}\tilde{c}_2 = -\sqrt{(q-q_0)^2+\alpha^2}\,\tilde{c}_2 - \frac{1}{2}\beta\tanh(\beta z)\,(q-q_{ini})\,\frac{\alpha}{(q-q_0)^2+\alpha^2}\,\tilde{c}_1,$$

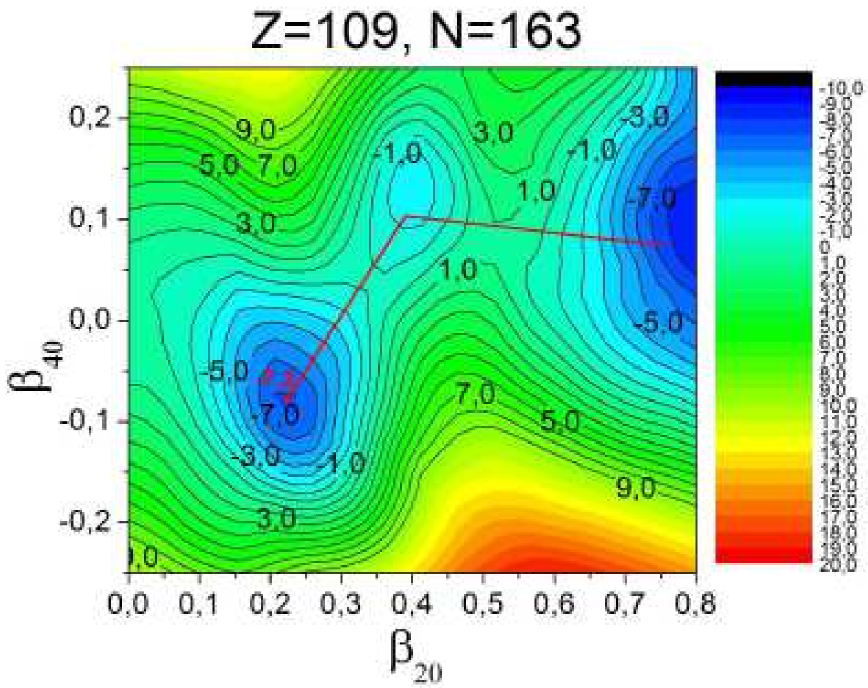
where $\beta = \hbar\Gamma/|E|$ and $z = \tau\frac{|E|}{\hbar}$.
We see that the instanton solutions and, therefore, corresponding action values depend on two parameters: $\alpha = V/E$ and β defined above, i.e. $S = S(\alpha, \beta)$.

Action $S(\alpha, \beta)$

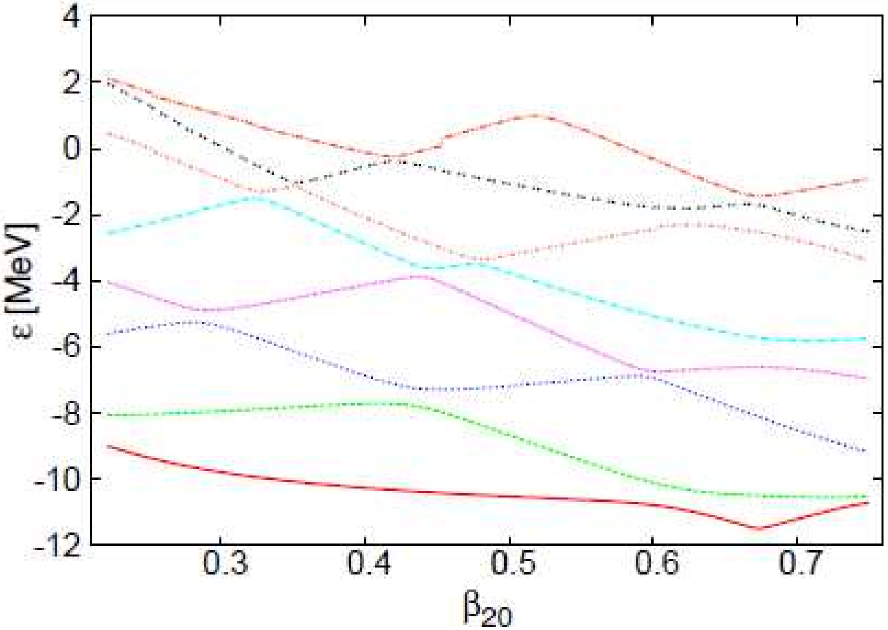


quasi-occupations(tau)

