Double beta decay within the Skyrme SV density functional theory

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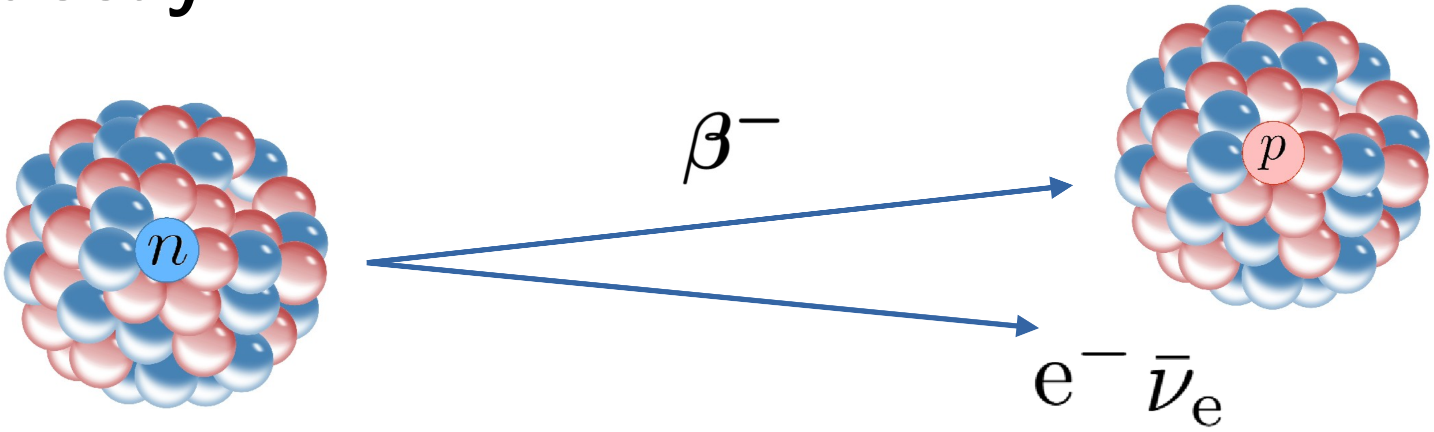
Seminar "Physics of Atomic Nucleus"

Outline of the presentation

I. Theory of the double beta decay
II. DFT-NCCI framework
III. Results of $2\nu\beta\beta$ analysis for 48Ca and 76Ge

I. Theory of the double beta decay

Beta decay



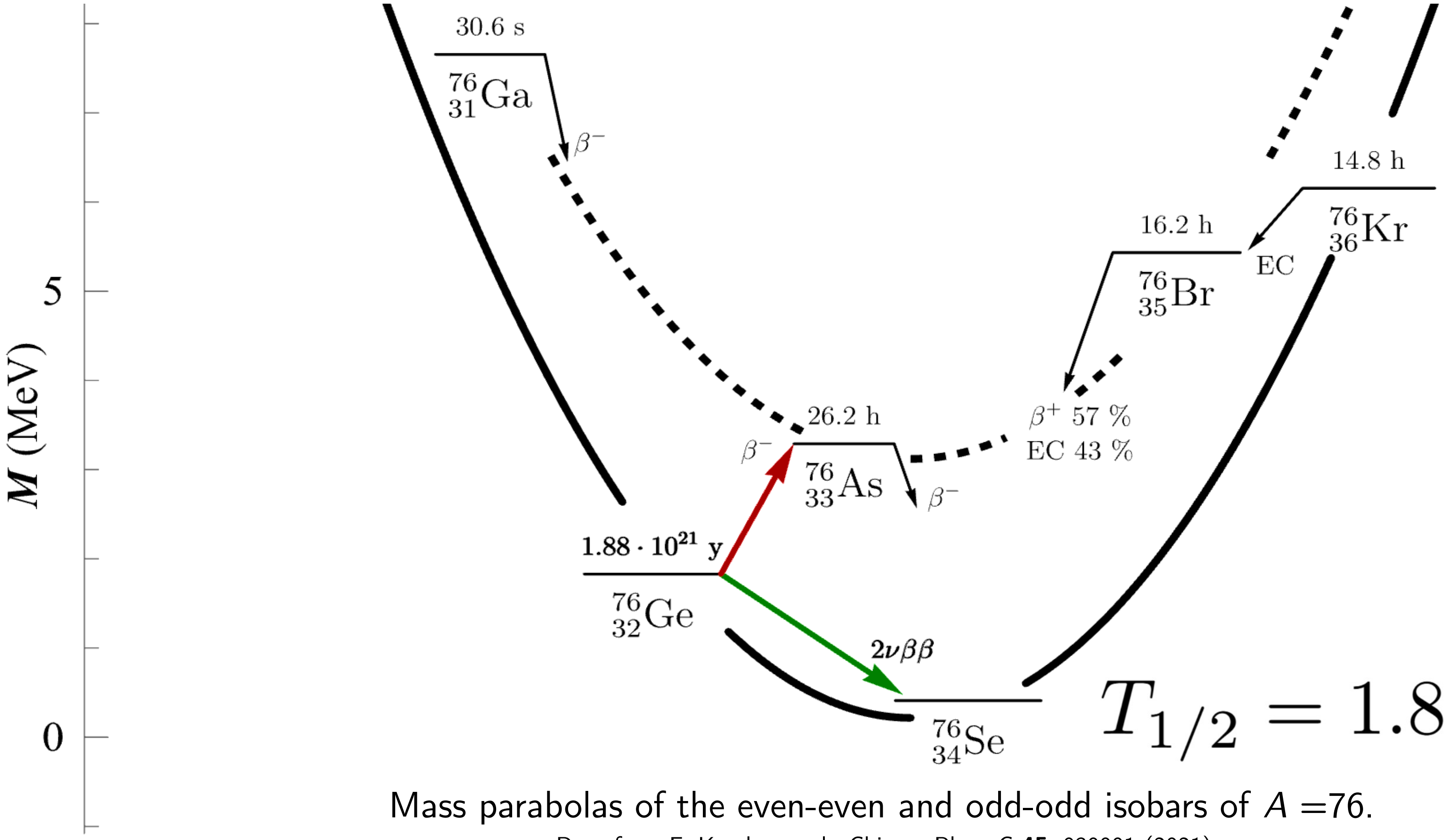
Quantum-wise:

$$\hat{\mathcal{H}}_\beta = \frac{G_F}{\sqrt{2}} J^{\mu\dagger} j_\mu + \text{h.c.}$$

Hadronic current (p → n)

Leptonic current (e → ν_e)

Double beta decay



$$T_{1/2} = 1.88 \cdot 10^{21}\text{y.}$$

Mass parabolas of the even-even and odd-odd isobars of A =76.

Data from F. Kondev et al., Chinese Phys. C **45**, 030001 (2021).

