Nuclear Collective Excitations and Realistic Models

John Sharpey-Schafer et al.

Dr. Rob Bark
iThemba LABS

Dr. Suzan Bvumbi
NRWDI, Pelendaba

Dr. Tshepo Dinoko
NMISA, Pretoria

Dr. Siyabonga Majola
Uni. Johannesberg

Review Article; EPJA55 (2019) 15
“Stiff” Deformed Nuclei, Configuration Dependent Pairing and the $\beta$ and $\gamma$ Degrees of Freedom.

4th June 2020

Warsaw Colloquium
Examples of collective Classical time-dependent Vibrations of the Mean-field

$\lambda = 2, a_{2,0}$  
Quadrupole $\beta$ vibration

$\lambda = 2, a_{2,2}$  
Quadrupole $\gamma$ vibration

$\lambda = 3, a_{3,0}$  
Octupole vibration

web-docs.gsi.de/~wolle/TELEKOLLEG/KERN/index-s.html
However!! Simple pictures can be Misleading

the Heisenberg Uncertainty Principle tells us that the Nucleon-Nucleon Interaction is not Strong Enough to Localise the Nucleons

You can only Measure $<R^2>$ for instance with electron scattering $(e,e)$
Assume axially symmetric \( (m=0) \) shape.

Expand shape in Legendre Polynomials

\[
\mathbf{R}(r,\theta) = R \{ 1 + a_2 P_2(\cos\theta) + a_3 P_3(\cos\theta) \ldots \} = R \{ 1 + \sum a_{\lambda} P_{\lambda}(\cos\theta) \}
\]

Consider the Dimensions supposing that \( \omega^2 \propto \gamma^x \rho^y R^z \)

\( \omega^2 = \text{frequency}^2 \) of vibration with dimensions \( [T]^2 \)

\( \gamma = \text{surface energy/unit area} \) with dimensions \( [M] [L]^2 [T]^{-2} [L]^{-2} \)

\( \rho = \text{density} \) with dimensions \( [M] [L]^{-3} \)

\( R = \text{radius} \) with dimensions \( [L] \)


So that

\[
\begin{align*}
-2 &= -2x & \text{ie.} & & x = 1 \\
0 &= x + y & \text{ie.} & & y = -1 \\
0 &= -3y + z & \text{ie.} & & z = -3
\end{align*}
\]

\( \omega^2 \propto \frac{\gamma}{\rho} R^3 \)
Classical Vibrations of a Liquid Drop

By considering a superfluid incompressible liquid sphere


\[ \omega^2 = \frac{(\lambda-1)\lambda(\lambda+2)\gamma}{\rho R^3} \]

For a charged spherical nucleus this becomes:

\[ \omega^2 = \frac{(\lambda-1)\lambda(\lambda+2)}{3} - \frac{C_s}{R_A m_A} - \frac{2(\lambda-1)\lambda}{(2\lambda+1)} \frac{e^2 Z^2}{4\pi\varepsilon_0 R_A^3 m_p A^2} \]

Where \( C_s \) is the SURFACE term in the Weizsäcker Binding Energy formula

\[ E_b = C_V - C_s A^{2/3} - C_C Z^2 A^{-1/3} - C_A (N-Z)^2 A^{-1} \pm \delta \]

and the second term has little effect for \( Z<80 \).
Quantization of the Vibrations of a Liquid Drop

Quantize Using \( E_x = \hbar \omega \) and

\[
\omega^2 = (\lambda - 1)\lambda(\lambda + 2)C_s/3R_A^2m_A
\]

Where \( C_S \sim 18 \text{ MeV} \) and \( R_A = 1.3 \text{ fm} \)

The Classical Result is that

Vibrations are WELL above the Pairing Gap

Pairing Energy \( \Delta \approx 12/A^{1/2} \text{ MeV} \)

From Bohr and Mottelson
Other Factors Affecting Vibrations

1. Moments–of-Inertia not superfluid \( \rightarrow \text{will put vibrational energy UP} \)

**Fig. 1.** Comparison of the different models for moment of inertia with experimental data.


Inglis-Belyaev Cranking code CONRAD
2. Shell Corrections → \textbf{will put vibrational energy UP}

\textbf{Figure 17.} Single-particle level energies calculated for an axially symmetric harmonic oscillator (from reference 18).
The Bohr and Mottelson Approach

