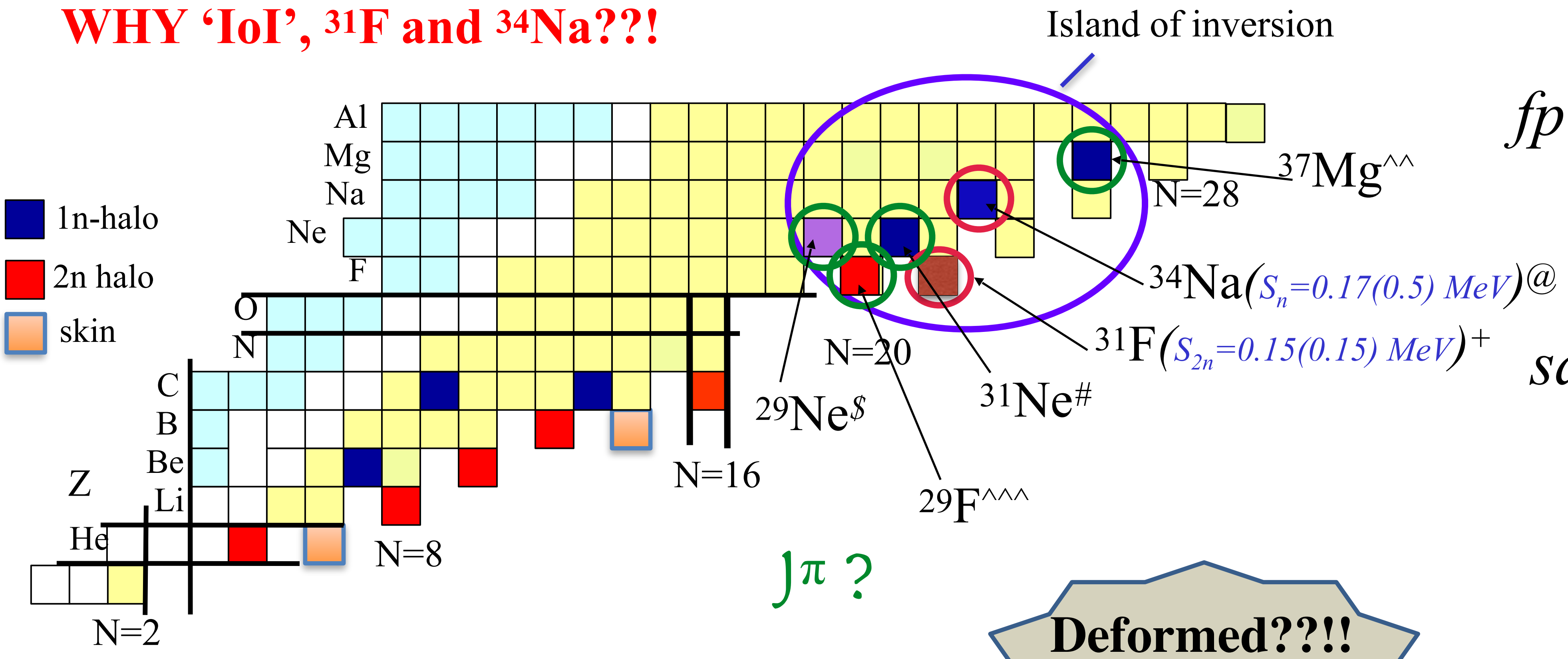
Low and medium mass neutron halos

G. Singh

Narodowe Centrum Badań Jądrowych
National Centre for Nuclear Research
ŚWIERK

The Whats and the Whys!

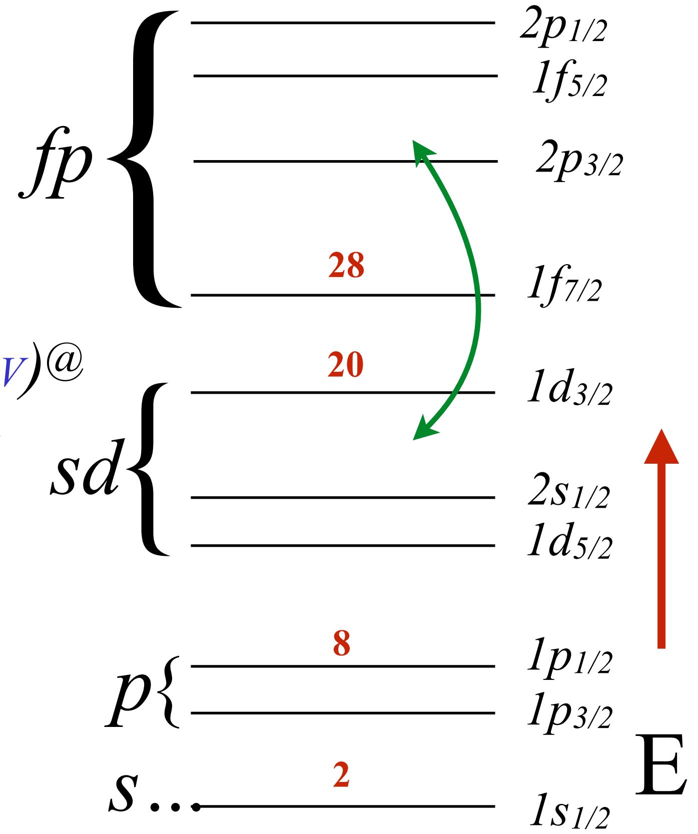
WHY ‘IoI’, ^{31}F and ^{34}Na??!



Exotic nuclei at/near the island of inversion

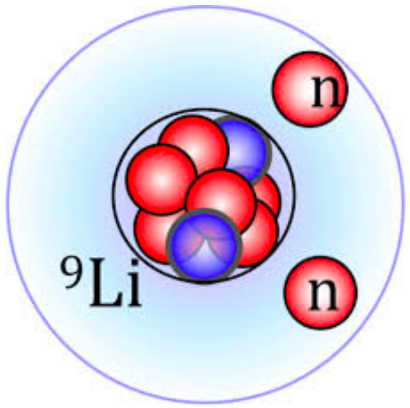
Deformed??!!

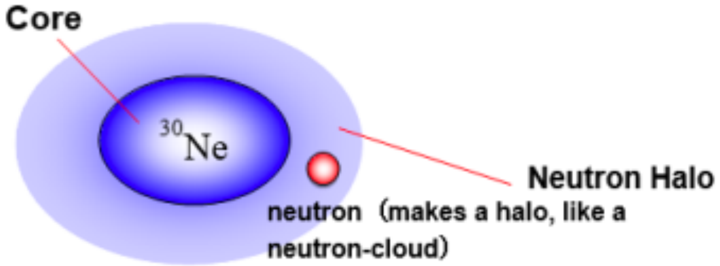
N = 20-28

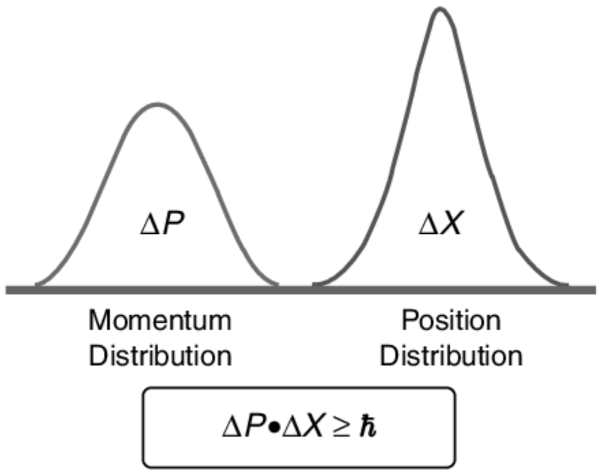


Shell model

Halo?!







Role in astro?

Do not follow $R = r_0A^{1/3}$ rule.

Radius of ^{11}Li ~ radius of ^{208}Pb

$^{+}$Wang M., et al. CPC 41 (2017) 030003.

$^{@}$Singh G., et al. PRC 94 (2016) 024606.

$^{\#}$Shubhchintak et al. NPA 922 (2014) 99.