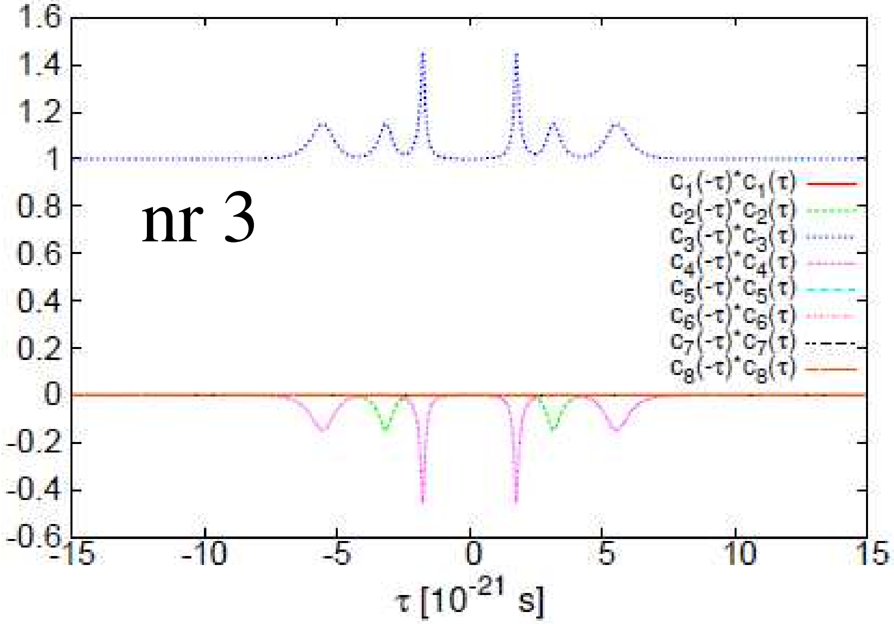


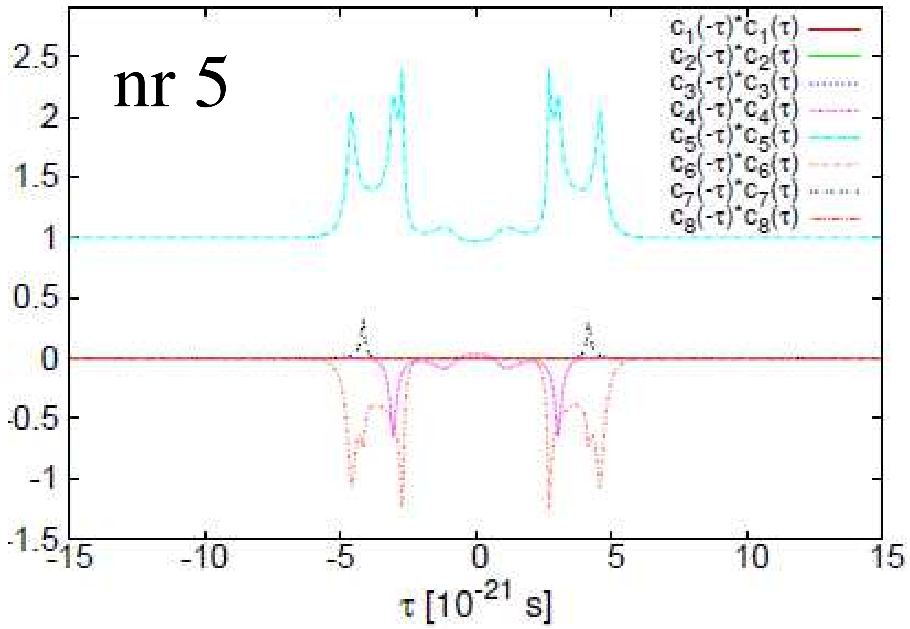


Levels n ½+



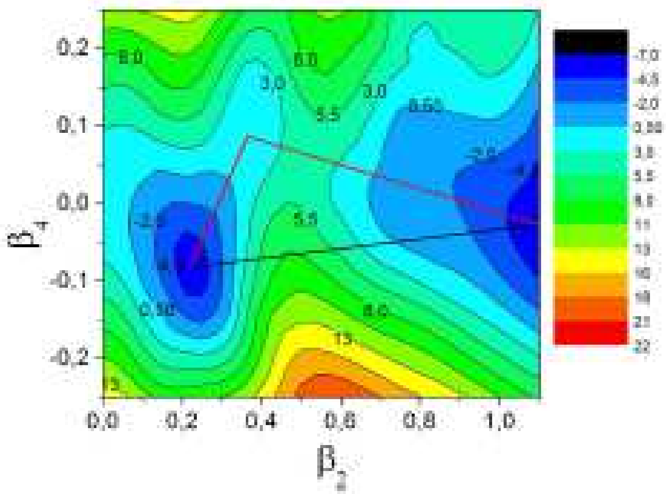
Quasi-occupations for levels 3&5



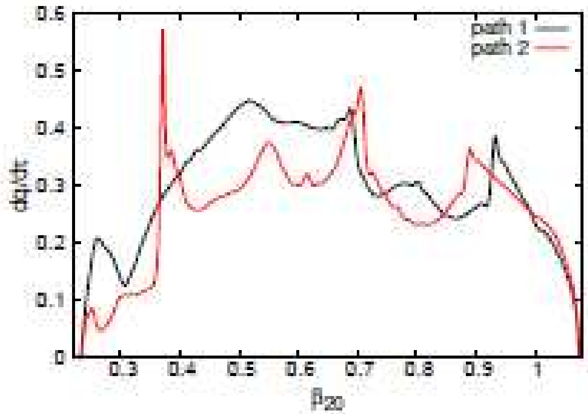