MODES OF NUCLEAR VIBRATION

\[ \nu = 0, \beta - \text{vibration} \]
\[ bR \propto (3 \cos^2 \theta - 1) \cos \omega t \]

\[ \nu = 1, \text{rotation} \]
\[ bR \propto \sin \theta \cos \theta \cos (\phi \pm \omega t) \]

\[ \nu = 2, \gamma - \text{vibration} \]
\[ bR \propto \sin^2 \theta \cos (2\phi \pm \omega t) \]

Figure 3. Ground-state, and \( \beta \)-, \( \gamma \)-, and octupole-vibrational bands in \( ^{152}\text{Sm} \).
The β and γ Quadrupole Degrees of Freedom

The Lund Convention

The β and γ Quadrupole Degrees of Freedom

$E_{SHO} = (n+\frac{1}{2}) \hbar \omega$
Bohr & Mottelson

$Iachello X(5)$

$E_{SqW} \propto n^2$
UNEXPECTED STRONG PAIR CORRELATIONS IN EXCITED 0⁺ STATES OF ACTINIDE NUCLEI

Argonne National Laboratory, Argonne, Illinois 60439
(Received 1 June 1970)

The (p, t) reaction has been studied with 17-MeV protons on targets of Th232, U234, 236, 238, and Pm241, 243. The results indicate unexpectedly strong 1 = 0 transitions to states at about 900-1000 eV excitation. Their cross sections are approximately 15% of the ground-state transitions; this percentage does not change appreciably with neutron number. This result, together with other available evidence, seems to suggest a simple and rather stable collective mode which has not yet emerged from any theoretical calculations.

2³⁸U(p, t)²³⁶U
17 MeV
θ = 30°

₀²⁺ NOT a β-vibration
NOR a pairing vibration
Two Neutron Transfer to $^{154}$Gd (N=90)

*Shiro Yoshida,* Nucl. Phys. 33, 685 (1962)
Showed that with Monopole Pairing ALL the TWO neutron Transfer strength will be Decanted into the Residual Ground State

**SEE ALSO**

**HENCE** Monopole Pairing is NOT Sufficient
Similar Structures Built on the Ground state $0_{1}^{+}$ and Second Vacuum $0_{2}^{+}$ state in $^{154}$Gd and $^{152}$Sm
What is the $|0^+_2>$ Configuration?

- $(t,p)$ & $(p,t)$ ➡️ $|0^+_2>$ is $2p_n - 2h_n$

- This gives $J^\pi$ but nothing on the orbit.

- Single particle transfer would give $l_n$ but does not populate $|0^+_2>$. 

In $\{ |0^+_2> + \text{neutron}\}$, look to see which orbit does NOT couple to $|0^+_2>$. 

4th June 2020
Configuration Dependent Pairing


Suggested that $\Delta_{pp} \approx \Delta_{oo} \gg \Delta_{op}$

for Actinide Nuclei where $0^2_2^+$ states were observed in (p,t) that were not pairing- or $\beta$-vibrations.

Suppose there are $n$ prolate and $n$ oblate degenerate levels at the Fermi Surface;

Assume that each pairing matrix element is the same for the same type - $a$

BUT the prolate-oblate matrix elements are very weak - $\varepsilon a$

Then if the prolate $n*n$ matrix is $A$, the oblate matrix is also $A$

The matrix for the total system is:

$$
\begin{bmatrix}
A & \varepsilon A \\
\varepsilon A & A
\end{bmatrix}
$$

Then there are $(2n-2)$ states with ZERO energy and 2 states with energies $E_{1,2} = -(1 \pm \varepsilon) na$

I. Ragnarsson and R. A. Broglia, Nucl. Phys. A263, 315 (1976). coined the term “pairing isomers” for these $0^2_2^+$ states
An analysis of the $(p,t)$ reaction on $^{158}$Dy

J. J. Kolata
Brookhaven National Laboratory, Upton, New York 11973

M. Oothoudt
Princeton University, Princeton, New Jersey 08540

(Received 28 February 1977)

**FIG. 1.** Typical spectrum for the $^{158}$Dy$(p,t)^{156}$Dy reaction at $E_p = 29.9$ MeV and $\theta_{\text{lab}} = 10^\circ$. This is a com-