Nuclear matrix element

$$[T_{1/2}^{2\nu}]^{-1} = \underbrace{G^{2\nu}}_{\textit{leptonic part}} \cdot \underbrace{|\mathcal{M}^{2\nu}|^2}_{\textbf{\textit{nuclear matrix element}}}$$

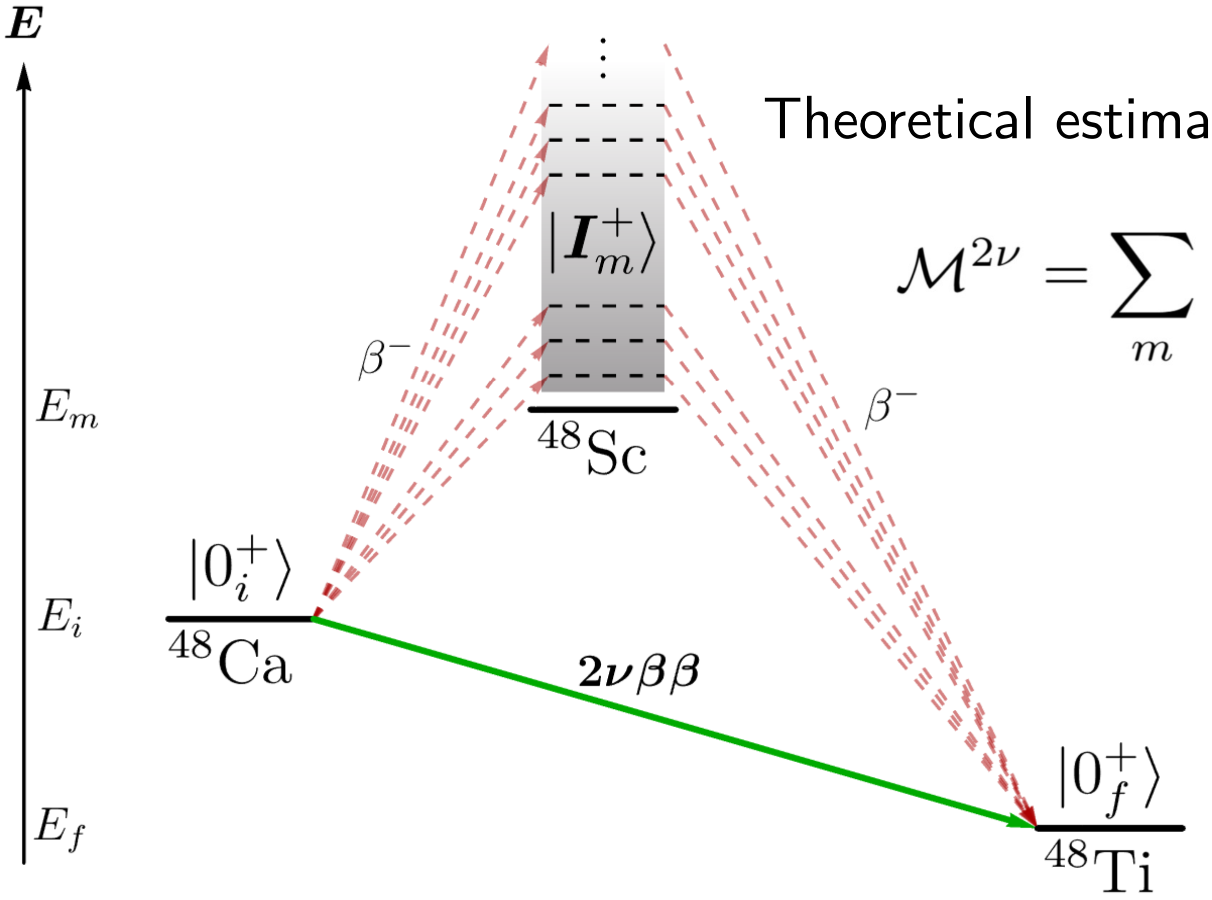
(all nuclear QM)

$$\sim G_F^4$$

Extremely rare!

(only **11** nuclei)

Fermi golden rule of the 2nd order



Theoretical estimation via **virtual** transitions:

$$\mathcal{M}^{2\nu} = \sum_m \frac{\langle 0_f^+ | \hat{H}_\beta | \boldsymbol{I}_m^+ \rangle \langle \boldsymbol{I}_m^+ | \hat{H}_\beta | 0_i^+ \rangle}{\boldsymbol{\Delta E_m} + \frac{1}{2} Q_{\beta\beta} + \Delta M}$$

Forbidden transitions contribute to the total decay rate.

Virtual transition selection rules

The most general form of weak nuclear current:

(T.Tomoda, Rep. Prog. Phys. 1991)

$$J^{\mu\dagger}(x) = \bar{\psi}(x)\tau^{+}\left(g_V\gamma^{\mu} - g_A\gamma_5\gamma^{\mu} - ig_M\frac{\sigma^{\mu\nu}}{2m_N}q_{\nu} + g_P\gamma^5 q^{\mu}\right)\psi(x)$$

Vector | Axial-Vector | Weak-magnetic | Pseudoscalar

<u>NME contains 2 copies of J^{μ}</u>

Only 3 combinations of NME are allowed within V-A theory:

$$\mathcal{M}^{2\nu} \sim \underbrace{\mathcal{M}^{2\nu}_{\mathrm{F}}, \quad \mathcal{M}^{2\nu}_{\mathrm{GT}},}_{\text{Pure VA}} \quad \underbrace{\mathcal{M}^{2\nu}_{\mathrm{T}}}_{\text{M + P admixtures}}$$

Virtual transition selection rules

Isospin conservation: $\cancel{\mathcal{M}^{2\nu}_{\mathrm{F}}}$

Low leptonic momentum transfer: $\cancel{\mathcal{M}^{2\nu}_{\mathrm{T}}}$

The dominant channel becomes double GT transition:

$$\mathcal{M}^{2\nu} = \sum_m \frac{\langle 0_f^+ | \vec{\sigma}\hat{\tau}^- | \mathbf{1_m^+} \rangle \langle \mathbf{1_m^+} | \vec{\sigma}\hat{\tau}^- | 0_i^+ \rangle}{\boldsymbol{\Delta E_m} + \frac{1}{2} Q_{\beta\beta} + \Delta M}$$