W-S neutron $1/2^+$ levels (8 states)

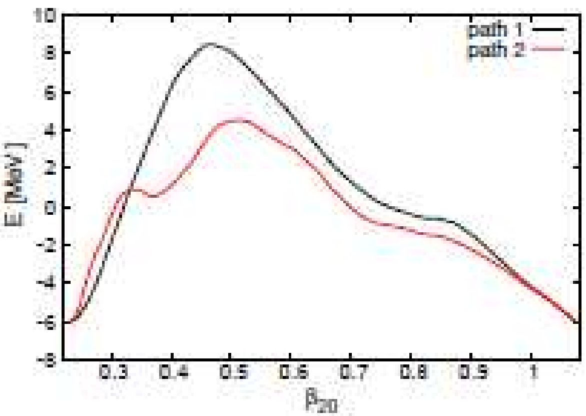
Energy landscape with blocked configuration:



Behaviour of the $\dot{q}$:



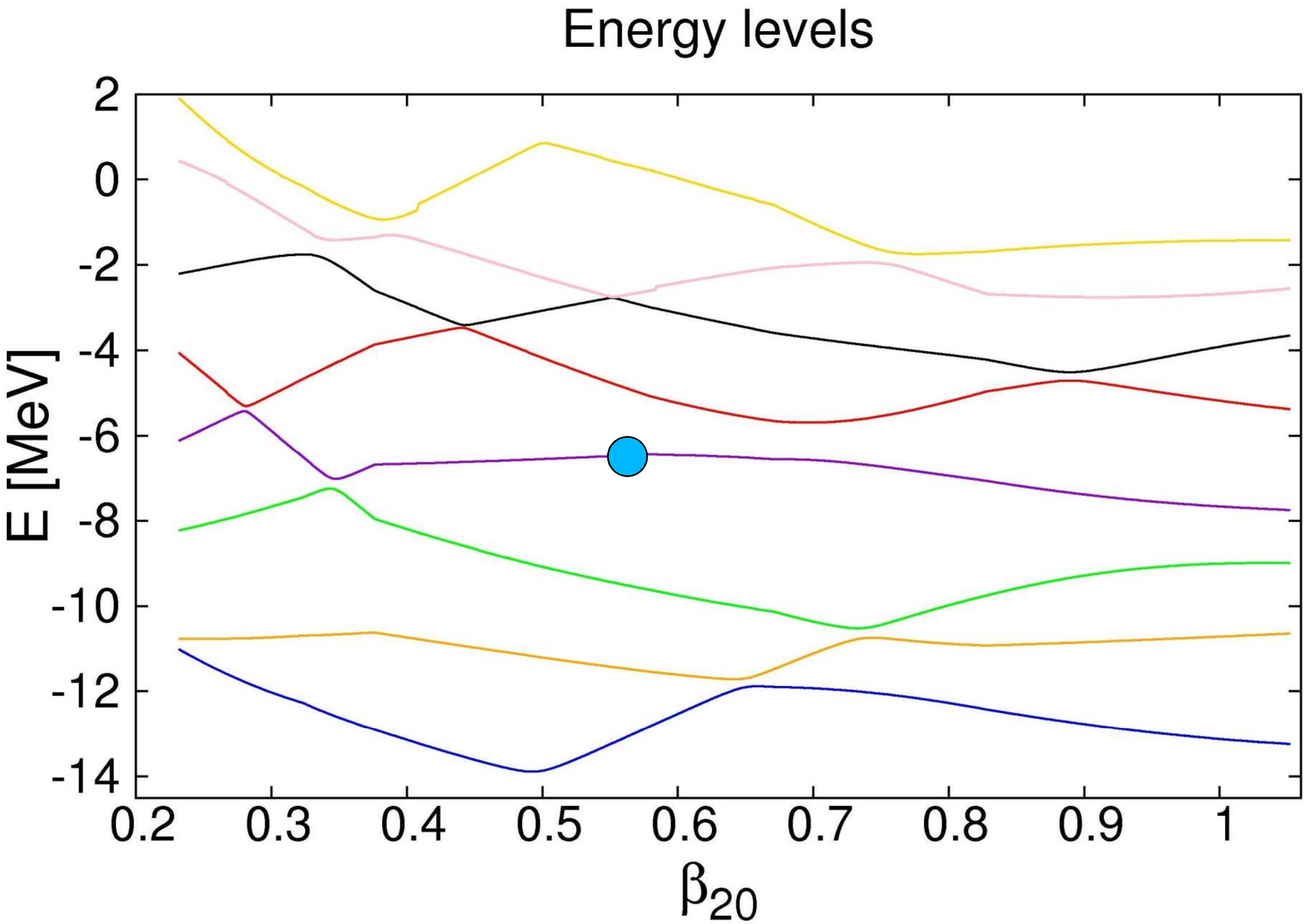
Shape of the potential along the chosen paths:



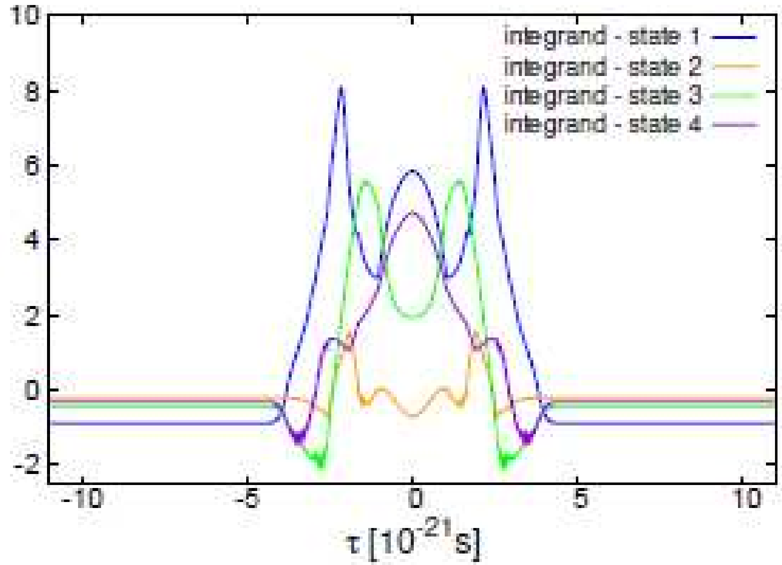
Comparison of the action values:

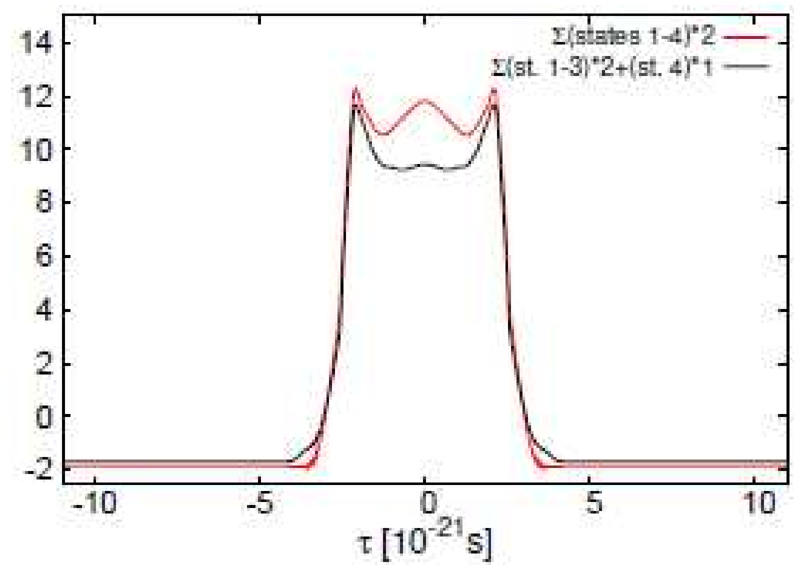
Nr	black path	red path
1	5.6803	1.6402
2	-2.1203	4.1620
3	4.6213	2.7241
4	-2.0957	1.2913
Sum	6.0857	9.8175