$^{158}$Dy$(p,t)^{156}$Dy

$\sim 30$ MeV $\theta = 10^\circ$

Ground state

Pairing Isomer
Configuration Dependent or Quadrupole Pairing; Assume \( \Delta_{pp} \approx \Delta_{oo} \gg \Delta_{op} \)

82 Neutrons

Prolate Deformation =>

"Flying Fish" Orbital

[505]11/2+ "Oblate"

[668]1/2+ "Prolate"
What you really need is SPLIT MONOPOLE PAIRING so that

\[- \mathcal{H}_{\text{pairing}} = G_{p-p} \hat{P}_p ^\dagger \hat{P}_p + G_{o-o} \hat{P}_o ^\dagger \hat{P}_o + \varepsilon G_{p-p} \hat{P}_{po} ^\dagger \hat{P}_{po}\]

\[\varepsilon \approx 0.05 \ ? ?\]
HOW TO MEASURE THE PAIRING ??

Use the Cranked Shell Model !!

Coriolis term in the Hamiltonian = \(- j_x \omega\)

Pairing

Gap \(2\Delta\)

\[ \Delta = 0.125 \quad \lambda = 6.420 \]

\[ \varepsilon_2 = 0.212 \quad \varepsilon_4 = -0.11 \]

\[ {}^{162}_{70}\text{Yb}_{92} \]

Coriolis term in the Hamiltonian = \(- j_x \omega\)
i_{13/2} neutron AB alignments from N = 88 to 98 and Z = 62 to 72

See:
Jerry Garrett et al. PL B118 (1982) 297

Cranked Shell Model
Routhians e'
Excitation Energies of $0_2^+$ band-heads and $\nu[505]11/2^-$ isomers

Excitation Energies of $K = 2^+ \gamma$ band-heads
Quantum Number $K = I_z$ the spin projection on the $\gamma = 0^\circ$ symmetry axis
Siyabonga Majola et al.
PR C91 (2015) 034333

$^{148}\text{Nd}(12\text{C},4\text{n})^{156}\text{Dy}$
Gammasphere Data

$^{156}_{62}\text{Dy}_{90}$
Positive Parity Bands
Tracking of $\gamma$ band through $i_{13/2}$ neutron $AB$ alignment and $h_{11/2}$ proton alignment $ab$
Ollier et al. PR C83 (2011) 044309
You cannot ignore the $\gamma$ degree of freedom!!

No $\gamma$ deformation

Nikšić et al., PRL 99 (2007) 092502

Li et al., PR C 79 (2009) 054301

Relativistic Energy-Density Functionals (REDF)
Hope in Shell Models ??

DATA FROM
Majola et al. PRC91 (2015) 034330

TPSM Successes
1. Predicts $\gamma$ and $\gamma\gamma$ bands
2. Predicts $S_n$-band and $S_n+S_p$ -band
3. Predicts observed $\gamma$ band built on $S_n$-band
4. Predicts an $S_n$-band built on $0_2^+$
5. Can show components of Wavefunctions

TPSM Failures
1. Pairing too crude, No Neutron Pairing Isomer $0_2^+$ too high in Energy
2. Signature Splitting not spot on

156\text{Dy} Triaxial Projected Shell Model
Javid Sheikh et al.

\[ S_n+S_p+K^+=2_1^+ \]
What you really need is $\epsilon \approx 0.05$ ??

SPLIT MONOPOLE PAIRING

\[ \hat{H}_{\text{pairing}} = G_{p-p} \hat{\rho}_p \hat{\rho}_p^+ + G_{o-o} \hat{\rho}_o \hat{\rho}_o^+ + \epsilon G_{p-p} \hat{\rho}_p \hat{\rho}_o^+ \hat{\rho}_o \hat{\rho}_p^+ \]

\[ S_n+S_p \]
\[ S_n+K^+=2_\gamma^+ \]

Jehangir et al.,
PRC97 (2018) 014310

4th June 2020
Warsaw Colloquium
The Bohr Hamiltonian
Uses a 5-D Space \((\theta, \varphi, \psi, \beta, \gamma)\) to
Characterize a Macroscopic Nuclear Drop
Rotating and Vibrating in Space