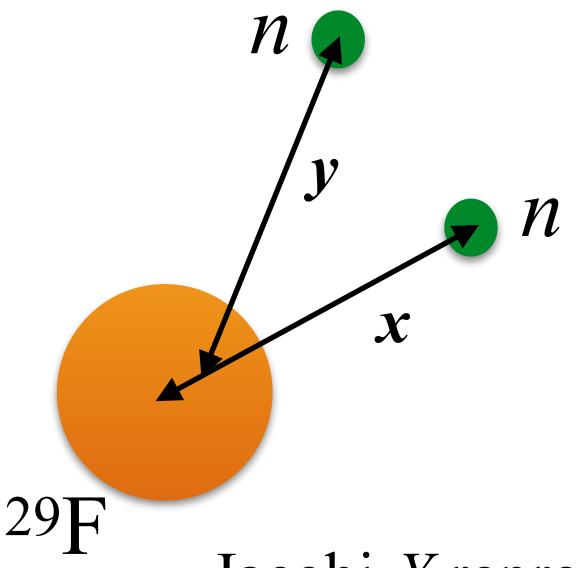
^^Shubhchintak et al. NPA 939 (2015) 101.

$^{\$}$Manju, Dan M., Singh G. et al. NPA 1010 (2021) 122194.

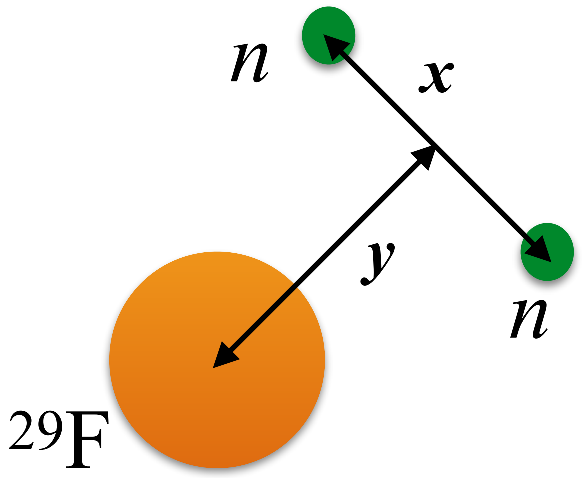
^^^Bagchi S. et al. PRL 124 (2020) 222504.

^^^Singh Jagjit, et al. PRC 101 (2020) 024310.

(^{31}F) The three-body formalism:



Jacobi-Y representation



Jacobi-T representation

The eigenstates of a three-body system:

$$\psi^{jm_j}(\rho,\Omega) = \frac{1}{\rho^{5/2}} \sum_\beta R^j_\beta(\rho) \mathcal{Y}^{jm_j}_\beta(\Omega),$$

β: Channel!

Hyperradius

Hyperangular functions

Expansion coefficients

Discretise the continuum!

The hyperradial functions can be expanded into a discrete basis via:

$$R^j_\beta(\rho) = \sum_i^{i_{max}} C^j_{i\beta} U_{i\beta}(\rho)$$

Under transformed harmonic oscillator (THO) basis,

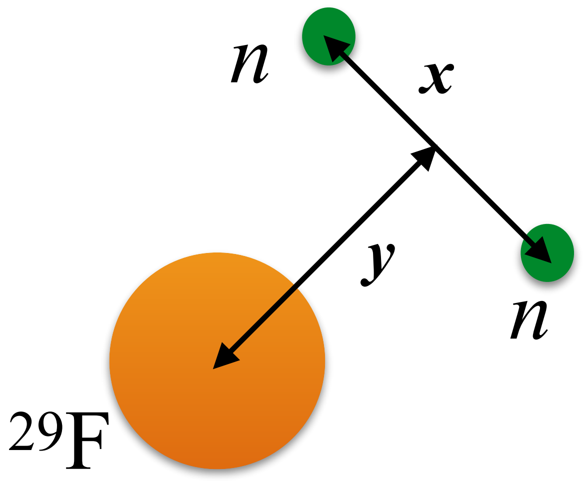
$$U^{\mathrm{THO}}_{i\beta}(\rho) = \sqrt{\frac{du}{d\rho}} U^{\mathrm{HO}}_{iK}[u(\rho)],$$

where, $U^{\mathrm{HO}}_{iK}(u) = N_{iK} u^{K+5/2} L_i^{K+2}(u) \exp\left(-u^2/2\right)$

Casal J., et. al, PRC 90 (2014) 044304.

Local scale transformation or LST:

$$u(\rho) = \frac{1}{b\sqrt{2}}\left[\left(\frac{1}{\rho}\right)^4 + \left(\frac{1}{\gamma\sqrt{\rho}}\right)^4\right]^{\frac{-1}{4}}$$



γ/b:
- Crucial to density of pseudostates!
- Gaussian asymptotic behaviour → exponential

The Potentials!

Two-body potentials: $V_{(^{29}F+n)} = -V_0 f(r) + V_{ls}\lambda_\pi^2 \frac{1}{r}\frac{df(r)}{dr}\vec{l}\cdot\vec{s},$

$r_0 = 1.25$ fm
$a = 0.75$ fm
$\lambda_\pi = 1.414$ fm

$n + n$: Gogny-Pires-Tourreil potential.

$$f(r) = \frac{1}{1 + \exp\left(\frac{r - R_c}{a}\right)}$$

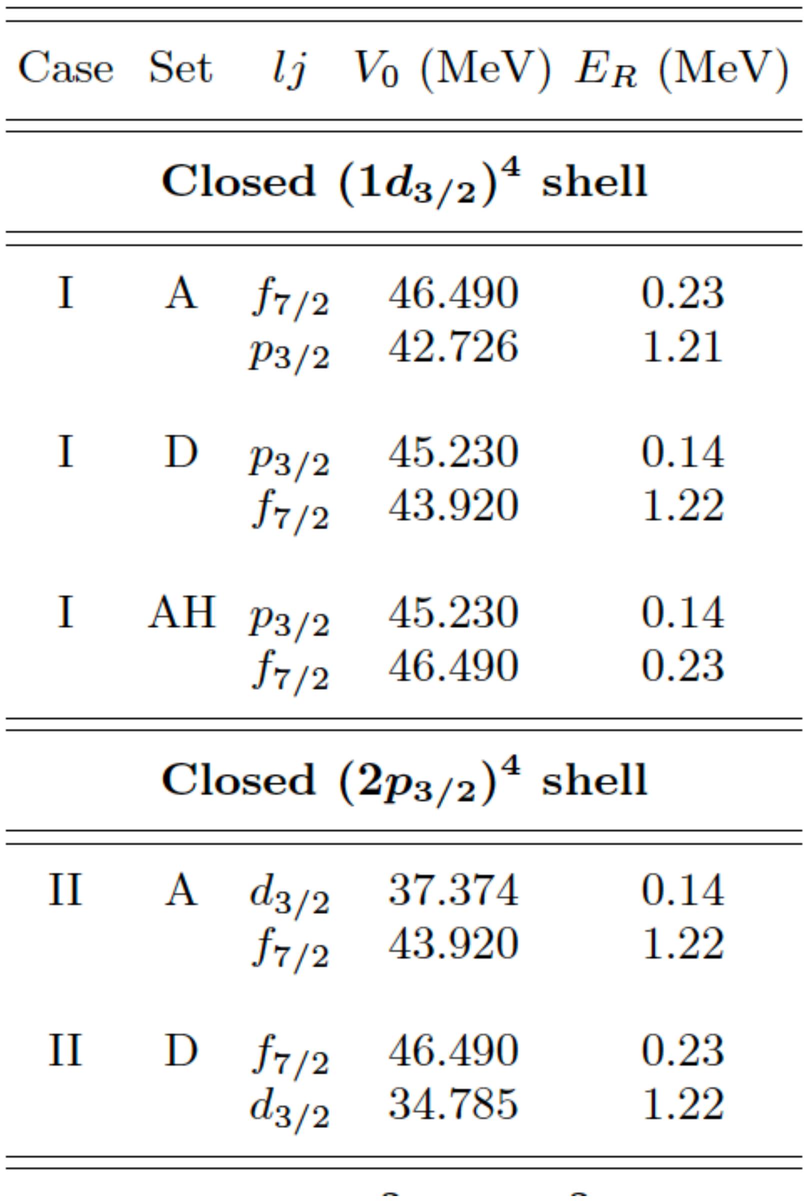
Spin-Orbit potentials: $V_{ls} = 22 - 14(N - Z)/A = 16.690$ MeV

Three-body potential: $V^{3b}(\rho) = V_0^{3b}\exp[-(\rho/r_0^{3b})^2],$ $r_0{}^{3b} = 6.0$ fm