Neutron 3/2+ levels along the axially-symmetric fission barrier
Z=109, N=163



Action integrands





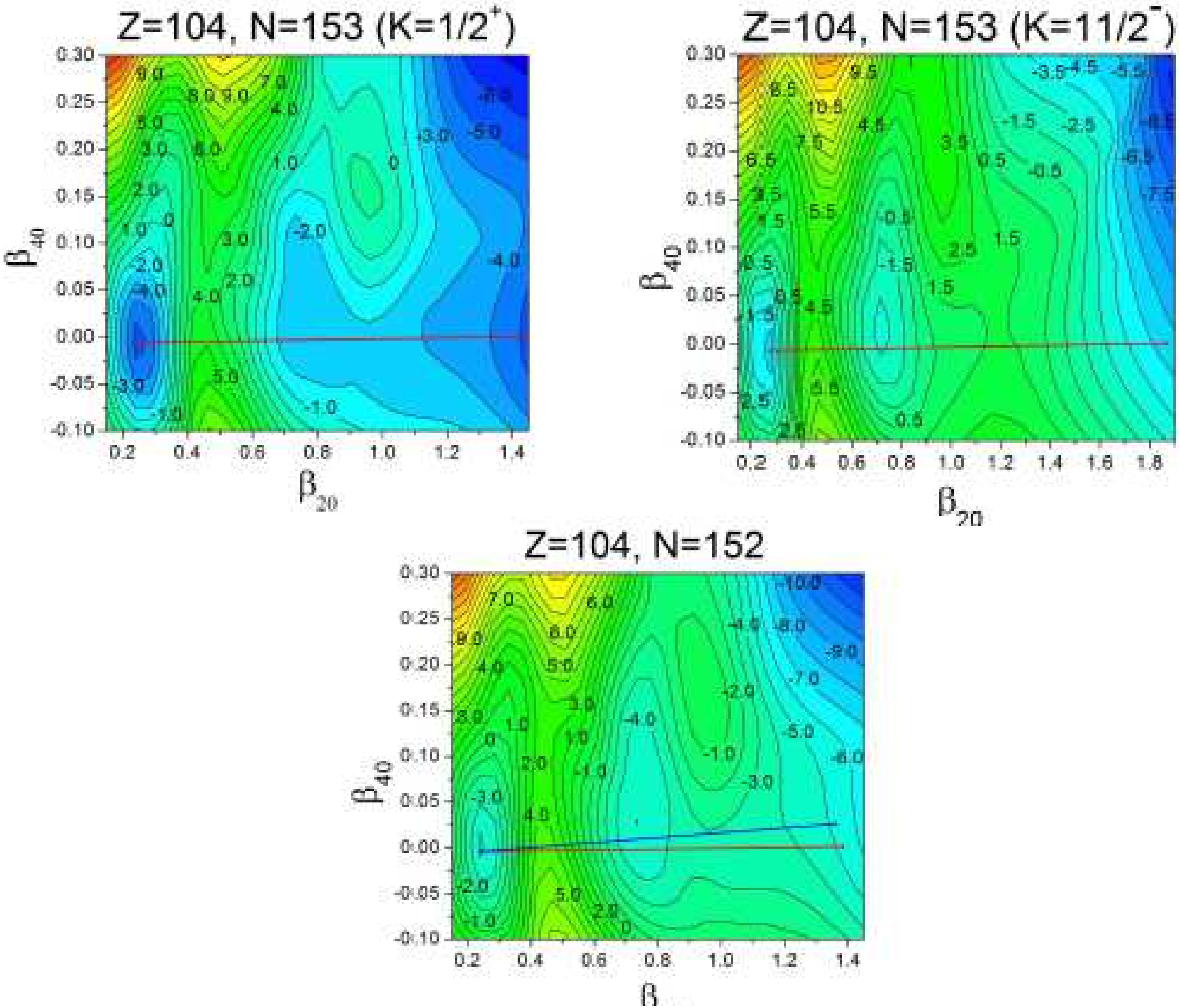
Question: Is it possible to extract mass parameter from the total action integrand: $\sum_{i,occ}\sum_{\mu=1}^{\mathcal{N}}[\zeta_i - \varepsilon_\mu(q)]p_{\mu i}(\tau) = B\dot{q}^2$?