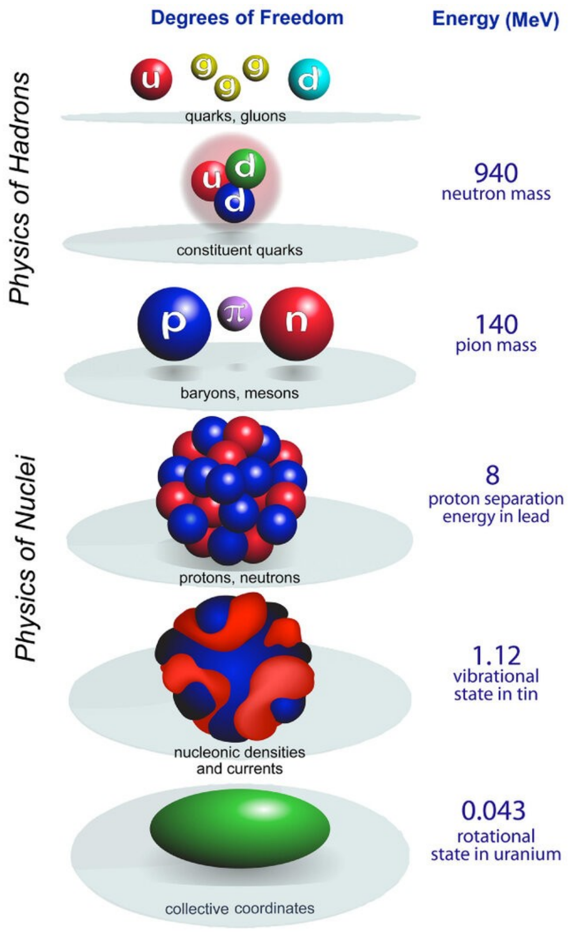
Key difficulty:

$|0_i^+\rangle, |1_m^+\rangle, |0_f^+\rangle$

(strong interaction)

II. DFT-NCCI framework

Dealing with strong interaction



We are here

Hierarchy of the nuclear degrees of freedom.

Picture taken from E. Litvinova and C. Robin (INPC2016).

Nuclear DFT in a nutshell

1) Kohn-Hohenberg theorem:

a) uniqueness of the ground state determined by the density

$$\rho_0(\vec{r}) \Longrightarrow |\Psi_0\rangle$$

b) existence of energy density functional

$$E_0 = \min_{\rho(\vec{r})} E[\rho(\vec{r})]$$

2) Skyrme energy density funtional:

Effective realization of strong interaction in EDF

$$E[\rho] = \int \left(\mathcal{E}_{\mathrm{kin}}[\rho] + \mathcal{E}_{\mathrm{Coul}}[\rho] + \mathcal{E}_{\mathrm{Skyrme}}[\rho]\right) d\vec{r}$$

Skyrme SV functional generator

$$
\begin{aligned}
V_{\text{Skyrme}}(\mathbf{x_1}, \mathbf{x_2}) = {} & t_0(1 + x_0\hat{P}_\sigma)\delta(\vec{r}_1 - \vec{r}_2) + \frac{1}{2}t_1(1 + x_1\hat{P}_\sigma)\left[\overleftarrow{k}^2 \delta(\vec{r}_1 - \vec{r}_2) + \delta(\vec{r}_1 - \vec{r}_2)\overrightarrow{k}^2\right] \\
& + t_2(1 + x_2\hat{P}_\sigma)\overleftarrow{k}\,\delta(\vec{r}_1 - \vec{r}_2)\overrightarrow{k} + \cancel{\frac{1}{6}t_3(1 + x_3\hat{P}_\sigma)\rho^\alpha\left(\frac{\vec{r}_1 + \vec{r}_2}{2}\right)\delta(\vec{r}_1 - \vec{r}_2)} \\
& + iW_0(\vec{\sigma}_1 + \vec{\sigma}_2)(\overleftarrow{k} \times \delta(\vec{r}_1 - \vec{r}_2)\overrightarrow{k}),
\end{aligned}
$$

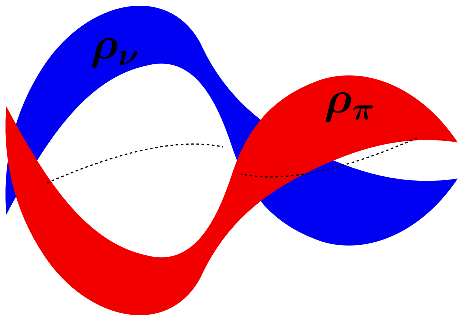
SV

$$\overrightarrow{k} = \frac{1}{2i}(\vec{\nabla}_1 - \vec{\nabla}_2), \qquad \overleftarrow{k} = -\frac{1}{2i}(\vec{\nabla}_1 - \vec{\nabla}_2)$$

The only density-independent parametrization →
no singularities during *I*/*T*-projection

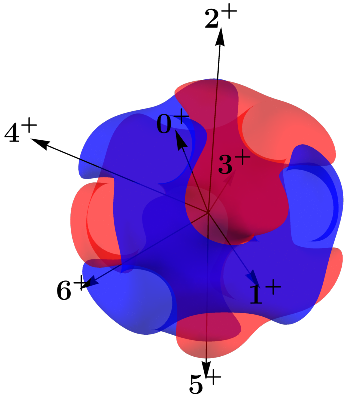
Nuclear DFT in a nutshell

Skyrme SV density functional



$\frac{\delta}{\delta\rho}$

Slater determinant (mean-field)

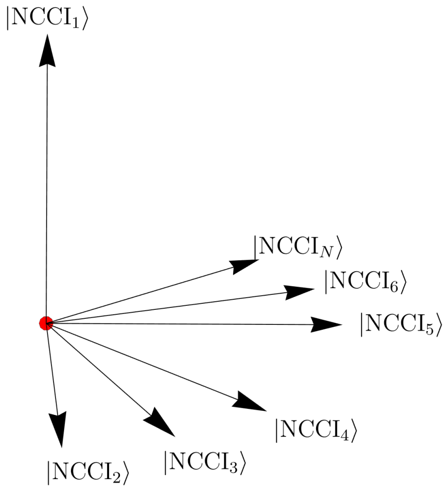


***I*-projection**

conf. mixing

Hill-Wheeler

NCCI basis



no good quantum numbers

~~γ, β transitions~~

good quantum numbers

γ, β **transitions**

DFT

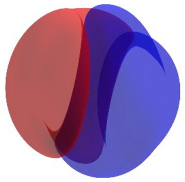
DFT-NCCI

2νββ in DFT-NCCI approach

1) Construction of mean-field configuration classes for each nucleus:

48Ca (parent)

1. Seniority-zero *nn*-pairs



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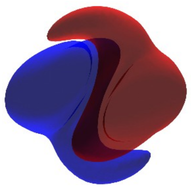
configurations

48Sc (virtual)

1. Seniority-two in $f_{7/2}$
2. Single *n*-excitations across N=28
3. Single *p*-excitations across Z=28



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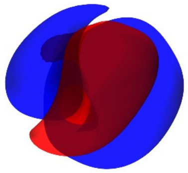
configurations

48Ti (daughter)

1. Seniority-zero *nn*- and *pp*-pairs in $f_{7/2}$
2. *np*-pairing in $f_{7/2}$
3. Seniority-zero *nn*-pairs across N=28 shell
4. Seniority-zero *pp*-pairs across Z=28 shell