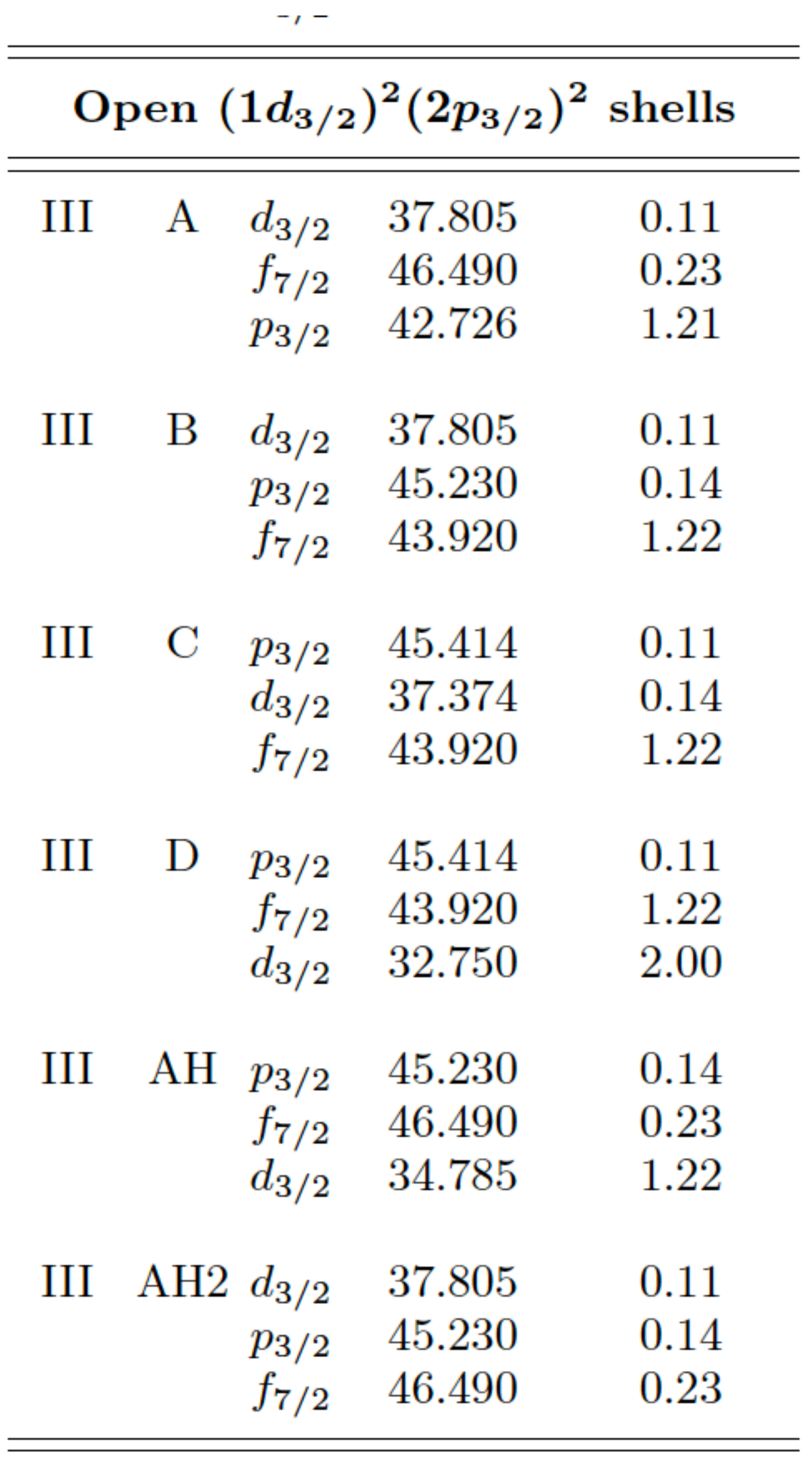
Karataglidis S., et. al, PRC 71 (2005) 064601. *Gogny D., Pires P., and Tourreil R. De, PLB 32 (1970) 591.* *Casal J., et. al, PRC 102 (2020) 064627.*

Bohr A., and Mottelson B. R., Nuclear Structure (Benjamin, New York, 1969).

The various configurations and two body potentials



Case	Set	lj	V_0 (MeV)	E_R (MeV)
Closed $(1d_{3/2})^4$ shell				
I	A	$f_{7/2}$	46.490	0.23
		$p_{3/2}$	42.726	1.21
I	D	$p_{3/2}$	45.230	0.14
		$f_{7/2}$	43.920	1.22
I	AH	$p_{3/2}$	45.230	0.14
		$f_{7/2}$	46.490	0.23
Closed $(2p_{3/2})^4$ shell				
II	A	$d_{3/2}$	37.374	0.14
		$f_{7/2}$	43.920	1.22
II	D	$f_{7/2}$	46.490	0.23
		$d_{3/2}$	34.785	1.22



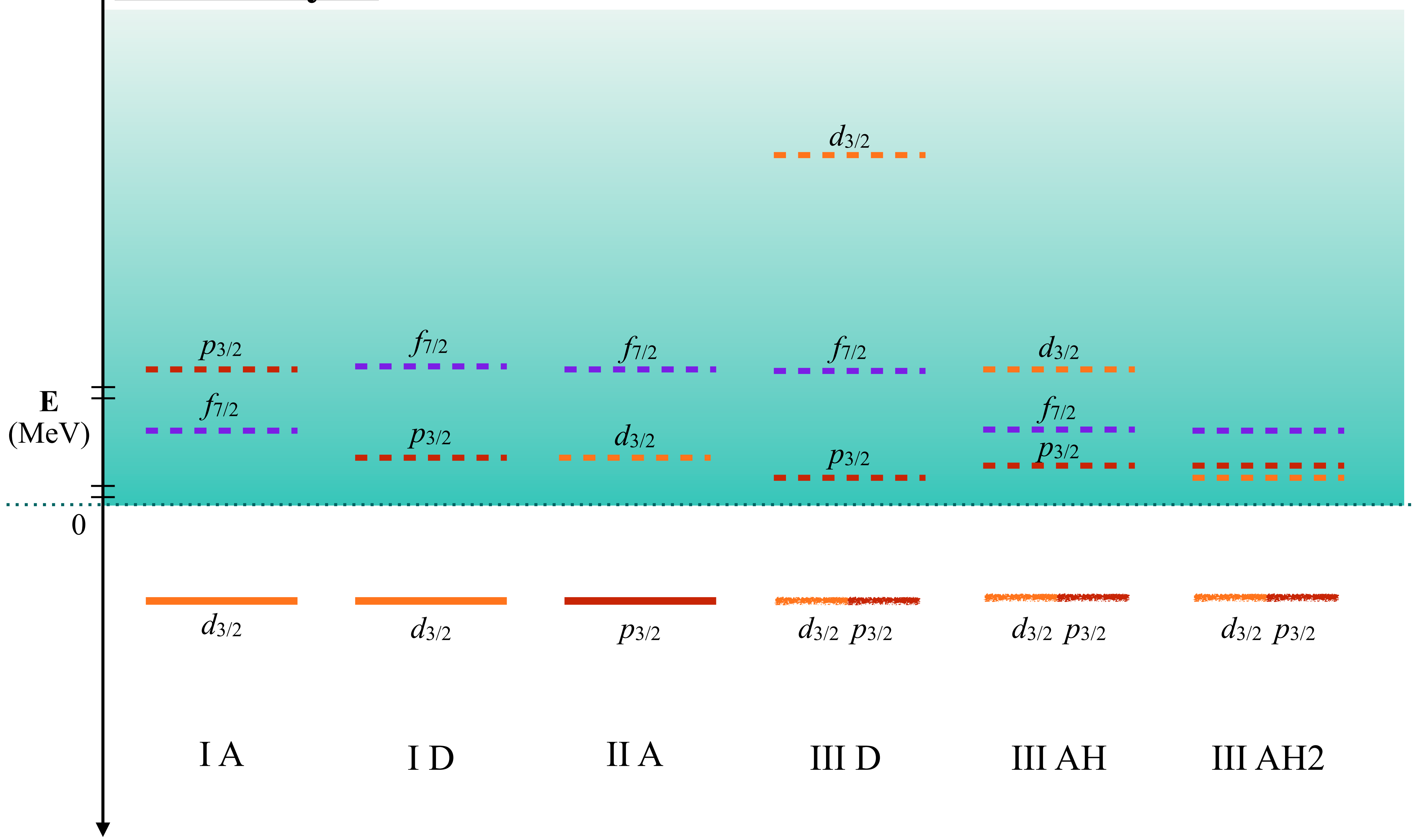
Open $(1d_{3/2})^2(2p_{3/2})^2$ shells

III	A	$d_{3/2}$	37.805	0.11
		$f_{7/2}$	46.490	0.23
		$p_{3/2}$	42.726	1.21
III	B	$d_{3/2}$	37.805	0.11
		$p_{3/2}$	45.230	0.14
		$f_{7/2}$	43.920	1.22
III	C	$p_{3/2}$	45.414	0.11
		$d_{3/2}$	37.374	0.14
		$f_{7/2}$	43.920	1.22
III	D	$p_{3/2}$	45.414	0.11
		$f_{7/2}$	43.920	1.22
		$d_{3/2}$	32.750	2.00
III	AH	$p_{3/2}$	45.230	0.14
		$f_{7/2}$	46.490	0.23
		$d_{3/2}$	34.785	1.22
III	AH2	$d_{3/2}$	37.805	0.11
		$p_{3/2}$	45.230	0.14
		$f_{7/2}$	46.490	0.23

A: Normal shell model, **B**: Intruder, **C**: Partially inverted, **D**: Inverted, **AH & AH2**: Anti-halo

Singh G., Singh Jagjit, Casal J., and Fortunato L., *PRC* **105** (2021) 014328.