Answer: Such "mass parameter" would depend on the path and on the collective velocity $\dot{q}$!

A hybrid model for odd – even fission hindrance factor, i.e. fission half-life ratios for seven SH even(A) – odd(A+1) pairs.

For an e-e nucleus - cranking action with pairing; for an odd one – the cranking action calculated with the odd-(A+1) energy and even-A mass parameter - for the e-e core (assumption of the collective character of tunneling); then the instanton action (without pairing) for the odd nucleon may be added.

Only axially - and reflection - symmetric shapes.
We are interested in fission hindrance factors HF.


Z=104, N=153 (K=1/2+)
Z=104, N=153 (K=11/2-)
Z=104, N=152
β40
β20
β

Even – even nuclei with adjusted E_zp:

Nucleus	$S_{crank}/\hbar$	T_{sf}^{exp} [s]	T_{sf}^{calc} [s]
^{258}No	21.60	1.2E-03	4.1E-03
^{254}Rf	18.46	2.3E-05	7.8E-06
^{256}Rf	21.91	6.4E-03	7.6E-03
^{260}Rf	22.97	2.2E-02	6.4E-02
^{258}Sg	21.92	2.6E-03	7.7E-03
^{260}Sg	23.62	7.0E-03	2.4E-01
^{282}Cn	18.82	9.1E-04	1.6E-05

Odd – A nuclei with adjusted E_zp;
very much overestimated action for fixed configurations.

Nucleus	K^π	$S^{conf}_{crank}/\hbar$	$S^{ad}_{crank}/\hbar$	$\Delta S_{crank}/\hbar$
^{259}Lr	7/2-	33.32	23.44	9.88
^{255}Rf	9/2-	56.06	25.31	30.75
^{257}Rf	1/2+	34.32	22.58	11.74
^{257}Rf (m)	11/2-	48.89	22.58	26.31
^{261}Db	9/2+	40.79	26.65	14.14
^{259}Sg	1/2+	32.44	23.23	9.21
^{261}Sg	3/2+	30.75	25.30	5.45
^{283}Cn	5/2+	24.52	21.56	2.96