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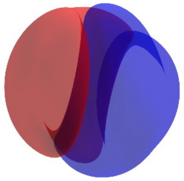
configurations

2νββ in DFT-NCCI approach

2) I^π - projection:

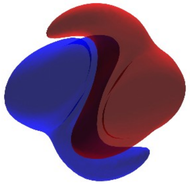
48Ca $|\Psi_i\rangle$



$\hat{P}^I_{MK}$

$$|\Psi_{i1}; 0^+\rangle \quad \vdots \quad |\Psi_{i\alpha}; 0^+\rangle \quad \vdots$$

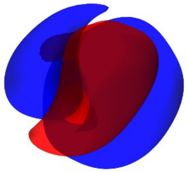
48Sc $|\Psi_m\rangle$



$\hat{P}^I_{MK}$

$$|\Psi_{m1}; 1^+\rangle \quad \vdots \quad |\Psi_{m\beta}; 1^+\rangle \quad \vdots$$

48Ti $|\Psi_f\rangle$



$\hat{P}^I_{MK}$

$$|\Psi_{f1}; 0^+\rangle \quad \vdots \quad |\Psi_{f\gamma}; 0^+\rangle \quad \vdots$$

3) Partial Gamow-Teller elements:

$$\mathcal{M}_{fm} = \langle \Psi_{m\alpha}; 0^+ | \hat{\mathcal{O}}_{\mathrm{GT}} | \Psi_{m\beta}; 1^+ \rangle$$

$$\mathcal{M}_{fm} = \langle \Psi_{m\alpha}; 0^+ | \hat{\mathcal{O}}_{\mathrm{GT}} | \Psi_{m\beta}; 1^+ \rangle$$

Watch out for orthogonality!

In the projection framework GT-matrix element is expressed via single-particle transition densities $\boldsymbol{\rho}$:

$$\langle \boldsymbol{\Psi_i}; I_i^\pi | \hat{O}_{\mathrm{GT}} | \boldsymbol{\Psi_j}; I_i^\pi \rangle \sim \int \langle \Psi_i | \hat{\tau}_{1\mu} \hat{\sigma}_{1\nu} | \tilde{\Psi}_j(\Omega) \rangle \cdot D^{I*}_{K\text{-num.}}(\Omega) d\Omega$$

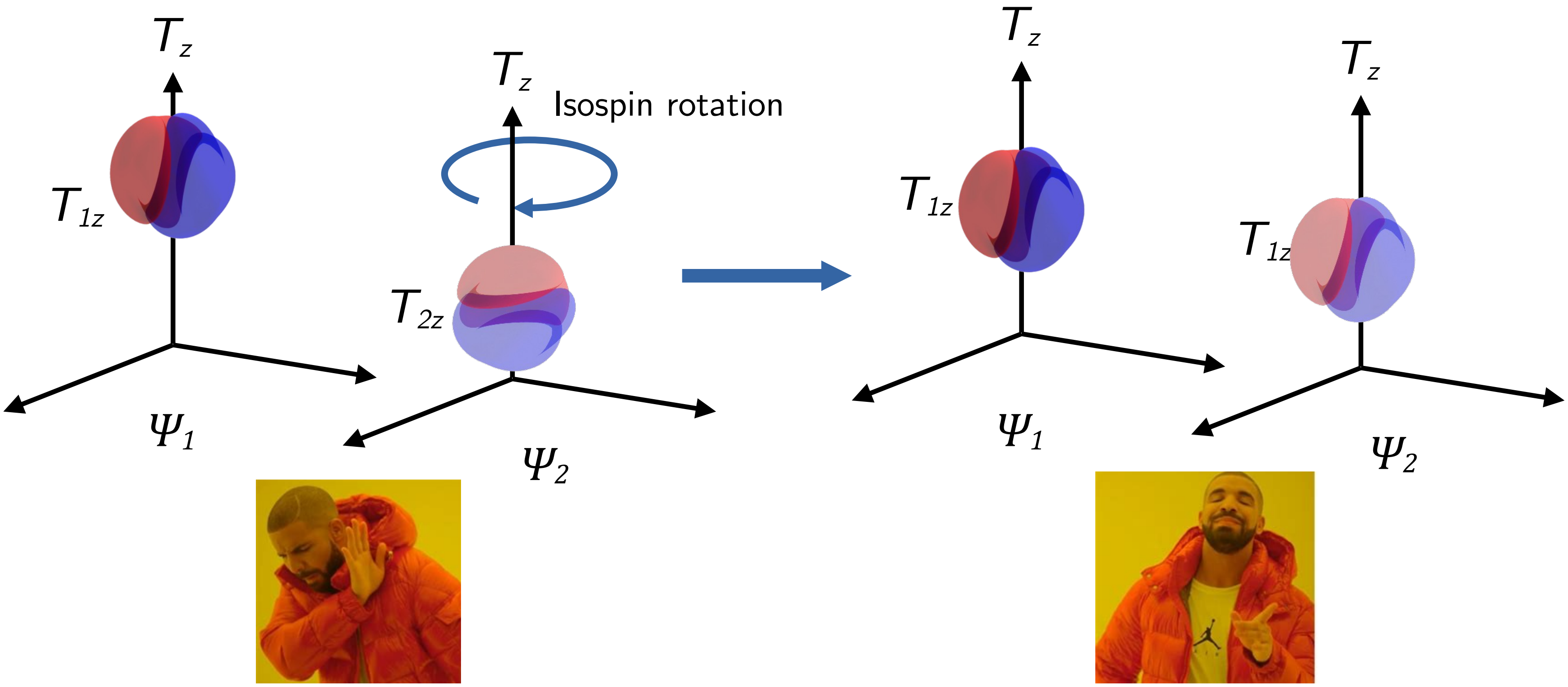
Generalized Wick's Theorem

$$\sim \underbrace{\langle \Psi_i(T_z') | \tilde{\Psi}_j(T_z) \rangle}_{\textbf{orthogonal!}} \sum_{aa';\sigma\sigma'} \langle a | \hat{\tau}_{1\mu} | a' \rangle \langle \sigma | \hat{\sigma}_{1\nu} | \sigma' \rangle \tilde{\boldsymbol{\rho}}(\vec{r}, a', \sigma'; \vec{r}, a, \sigma)$$

But we've got a workaround for that ...

Watch out for orthogonality!