Pictorially…



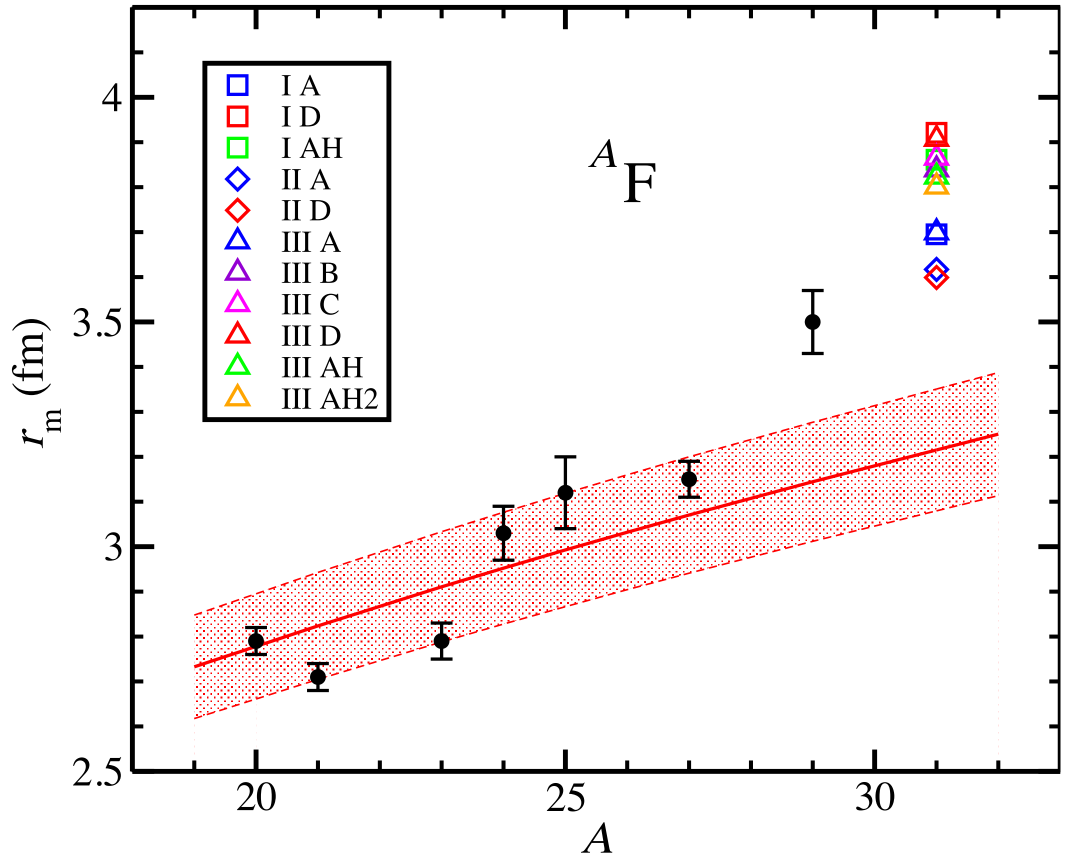
I, II: Closed; III: Open

A: Normal shell model, **B**: Intruder, **C**: Partially inverted, **D**: Inverted, **AH** & **AH2**: Anti-halo

The observables computed…

Case	Set	r_m (fm)	r_{nn} (fm)	r_{c-nn} (fm)	Δr (fm)	$E1$ Sum rule (e^2fm^2)
I	A	3.695	6.881	4.881	0.195	1.929
	D	3.921	9.640	6.340	0.421	3.254
	AH	3.861	8.963	5.986	0.361	2.902
II	A	3.617	5.728	4.270	0.117	1.477
	D	3.599	5.732	4.017	0.099	1.306
III	A	3.699	6.264	4.901	0.199	1.945
	B	3.839	8.341	5.995	0.339	2.910
	C	3.865	8.698	6.125	0.365	3.038
	D	3.907	9.488	6.253	0.407	3.166
	AH	3.823	8.400	5.793	0.323	2.718
	AH2	3.801	7.905	5.742	0.301	2.670

How do they compare with other F isotopes?



Variation with S_{2n}?

S_{2n} = 0.30 MeV: r_m = 3.817 fm.

S_{2n} = 0.01 MeV: r_m = 4.084 fm.

Meanwhile, $\triangle r$ (^{29}F) = 0.20 fm[#]

$\triangle r$ (^{29}F) = 0.35fm[$]

$\triangle r$ (^{31}F) is large enough to form a halo!

What about orbital angular momentum?

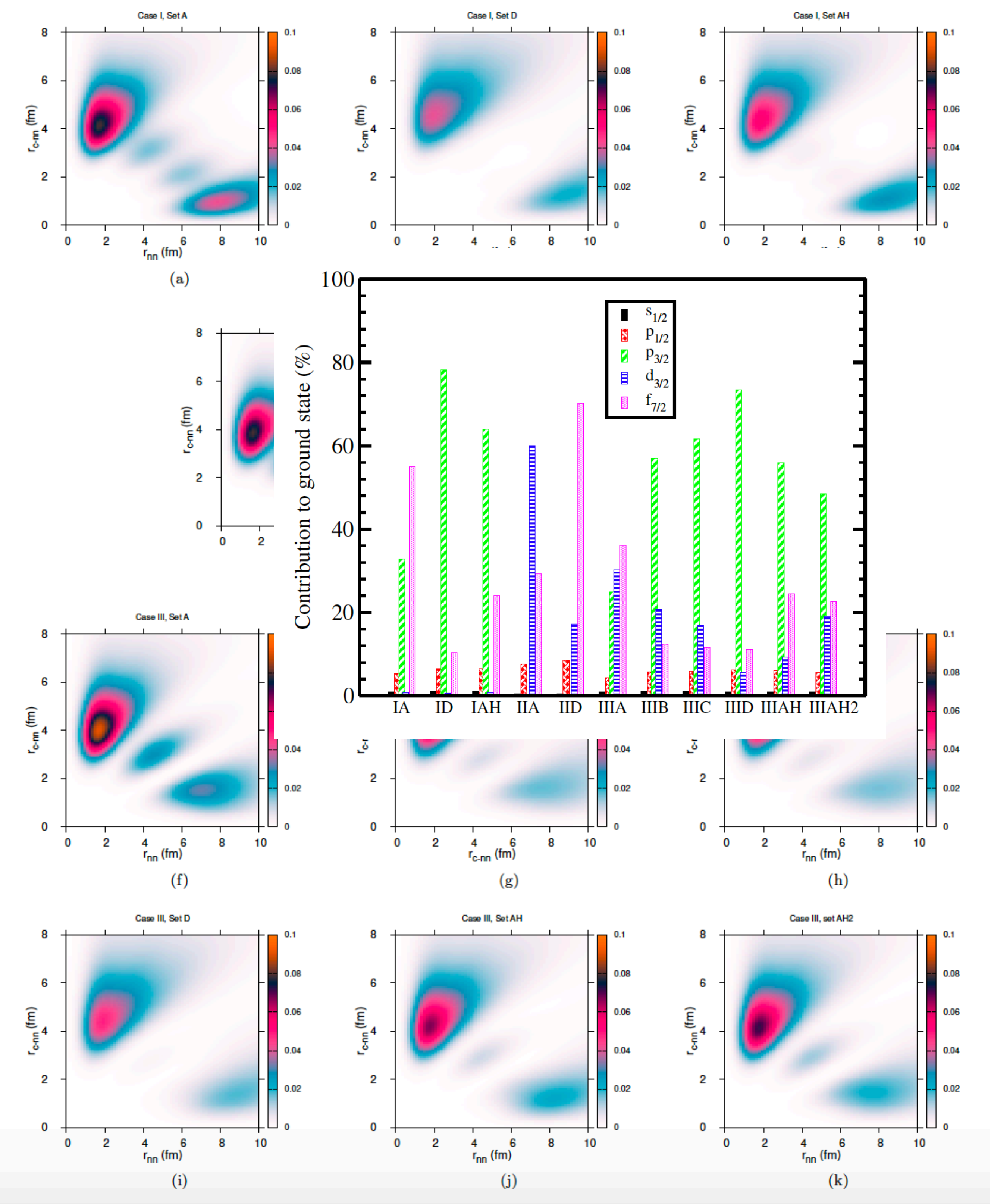
No evidence of anti-halo?!