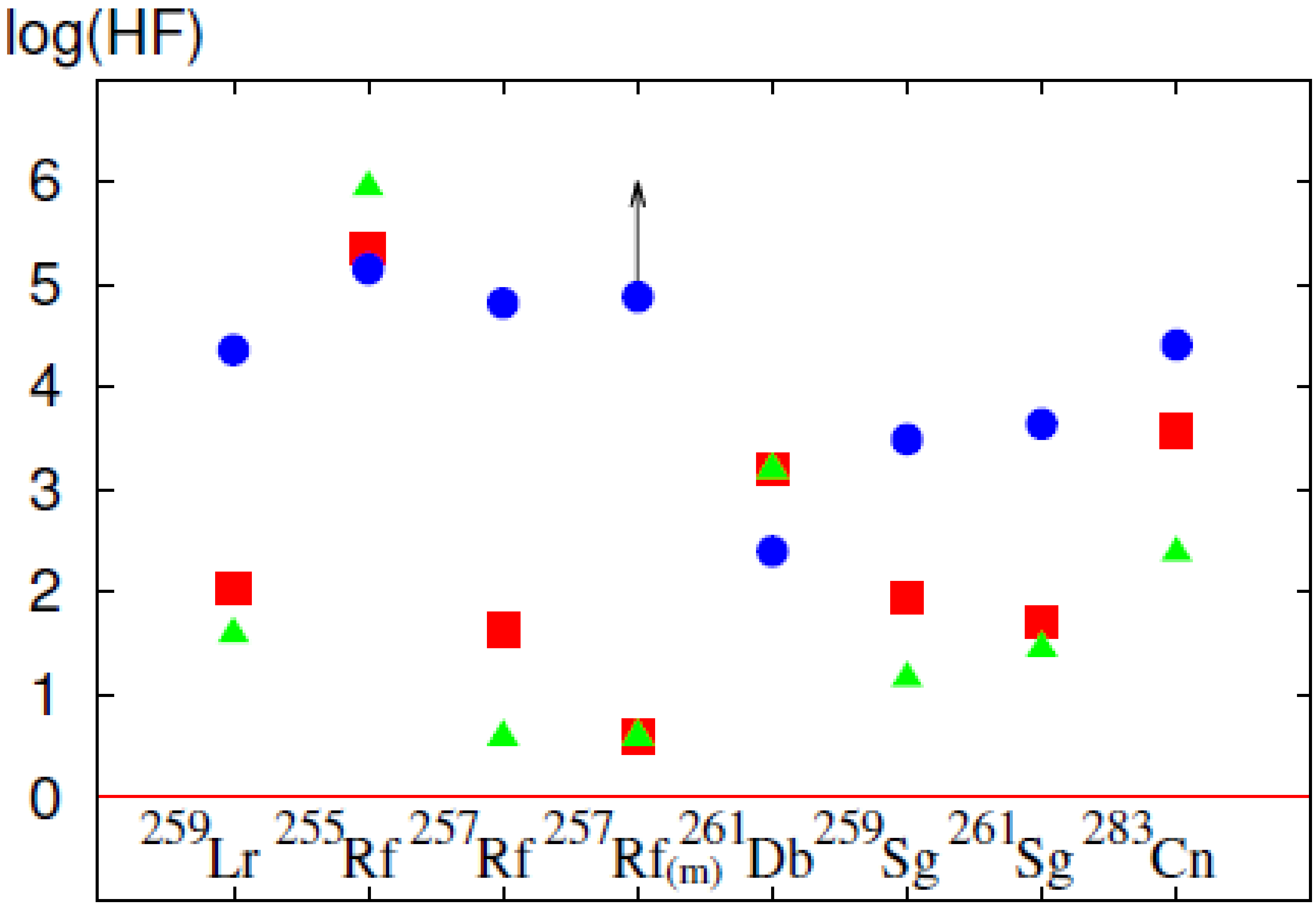
Nucleus data				Adiabatic blocking				
^{A}X	I^{π}	T_{sf}^{exp} [s]	HF_{exp}	$S_{s.p.}^{inst}/\hbar$	T_{sf}^{cr} [s]	$T_{sf}^{cr+inst}$ [s]	HF_{calc}^{cr}	$HF_{calc}^{cr+inst}$
^{259}Lr	7/2-	27.4	2.3E+04	1.02	0.16	0.45	3.9E+01	1.1E+02
^{255}Rf	9/2-	3.15	1.4E+05	-1.37	6.83	1.73	8.8E+05	2.2E+05
^{257}Rf	1/2+	423	6.6E+04	2.43	0.03	0.33	3.9E+00	4.34E+01
^{257}Rf (m)	11/2-	>490	>76562.5	0.03	0.03	0.03	3.9E+00	3.9E+00
^{261}Db	9/2+	5.6	2.5E+02	0.04	99.6	103.6	1.56E+03	1.62E+03
^{259}Sg	1/2+	8	3.1E+03	1.85	0.11	0.68	1.43E+01	8.83E+01
^{261}Sg	3/2+	31	4.4E+03	0.61	6.7	12.32	2.79E+01	5.13E+01
^{283}Cn	5/2+ (*)	24	2.6E+04	2.76	0.0038	0.06	2.38E+02	3.75E+03

↑ ↑

With instanton contribution

$$HF = \frac{T^{o}_{sf}}{T^{c}_{sf}},$$

b: exp., g: ad. crank. for a core, r: with odd nucleon instanton contribution;



Conclusions

- Experimental data suggest a mechanism for fission hindrance in both odd-A nuclei and isomers.
- Such states can have longer fission half-lives in the SHN region.
- The instanton method adapted to the mean-field formalism may provide a basis for the minimization of action.
- The non-selfconsistent studies indicate that: the action is well defined for an arbitrary path & is determined by states close to the Fermi level & has the adiabatic limit for sufficienly small velocities.
- If mass parameters of odd and even nuclei are similar, fixing configuration largely overestimates half-lives of the former & adding instanton contribution without pairing is not sufficient.
- Instantons with pairing require a selfconsistent determination of Delta(tau) – work in progress.