Workaround: rotate in isospace, so both Ψ_1 and Ψ_2 have the same T_z:



2νββ in DFT-NCCI approach

4) Configuration mixing of the projected states:

48Sc:

$$|\Psi_{m1};1^+\rangle \quad \vdots \quad |\Psi_{m\beta};1^+\rangle \quad \vdots$$

mixing

$$|\mathrm{NCCI}_{m1};1^+\rangle \quad |\mathrm{NCCI}_{m2};1^+\rangle \quad \vdots \quad |\mathrm{NCCI}_{m\beta};1^+\rangle \quad \vdots$$

+ same for ***48Ca*** *and* ***48Ti***

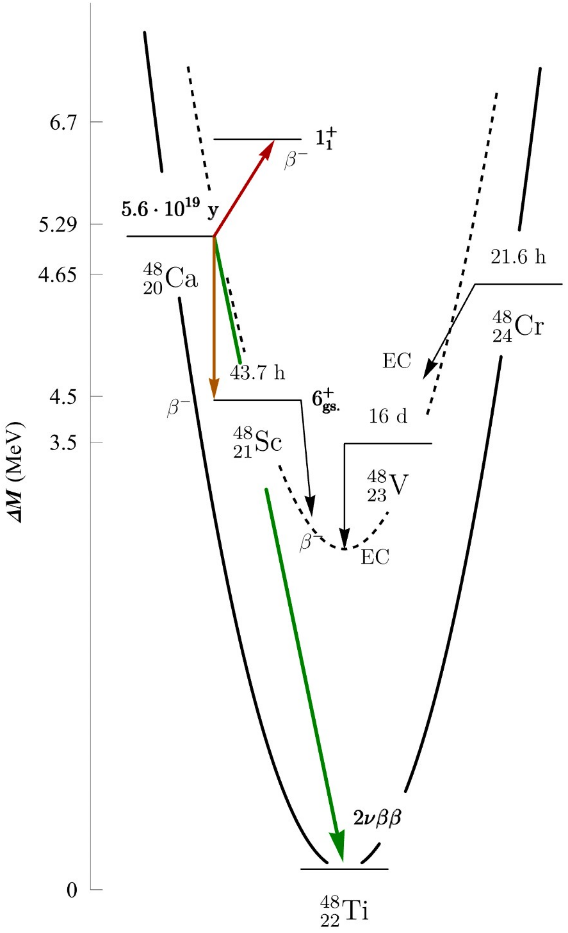
5) Calculation of the cumulative 2νββ matrix element :

$$\mathcal{M}^{2\nu} = \sum_m \frac{\langle \mathrm{NCCI}_f|\hat{O}_{\mathrm{GT}}|\mathrm{NCCI}_m\rangle\langle \mathrm{NCCI}_m|\hat{O}_{\mathrm{GT}}|\mathrm{NCCI}_i\rangle}{\underbrace{\Delta E_m + \frac{1}{2}Q_{\beta\beta} + \Delta M}_{\textit{spectrum shifted to experiment}}}$$

The whole procedure is performed ***numerically*** *within DFT-based* ***HFODD*** *code (open access).*

III. Results of $2\nu\beta\beta$ analysis for 48Ca and 76Ge

48Ca is unique among the 2νββ nuclei

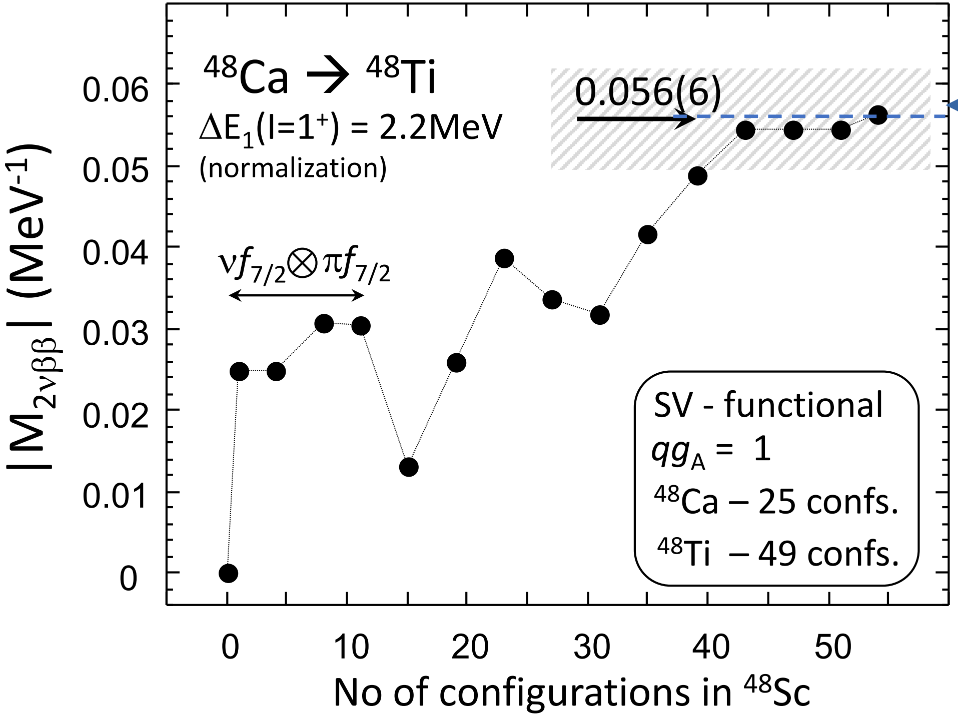


- Doubly magical
- The only spherical nucleus in the **2νββ** set
- Single **β** to 4^+, 5^+, 6^+ possible (orange), but suppressed by **2νββ** (green)

NME(48Ca → 48Ti) : virtual dependence

Resulting cumulative $|\mathcal{M}^{2\nu}|$ **:**

$\Delta E_1(I^\pi{=}1^+){=}2.2$MeV (normalization)



saturation at **0.056** (MeV^{-1})

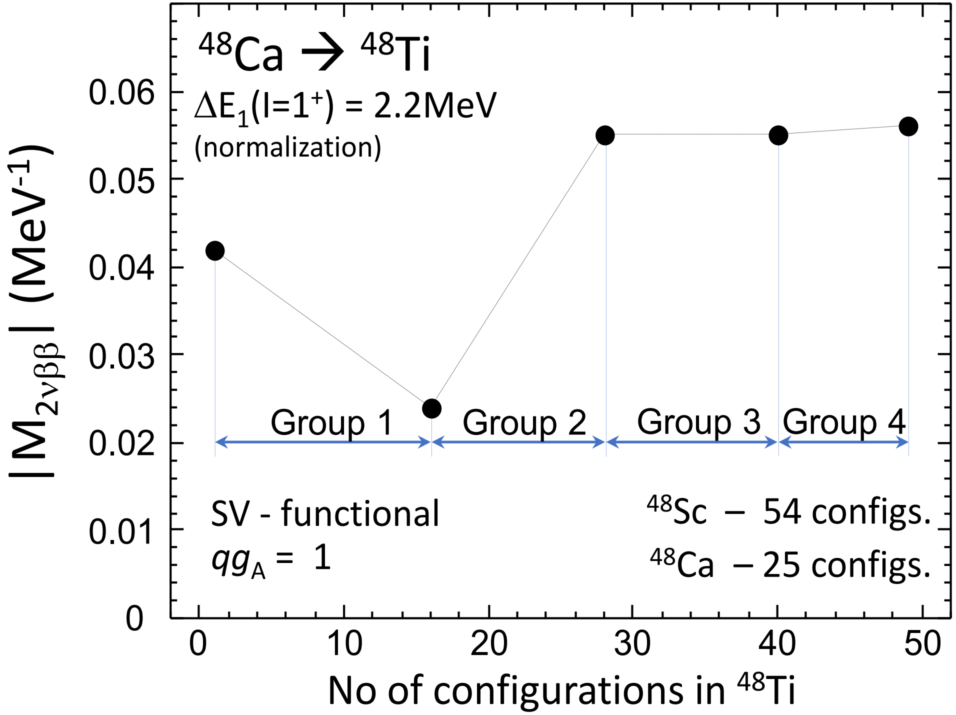
0.029 (dimless)