Singh G., Singh Jagjit, Casal J., and Fortunato L., *PRC* **105** (2021) 014328.

Ozawa A.. et al, NPA 691 (2001) 599.
[$]Bagchi S. et al, PRL 124 (2020) 222504.

[#]Casal J., Singh Jagjit, Fortunato L., Horiuchi W., and Vitturi A., PRC 102 (2020) 064627.

Density Distributions!



- Number of peaks depends on the dominance of orbital angular momentum.
- Higher l, more peaks!
- Evidently, the $p_{3/2}$ dominant plots have a larger dineutron tail, a feature consistent with other two-neutron halos like ^{29}F!
- Case III configurations also have a larger dineutron peak to cigar-like peak density ratio!
- Anti-halo effect is present, but not large enough to prevent a halo!
- **Good enough to be a two-neutron halo!**

Singh G., Singh Jagjit, Casal J., and Fortunato L., *PRC* **105** (2021) 014328.

Pairing enhancement in a 2n transfer through the intermediate continuum!

Transfer reactions:

- One particle transfer
- Two particle transfer
- Cluster transfer

Two neutron transfer → Correlations between valence neutrons. PAIRING!

Study $2n$ transfer to form ^{6}He via the reaction ^{18}O(^{4}He,^{6}He)^{16}O.

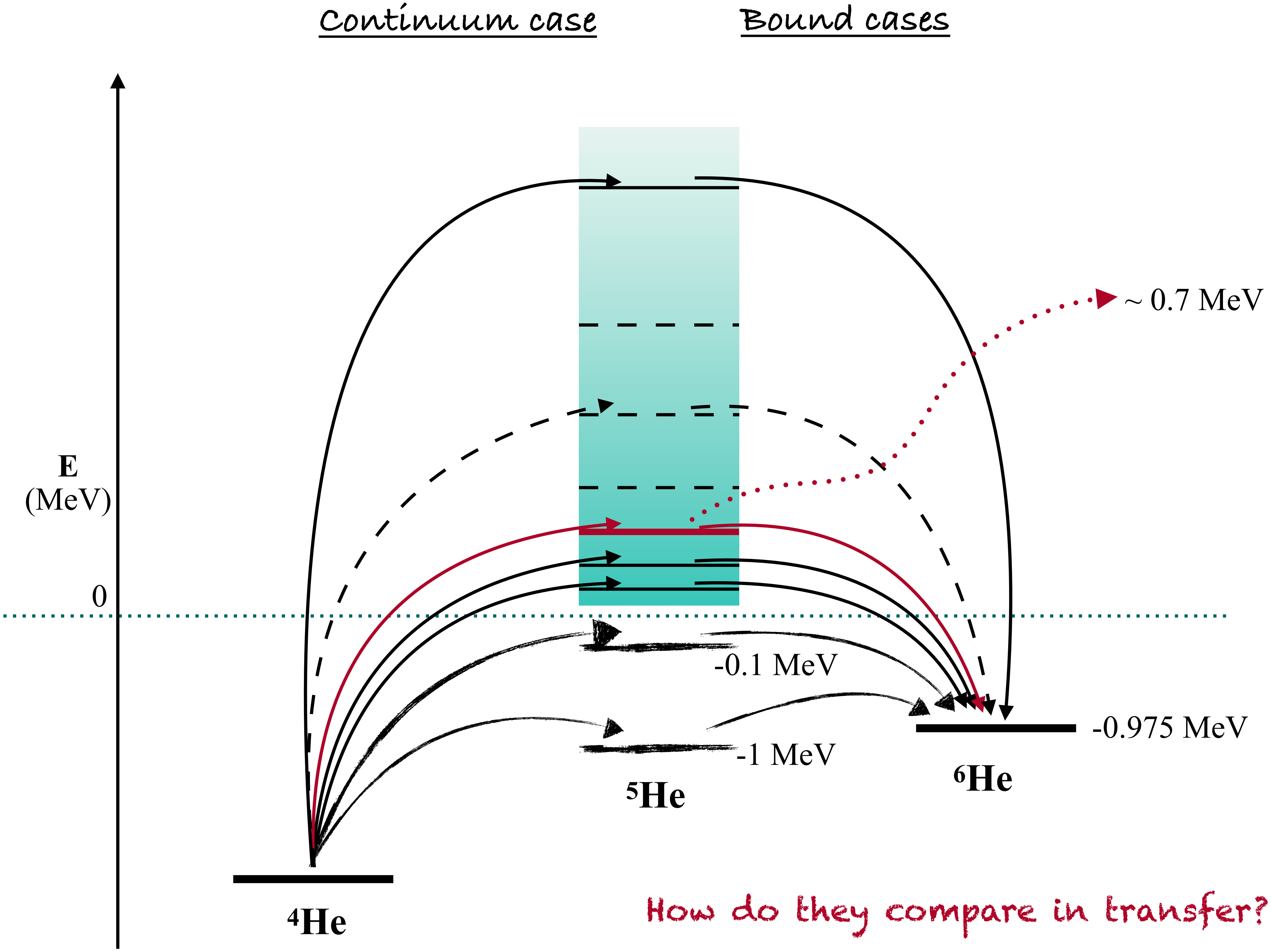
WHY ^{6}He?!

- Compare the case of a $2n$ transfer through a natural (unbound) ^{5}He with a hypothetically bound ^{5}He.
- See the effect of the continuum on the pairing enhancement in both the cases.

^{5}He with S_n = -1 MeV ^{5}He with S_n = -0.1 MeV ^{5}He with E_R = 0.69 MeV

No excitation of de-excitation of Helium or Oxygen isotopes. Only the $d_{5/2}$ state in ground state of ^{17}O at -4.1433 MeV. Only $p_{3/2}$ state in ^{5}He.

Pictorially…

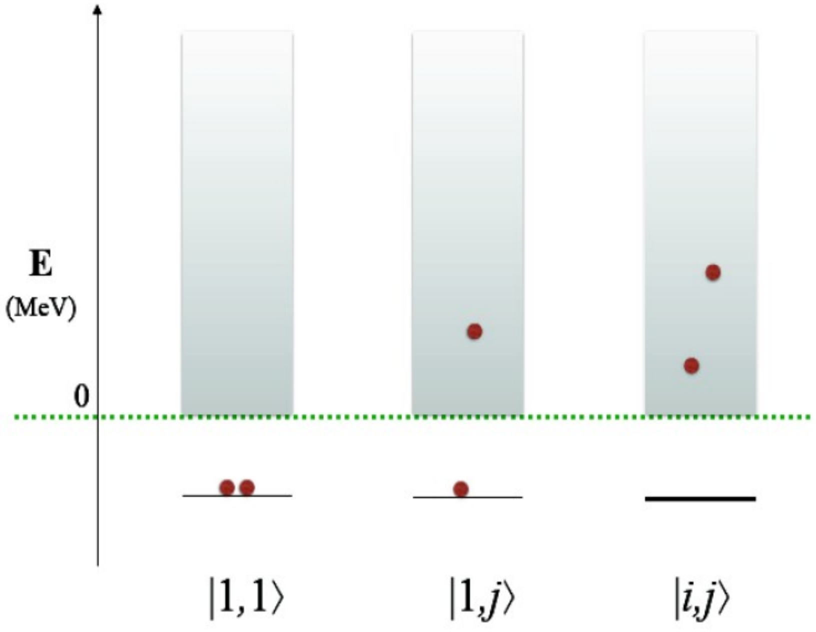


How do they compare in transfer?

Two-neutron transfer

Three possibilities:

- Both particles in the ground state → $|1,1\rangle$.
- One particle in the continuum → $|1,j\rangle$; $j \neq 1$.
- Both particles in the continuum → $|i,j\rangle$; $i, j \neq 1$.



Pairing:

Hamiltonian: $H = H_u + H_p$

Perturbation introduced via contact Delta interaction!