Quantization is achieved by the usual Pauli prescription:

\[ E(\alpha, \dot{\alpha}) = \frac{1}{2} C \alpha^2 + \frac{1}{2} D \dot{\alpha}^2 \]

\[ \pi = \frac{\partial}{\partial \dot{\alpha}} (T - V) = D \dot{\alpha} \]

\[ H = \frac{1}{2} D^{-1} \pi^2 + \frac{1}{2} C \alpha^2 \]

\[ E(n) = (n + \frac{1}{2}) \hbar \omega \]

\[ \omega = \left( \frac{C}{D} \right)^{1/2} \]

Bohr & Mottelson II Ch.6

\[ \{ \pi, \alpha \} = -i \hbar \]

Li et al., PR C79 (2009) 054301
Can you calculate $V(\beta, \gamma)$ and use Bohr-Type Hamiltonians to calculate the level structure??
Warsaw Colloquium 29

SUCCESSES of the BH-SA-China Collaboration

SYSTEMATICS of Z=62-70, N=88,90,92

South Africa (Experiments) – China (5DCH+CDFT)

Beijing May 2017

Signature Splitting

\[ S(I) = \frac{[E(I) - E(I-1)] - [E(I-1) - (E(I-2))]}{E(2,1^+)} \]

Due to BAND MIXING!!

Majola et al., PRC100, 044324 (2019) +
Zhi Shi, Zhipan Li, Shuangquan Zhang et al.

South Africa (Experiments) – China (5DCH+CDFT)

Linda Mdletshe et al.
EPJA54 (2018) 176

4th June 2020
Pairing Gap = 2Δ

Pairing Gap = 2Δ

Pairing Energy $\Delta \approx 12/A^{1/2}$ MeV

From Bohr and Mottelson
Experimental Pairing Gap for Semi-Magic Nuclei holding either Z or N = constant

\[ \text{B&M GAP} = \frac{24}{A^{1/2}} \text{ MeV} \]
They conclude: “….the existence of low-energy quadrupole vibrations in nuclei must be seriously questioned.”

Recently Paul Garrett told us: “These are NOT Vibrations or Phonons” see Garrett et al. PRL123, 142502 (2019)
Atomic nuclei can “organize” their single-particle energies by taking particular configurations of protons and neutrons optimized for each eigenstate, thanks to orbit-dependences of monopole components of nuclear forces (e.g., tensor force).

→ an enhancement of Jahn-Teller effect.

Nilsson-type effects can be enhanced by this optimization.
Monte Carlo Shell Model using $^{110}_{40}\text{Zr}_{70}$ Core, Single Particle Energies (SPE) from $^{123}_{51}\text{Te}_{82}$ and $^{123}_{50}\text{Sn}_{83}$

- Effective interaction: 
  G-matrix* + $V_{\text{MU}}$

  * Brown, PRL 85, 5300 (2000)

Nucleons are excited fully within this model space (no truncation)

We performed Monte Carlo Shell Model (MCSM) calculations, where the largest case corresponds to the diagonalization of $3.9 \times 10^{31}$ dimension matrix.

Otsuka et al., PRL123, 222502 (2019)
Effective Single-Particle Energies (ESPE)

$^{154}\text{Sm}$

$0^+_1$ and $0^+_2$ eigenstates

$0^+_1$ prolate
$0^+_2$ triaxial

ESPEs show very different patterns between eigenstates
Monte Carlo Shell Model

Shape evolution in Sm isotopes (very preliminary)
T. Otsuka NuSpin2018 Valencia

- Energy levels
  - Experimental (exp.)
  - Calculated (calc.)

- Quadrupole moment of 2^+ state

- Monopole frozen original Hamiltonian
- Strong triaxiality
- Prolate minimum
$^{154}$Sm$_{92}$

Calculation

$^{154}$Sm$_{92}$

Experiment

Otsuka et al., PRL123, 222502 (2019)
**Summary**

Nuclear forces are rich enough to optimize single-particle energies for each eigenstate (especially in the cases of collective-mode states), as referred to as *quantum self-organization*. It produces sizable effects with

(i) two quantum fluids (protons and neutrons),
(ii) two major forces: *e.g.*, quadrupole interaction to drive collective mode monopole interaction to control resistance

*This feature fits well the general concept of the self organization.*

“The $0^+_{2}$ and $2^+_2,3$ may not be members of $\beta$ or $\gamma$ vibration, but are triaxially deformed states with stronger fluctuation.”

Effective Single Particle Energies show different patterns to produce such shapes.
LASTLY

What, NO Vibrations ??

or Phonons or Bosons ??!!