$q \simeq 0.73$ – assumed from other DFT-NCCI works in the $A{\simeq}40$ region

NME(48Ca → 48Ti) : daughter dependence

Resulting cumulative $|\mathcal{M}^{2\nu}|$ **:**

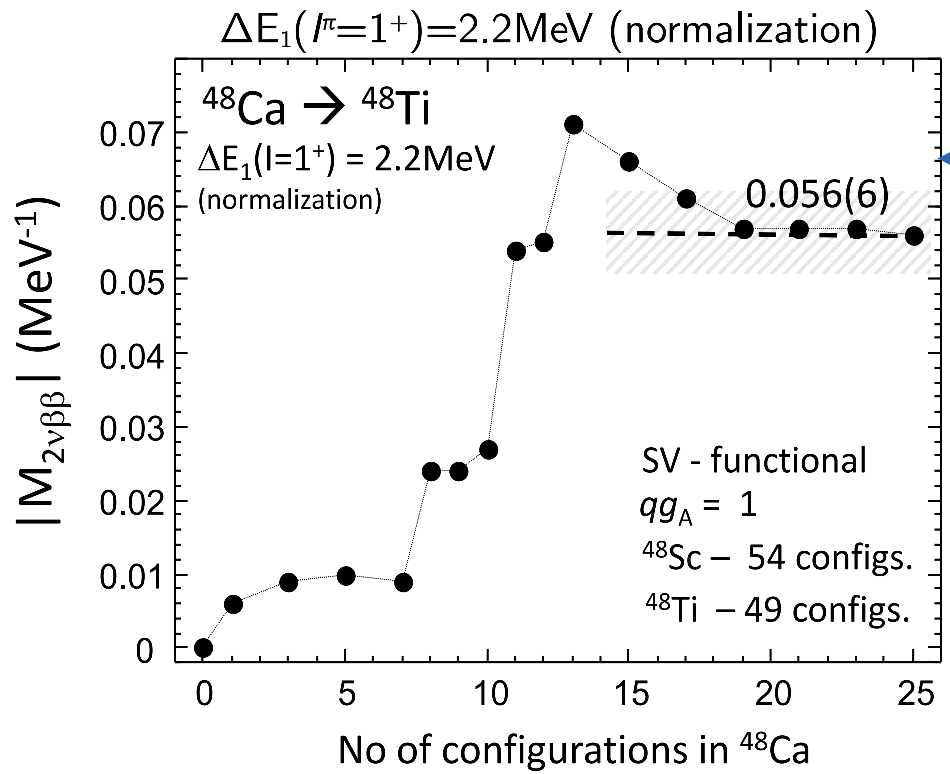
$\Delta E_1(I^\pi\!=\!1^+)\!=\!2.2$MeV (normalization)



Mixing of **crumble** GTs:
88% of the
total matrix element!

NME(48Ca → 48Ti) : mother dependence

Resulting cumulative $|\mathcal{M}^{2\nu}|$ **:**



saturation at **0.056** (MeV^{-1})

0.029 (dimless)

Very surprising result in a spherical nucleus!

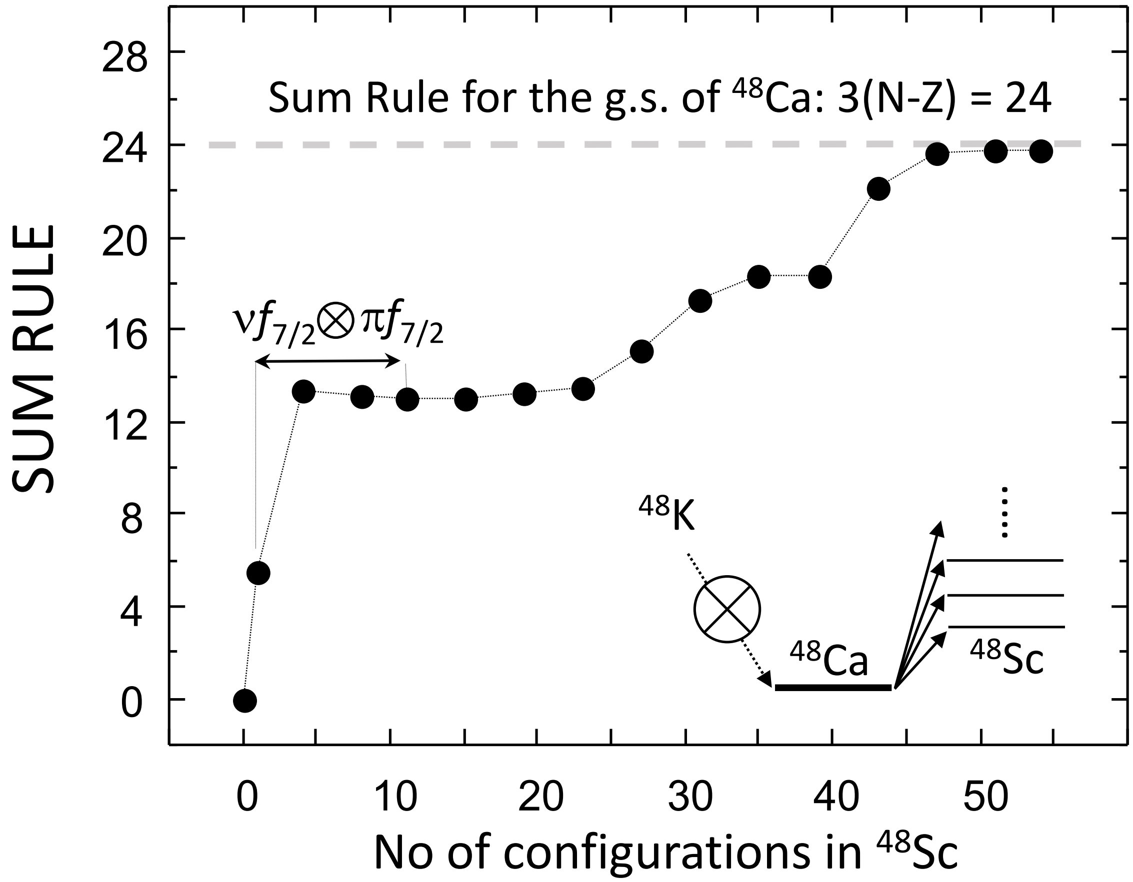
PRC 112, 055502 (Nov 2025)