$\Delta = -g(\boldsymbol{r}_1 - \boldsymbol{r}_2)$ = Attractive interaction!

Vary $\boldsymbol{g}$, vary Δ!

Lower the system energy to -0.975 MeV.

Neglect $S_n = 1$ MeV case.

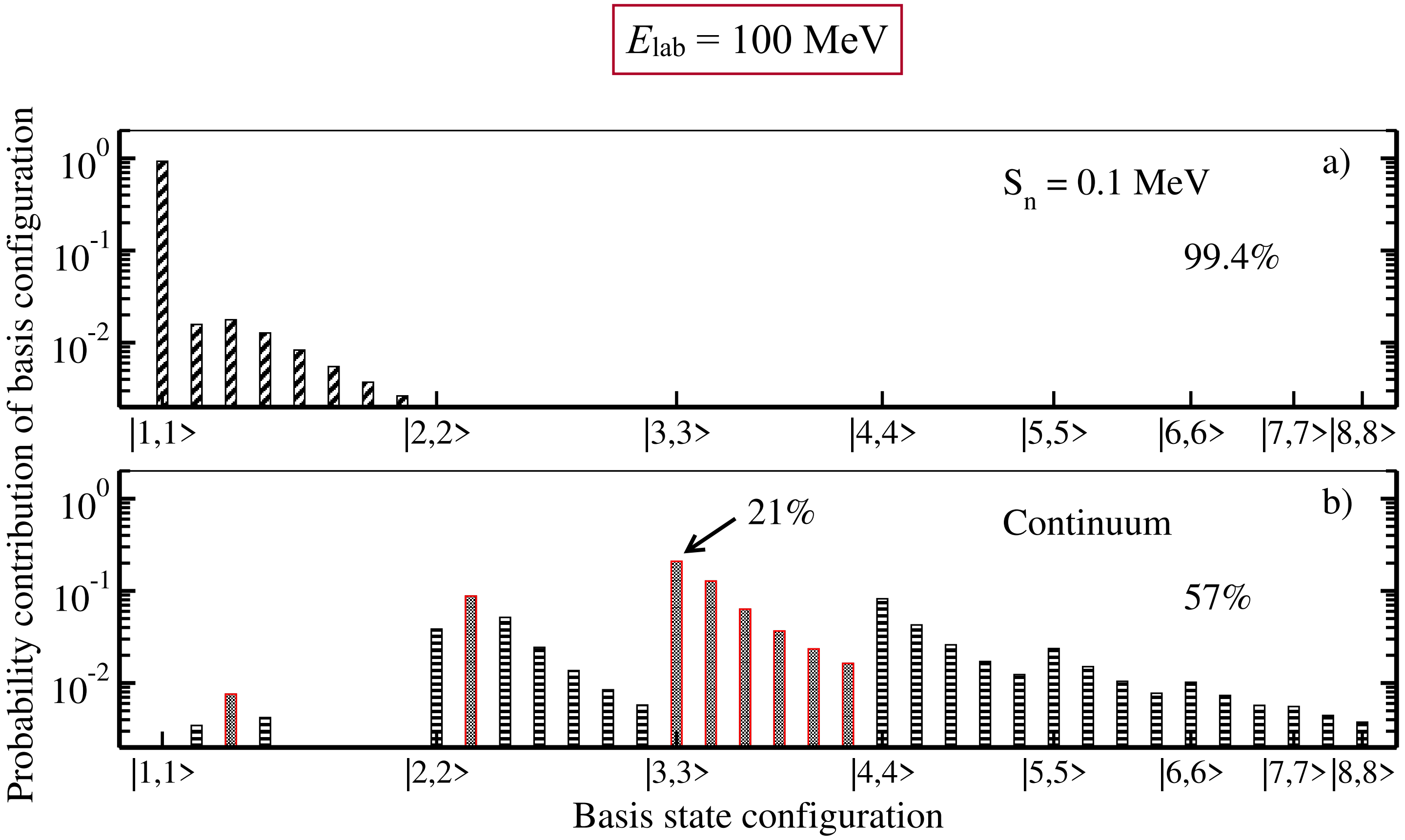
Compare $S_n = 0.1$ MeV bound case with continuum scenario.

Note: Neglect Simultaneous and Non-orthogonality terms as they cancel each other due to 0^+ spins of projectile and target!

Broglia R. A., and Winther A., *Heavy Ion Reactions* (Addison-Wesley Publishing Co., 1991).

Probability of each configuration!

- ✓ Bound state has most contribution in bounds case.
- ✓ Resonant state has most contribution in continuum case.
- ✓ Paired configurations have most important contributions (?).
- ✦ How does this pairing affect the two-neutron transfer?



The Numbers!

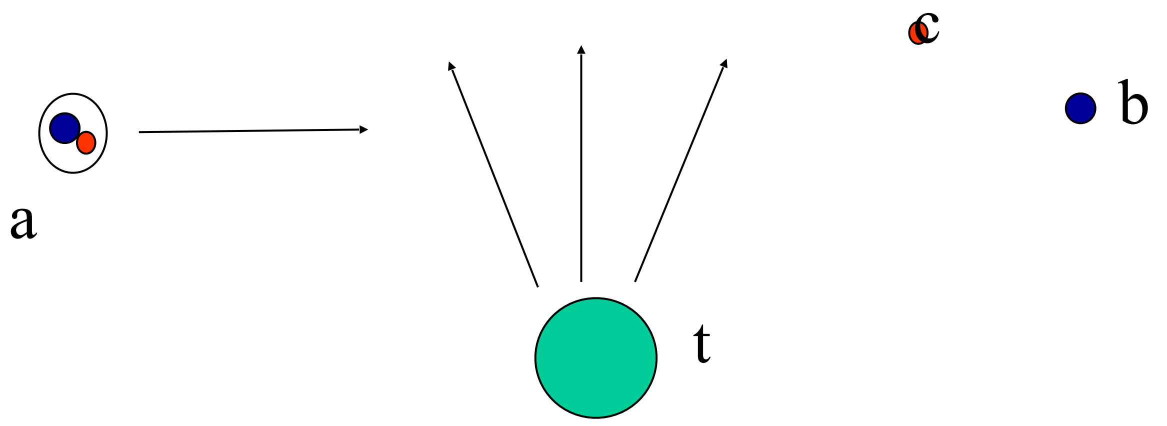
Case	Δ (MeV)	E_l=21 MeV				E_l=100 MeV			
		$-g$	σ_{2n}	σ_{2n} (u)	σ_{2n}/σ_{2n}(u)	$-g$	σ_{2n}	σ_{2n} (u)	σ_{2n}/σ_{2n}(u)
S_n = 0.1 MeV	0.775	1037	6.95	6.89	1.01	992	147	125	1.18
Continuum	2.356	10430	0.94	0.51	1.84	7827	44	8.6	5.12

Pairing leads to significant enhancement in the transfer when there is no bound state in the intermediate continuum!

Singh G., Fortunato L., and Vitturi A. *Phys. Lett. B* **834**, 137413 (2022), doi: https://doi.org/10.1016/j.physletb.2022.137413.

(^{34}Na) Coulomb dissociation and the finite range distorted wave Born approximation (FRDWBA) theory:

Coulomb dissociation: an elegant method to study nuclear halos!!



The triple differential cross-section:

Phase space factor

$$\boxed{\frac{d^3\sigma}{dE_b d\Omega_b d\Omega_c} = \frac{2\pi}{\hbar v_{at}} \rho \sum_{lm} |\beta_{lm}|^2}$$

Reduced transition matrix

For a neutron,

$$\chi_c^{(-)*}(\mathbf{q}_c,\mathbf{r}_c) = e^{-i(\mathbf{q}_c,\mathbf{r}_c)}$$

The reduced transition matrix under the FRDWBA theory:

Structure part! *Includes deformation!!*

Dynamics part!

$$\boxed{\hat{l}\beta_{lm} = \int d\mathbf{r}_1 e^{-i\mathbf{W}.\mathbf{r}_1} V_{bc}(\mathbf{r}_1)\phi_a^{lm}(\mathbf{r}_1) \int d\mathbf{r}_i e^{-i\delta\mathbf{q}_c.\mathbf{r}_i} \chi_b^{(-)*}(\mathbf{q}_b,\mathbf{r}_i)\chi_a^{(+)}(\mathbf{q}_a,\mathbf{r}_i)}$$

Can be evaluated analytically in terms of the Bremsstrahlung integral. ☺

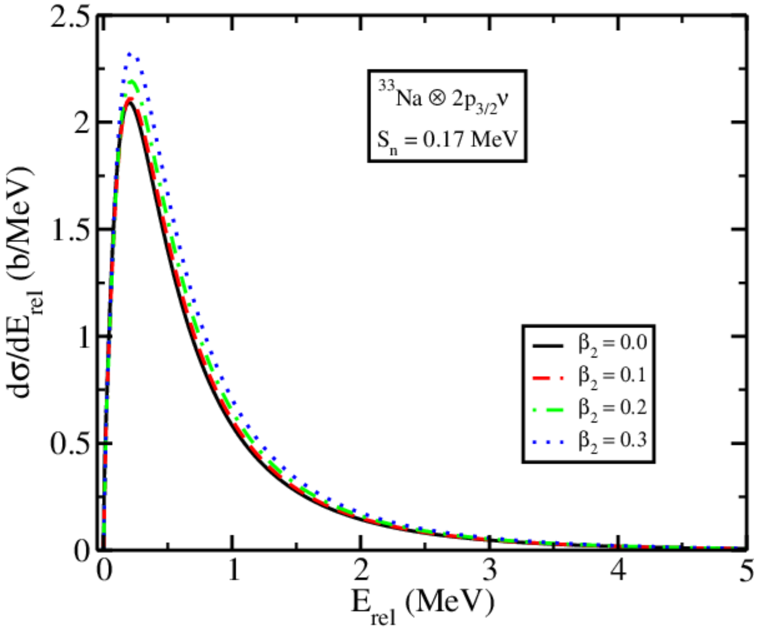
Chatterjee R, Banerjee P, and Shyam R, Nucl. Phys. A **675** (2000) 477.

Results (^{34}Na)

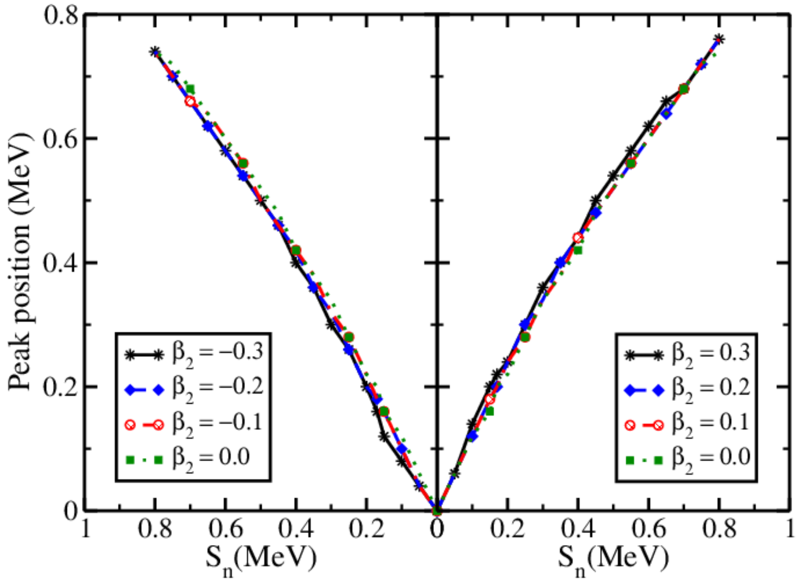
Singh G., Shubhchintak, and Chatterjee R, Phys. Rev. C **94** (2016) 024606.

$$^{34}\mathrm{Na} + {}^{208}\mathrm{Pb} \xrightarrow{100\ \mathrm{MeV/u}} {}^{33}\mathrm{Na} + {}^{1}\mathrm{n} + {}^{208}\mathrm{Pb}$$

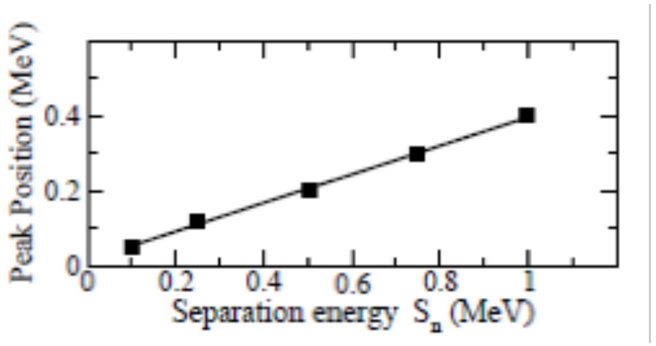
Relative energy spectra



Peak position of relative energy spectrum dependent on separation energy!

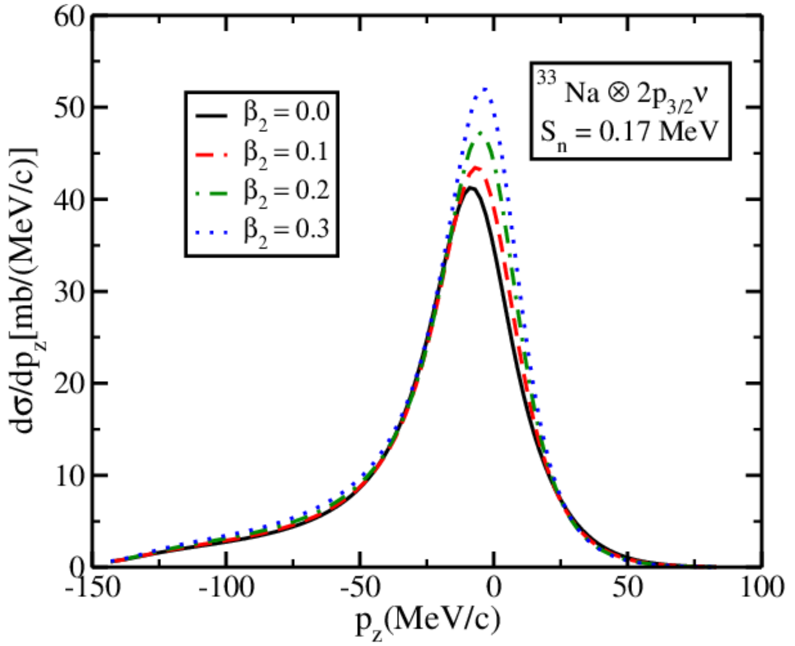


Pattern independent of sign of deformation.
Scaling valid!!



Chatterjee R, Fortunato L and Vitturi A, Eur. Phys. J. A **35** (2008) 213.

Parallel momentum distribution



β_2	FWHM (MeV/c)
0.0	36.82
0.1	35.88
0.2	34.76
0.3	33.83

FWHM for normal nuclei ~ 120 MeV/c
while
FWHM for ^{11}Be and ^{19}C ~ 44 MeV/c

Chatterjee R, Banerjee P, and Shyam R, Nucl. Phys. A **675** (2000) 477.
Banerjee P, Thompson I J, and Tostevin J A, Phys. Rev. C **58** (1998) 1042.

Definitely a HALO!!!

Role of ^{34}Na in the ^{33}Na(n,γ)^{34}Na radiative capture reaction and Sodium isotopic chain

Important for *r*-process pathway!!

Light nuclei play an important role as seed nuclei/progenitors of seed nuclei in stellar plasma.

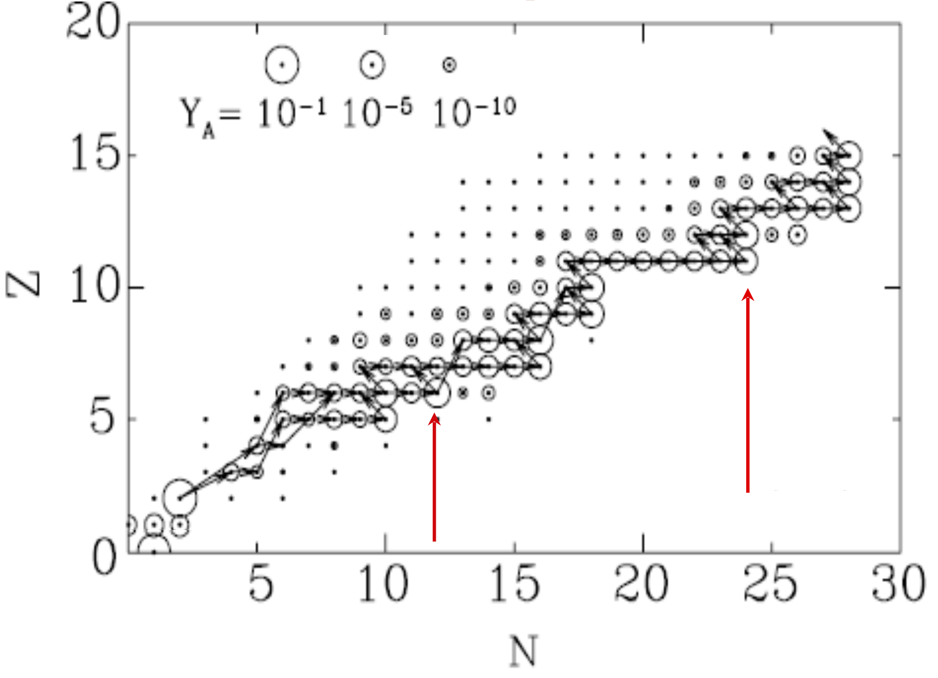
Competition between α-capture and *n*-capture!