Comparison with other models

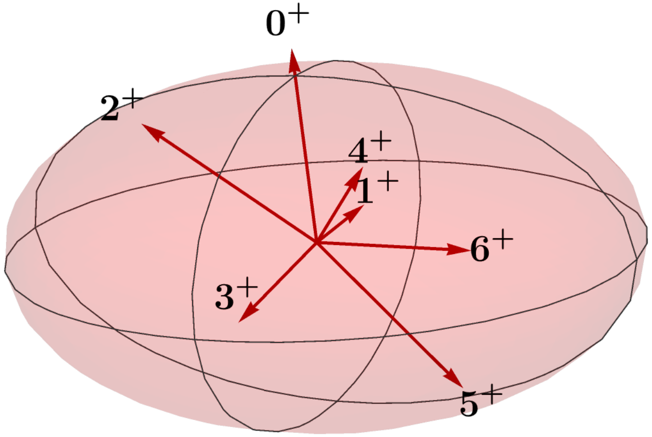
Reference	Method	$\lvert\mathcal{M}^{2\nu}\rvert$ (MeV^{-1})
A. S. Barabash (2020)	Experiment	0.068 ± 0.006
Y. Iwata et al. (2015)	Shell model	0.0539
F. Šimkovic et al. (2018)	ChER	0.0832
J. Kostensalo and J. Suhonen (2019)	Shell model	0.100
S. Novario et al. (2021)	Coupled cluster	0.082
J. Terasaki and Y. Iwata (2021)	Shell model	0.0515
	QRPA	0.0745
P. Veselý et al. (2024)	STDA	0.1668
J. Miśkiewicz, M. Konieczka and W. Satuła (2025)	**DFT-NCCI**	**0.056**

Comparison of DFT-NCCI matrix element magnitude with other nuclear models. Values for other models converted from dimensionless units to [MeV^{-1}] assuming qg_A=1.

48Ca Ikeda sum rule

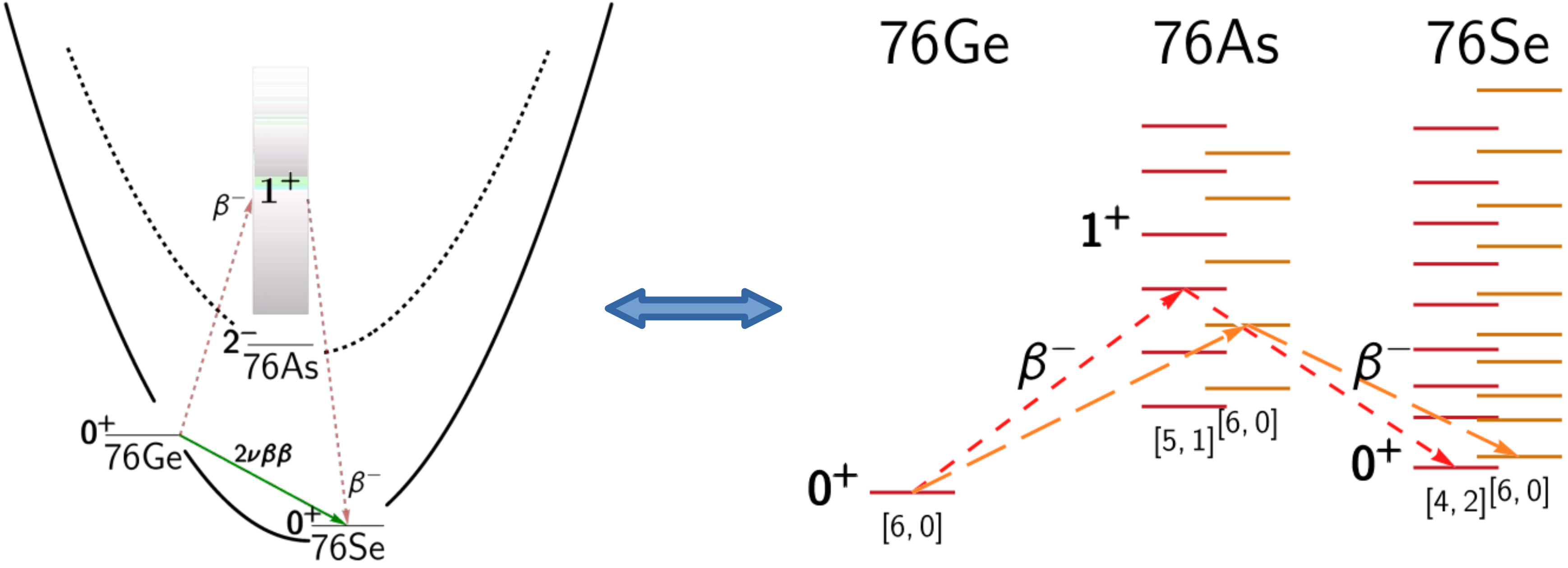


76Ge – 76Se: triaxial system



- Significant deformation (β~0.2-0.3)
- High triaxiality (γ~27-30°) in each nucleus
- Single-state dominance scenario

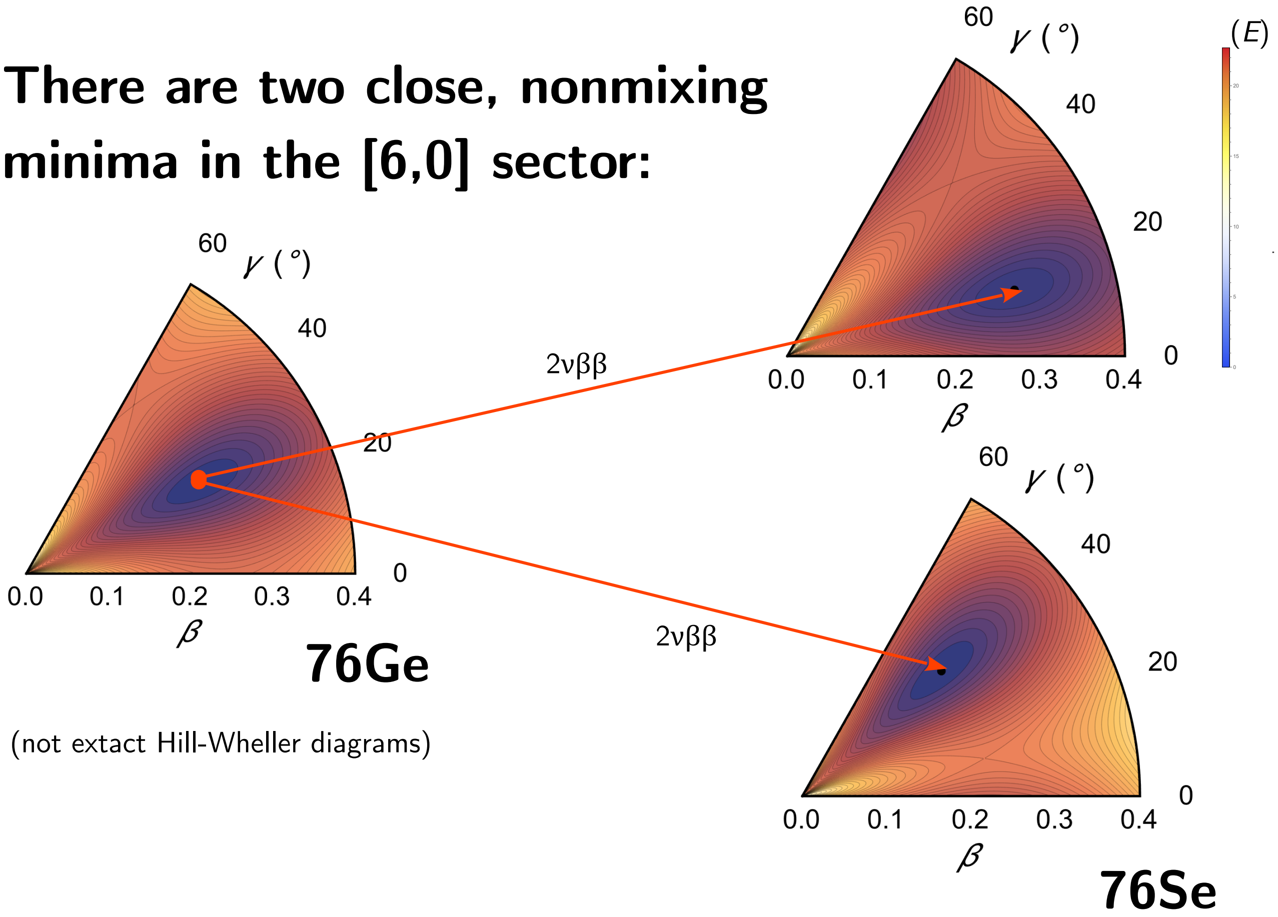
Possible decay scenarios



Virtual transitions split into two, nonmixing paths of different [ν,π] $0g_{9/2}$ orbital occupation. Red/orange levels denote active orbitals of positive/negative parity.

Shape coexistence in configuration space

There are two close, nonmixing minima in the [6,0] sector:

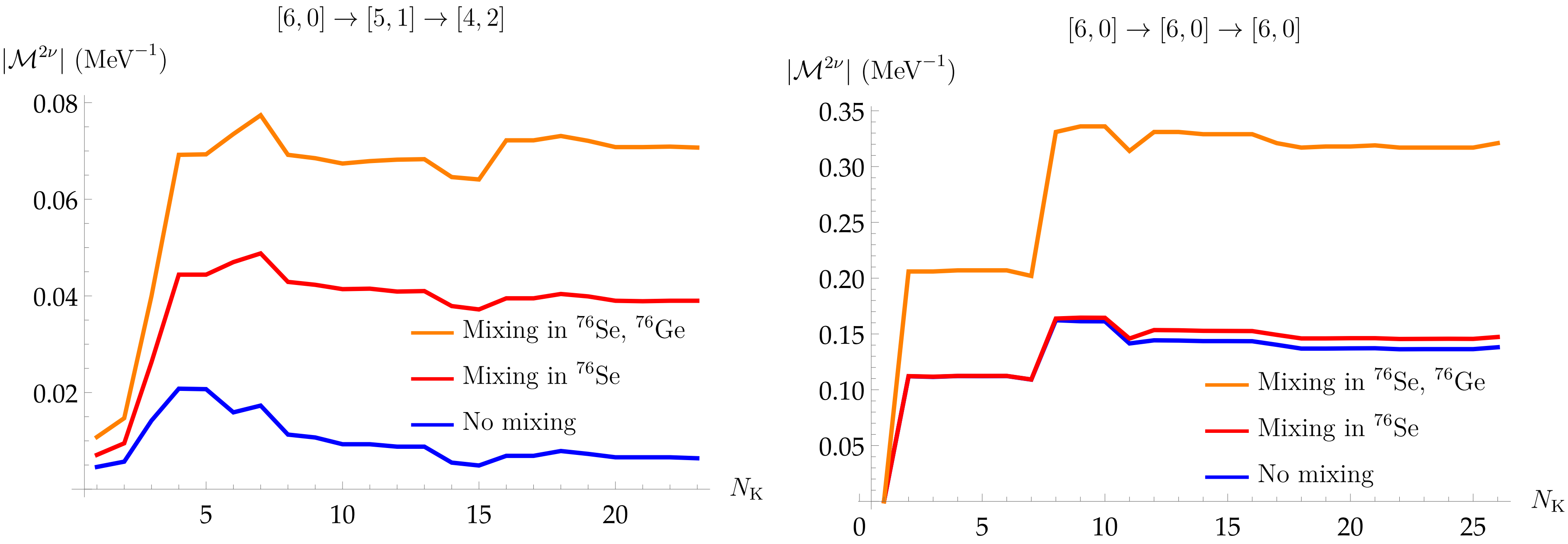


$$\mathcal{M}^{2\nu} = 0.06 \text{ [MeV}^{-1}\text{]}$$

$$\mathcal{M}^{2\nu} = 0.32 \text{ [MeV}^{-1}\text{]}$$

(not extact Hill-Wheller diagrams)

NME(76Ge → 76Se): two scenarios

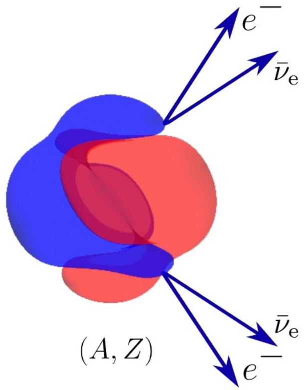


Cimilative matrix element wrt to virtual configurations included. Spectrum shifted to experimental $\Delta E_1(I^{\pi}{=}1^{+}){=}44$keV (normalization). **Single state dominance** predicted.

Summary: Status and future plans

2νββ
(~ completed)

- **48Ca** (spherical)
- **76Ge** (deformed)
- 136Xe (heavy)



0νββ

- Extension of HFODD code: **forbidden transitions** (GWT)
- NMEs for 48Ca and 76Ge (closure approximation)
- Half-life estimation with modern constraints on $\langle m_{\beta\beta} \rangle$