α-capture > *n*-capture ⇒ new elements

n-capture > α-capture ⇒ drip line!

We compare rates of ^{33}Na(n,γ)^{34}Na with ^{33}Na(α,n)^{36}Al reaction!

Terasawa M, *et al.*, ApJ **562** (2001) 470.



^{33}Na(n,γ) important to ^{34}Na, ^{35}Na abundance!

FRDWBA!!

$$\frac{d^3\sigma}{dE_b d\Omega_b d\Omega_c} = \frac{2\pi}{\hbar v_{at}} \rho \sum_{lm} |\beta_{lm}|^2$$

For a **single multipole dominated reaction,**

$$\sigma^{\pi\lambda}_{\gamma,n} = \frac{E_\gamma}{n_{\pi\lambda}} \boxed{\frac{d\sigma}{dE_{bc}}}$$

Bertulani C A, and Baur G, Phys. Rep. **163** (1988) 299.

Rauscher, At. Data Nucl Data Tables **79** (2001) 47: http://nucastro.ord/nonsmoker.html.

$$\sigma_{n,\gamma} = \frac{2\hat{j}_a{}^2}{\hat{j}_b{}^2 \hat{j}_c{}^2} \frac{k_\gamma^2}{k_{bc}^2} \sigma^{\pi\lambda}_{\gamma,n}$$

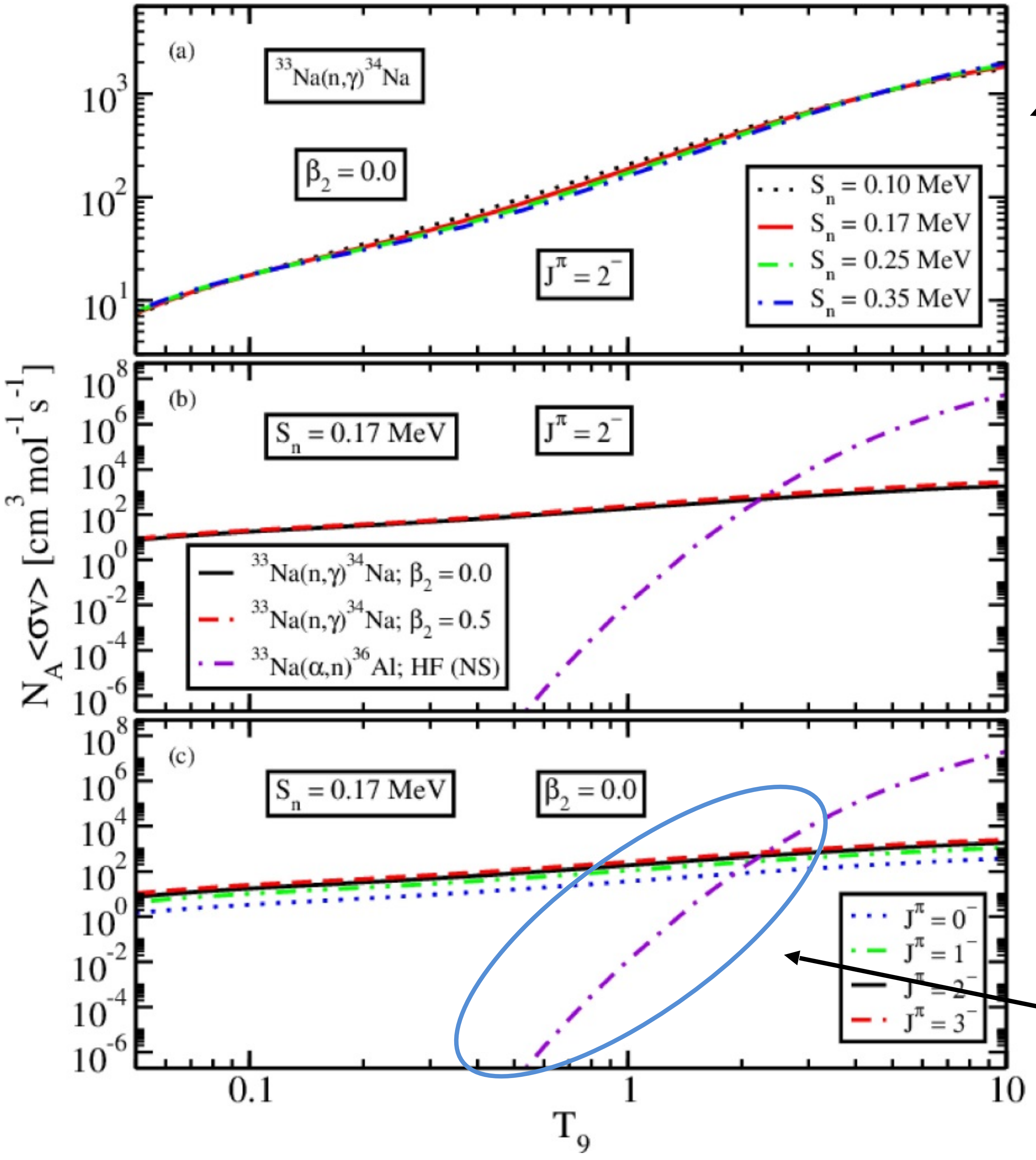
$$\langle \sigma(v_{bc}) v_{bc} \rangle = \sqrt{\frac{8}{\pi \mu_{bc} (k_B T)^3}} \int_0^\infty dE_{bc} \sigma_{(n,\gamma)}(E_{bc}) E_{bc} \exp\left(-\frac{E_{bc}}{k_B T}\right)$$

Then, reaction rate,

$$R = N_A \langle \sigma(v_{bc}) v_{bc} \rangle$$

Reaction rates!

$T_9 = 0.62$



Reaction rate with various S_n values.

Rate with different β_2.

Rate with $J\pi$

Comparison of $^{33}Na(n,\gamma)^{34}Na$ and $^{33}Na(\alpha,n)^{36}Al$.

Rates follow cross section patterns!

n-capture highly dominant over α-capture at the relevant temperature!!

Singh G., Shubhchintak, and Chatterjee R, Phys. Rev. C **95** (2017) 065806.

Conclusions!

★ Three-body analysis of ^{31}F was done with the pseudostate (PS) method using the transformed harmonic oscillator (THO) basis and a strong $2p_{3/2}$ contribution to the ground state was observed.

★ Observations of larger density tails of dineutron configuration and large dipole response coupled with radius calculations suggest that ^{31}F should be a strong ***two-neutron halo*** nucleus in its ground state!

★ ***Pairing Enhancement*** is observed in the two-neutron transfer process due to the intermediate continuum!

★ This enhancement is a result of the constructive interference between different paths passing through the intermediate continuum.

★ FRDWBA analysis of ^{34}Na suggests that it is a ***p*-wave one-neutron halo** nucleus.

★ Reaction rates for formation of ^{34}Na confirmed that in stellar environment at the equilibrium temperature, neutron-capture dominated the alpha-capture process, favouring ^{34}Na and pushing the isotopic abundance of Na isotopes towards the neutron drip line!

★ Strong encouragement to experiments to put our results on firmer footings!!!

Future Outlook!

★ Theoretical support for experimental observations!!!

★Inclusion of non-zero angular momenta in the two-neutron transfer mechanism to enable study of higher mass Borromean systems.

★ Using the FRDWBA theory for three-charged particles in the final channel, enabling the understanding of proton halos and their reaction rates.

★ ‘Deblurring’ the experimental data for nuclei, for a better resolution and theoretical interpretation.

★ Studying the ‘island of inversion’ halos to try and predict their importance in astrophysical scenarios such as nucleosynthesis. Neural networks??

★ Need for a relativistic breakup reaction theory. Eikonal-FRDWBA??

Collaborators:

L. Fortunato
A. Vitturi
M. Kimura
J. Casal
M. Dan
W. Horiuchi
A. Moro
Manju
N. Keeley
M. Rodriguez-Gallardo
Shubhchintak
K. W. Kemper
R. Chatterjee
Jagjit Singh